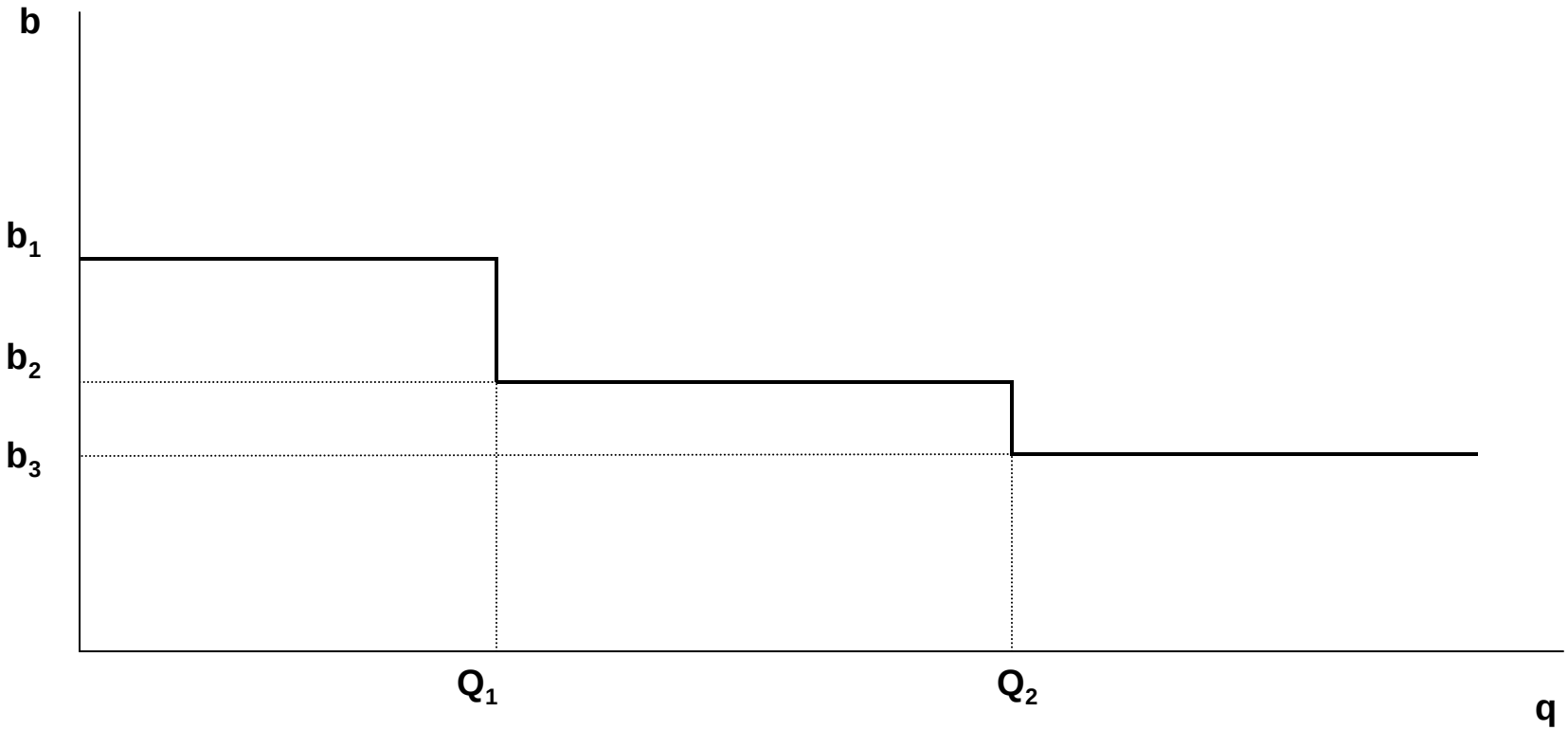
PARÁMETROS VARIABLES CON CANTIDAD A ADQUIRIR

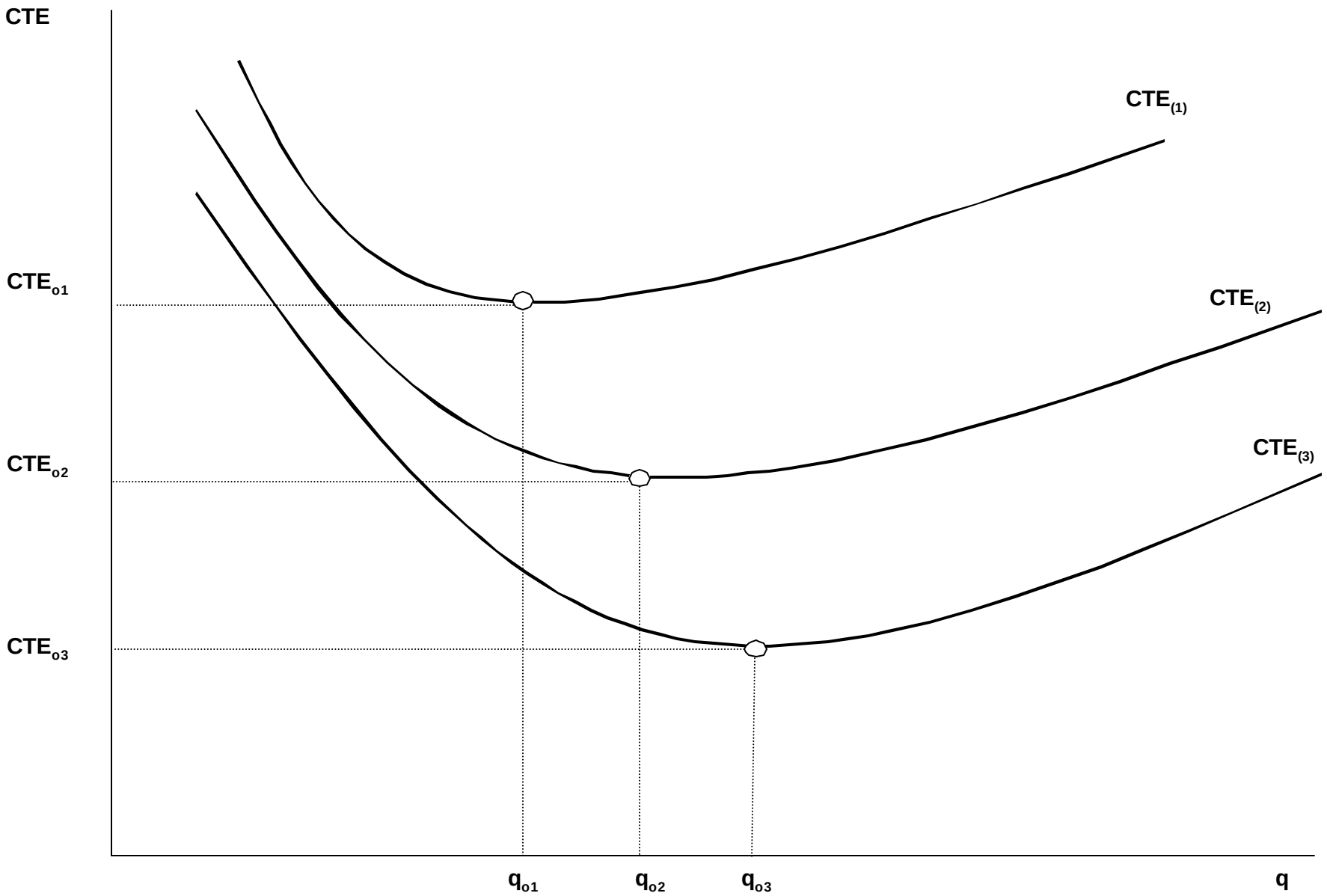

b
b1
b2
b3
Q1
Q2
q

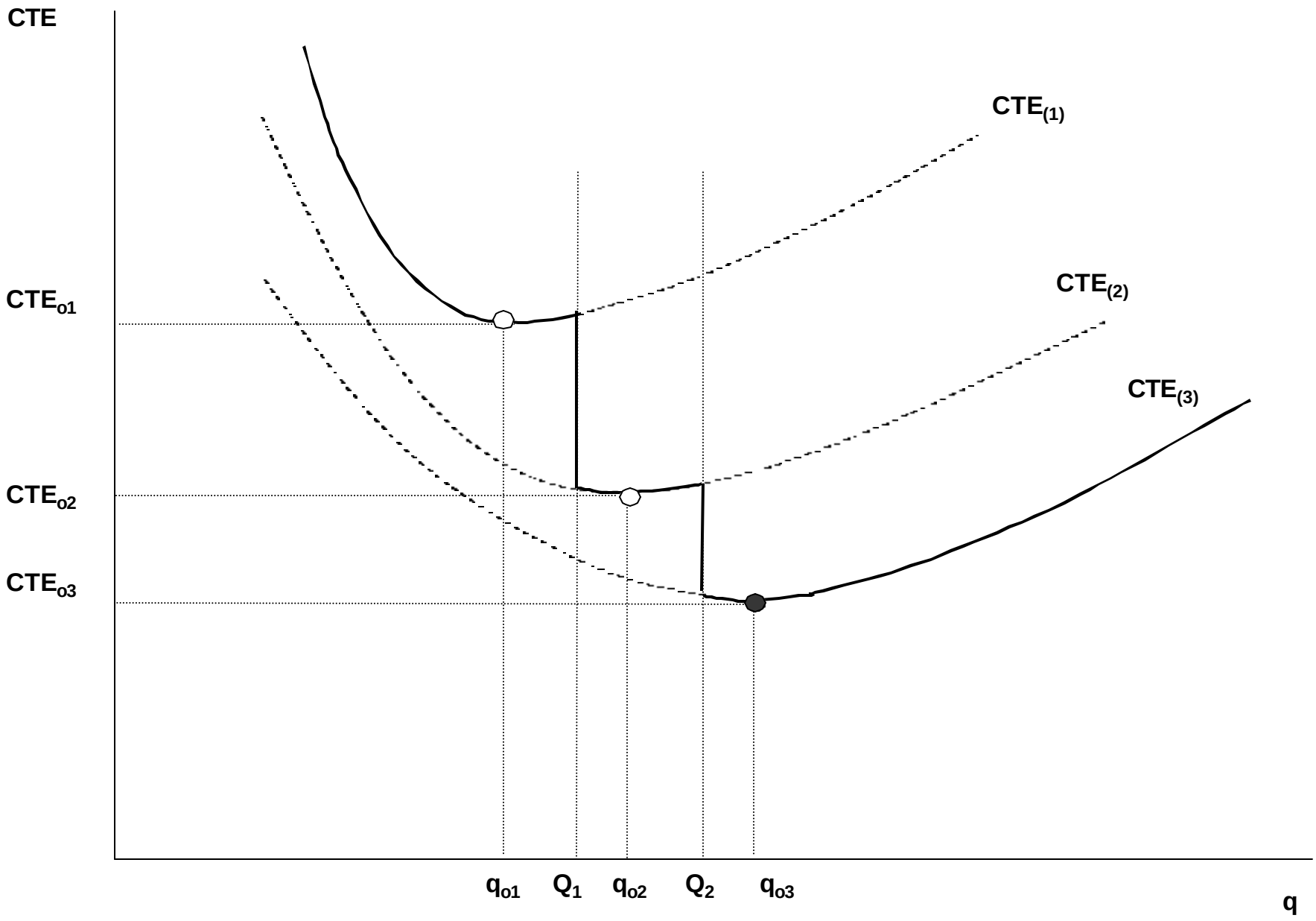
$$b_1 > b_2 > b_3$$

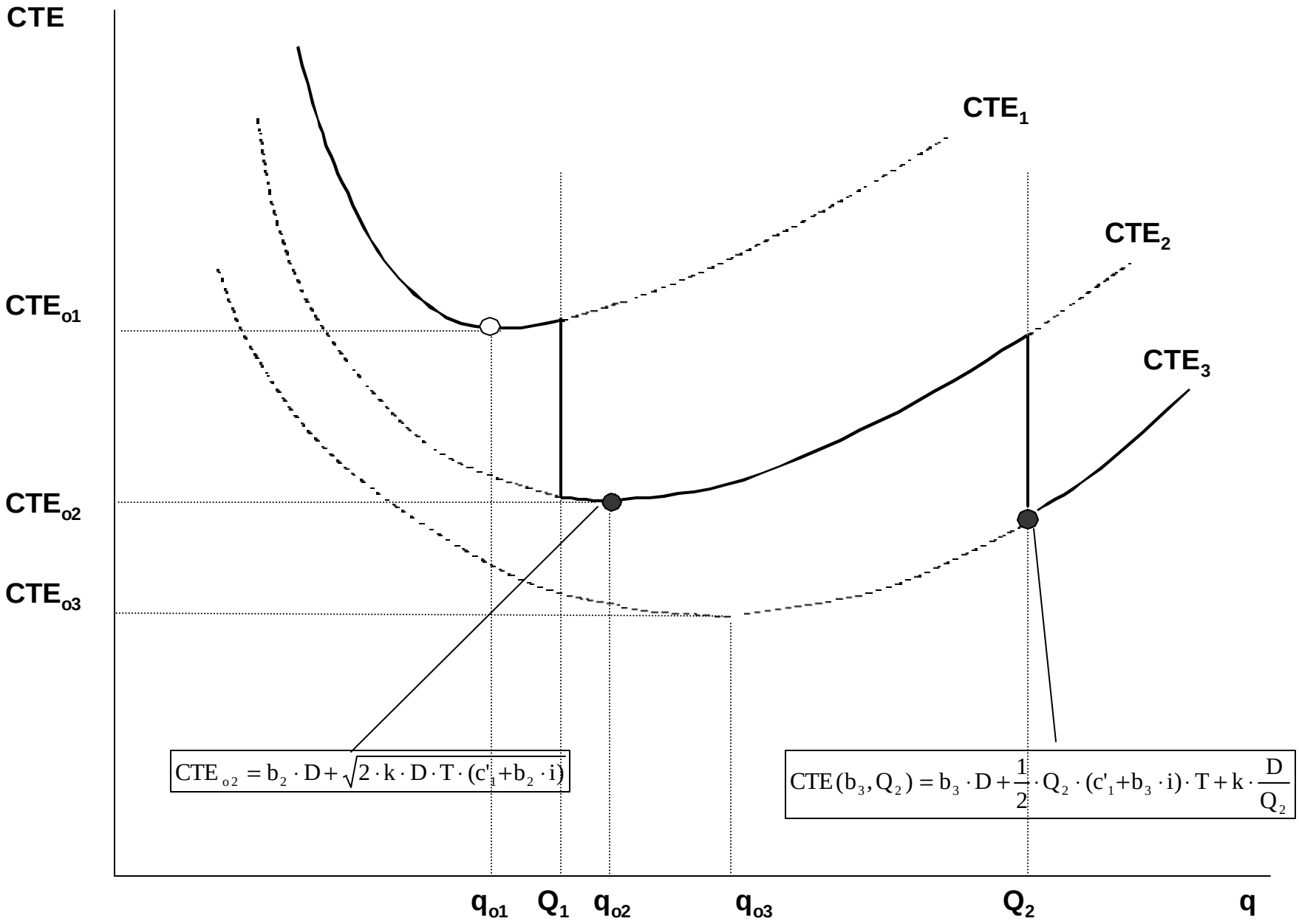
$$CTE_{(i)} = b_i \cdot D + \frac{1}{2} \cdot q \cdot (c'_1 + b_i \cdot i) \cdot T + k \cdot \frac{D}{q} \quad \Rightarrow \quad CTE_{(1)} > CTE_{(2)} > CTE_{(3)}$$

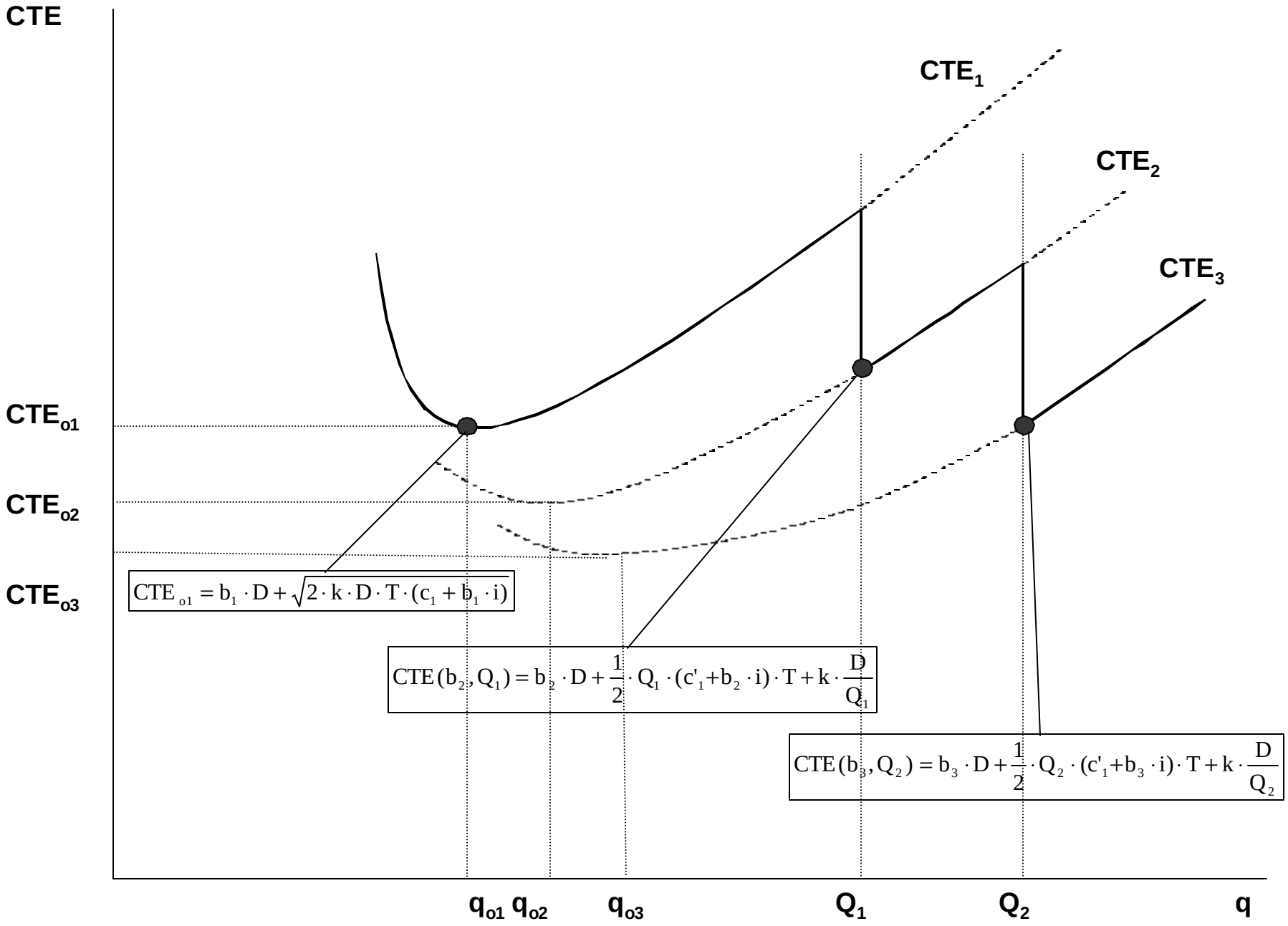
$$q_{oi} = \sqrt{\frac{2 \cdot k \cdot D}{T \cdot (c'_1 + b_i \cdot i)}} \quad \Rightarrow \quad q_{o1} < q_{o2} < q_{o3}$$

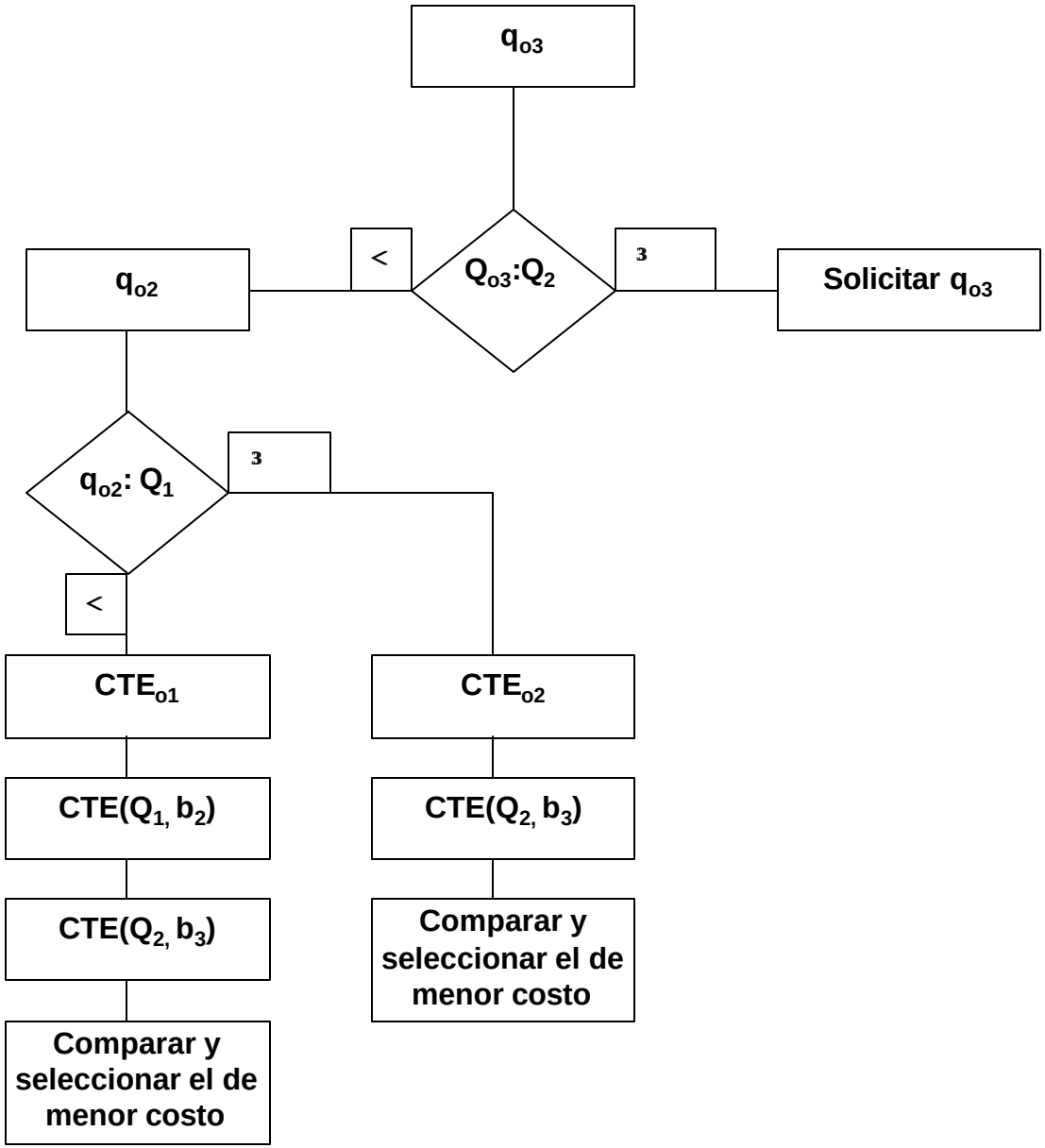
$$CTE_{oi} = b \cdot D + \sqrt{2 \cdot k \cdot D \cdot T \cdot (c'_1 + b_i \cdot i)} \quad \Rightarrow \quad CTE_{o1} > CTE_{o2} > CTE_{o3}$$

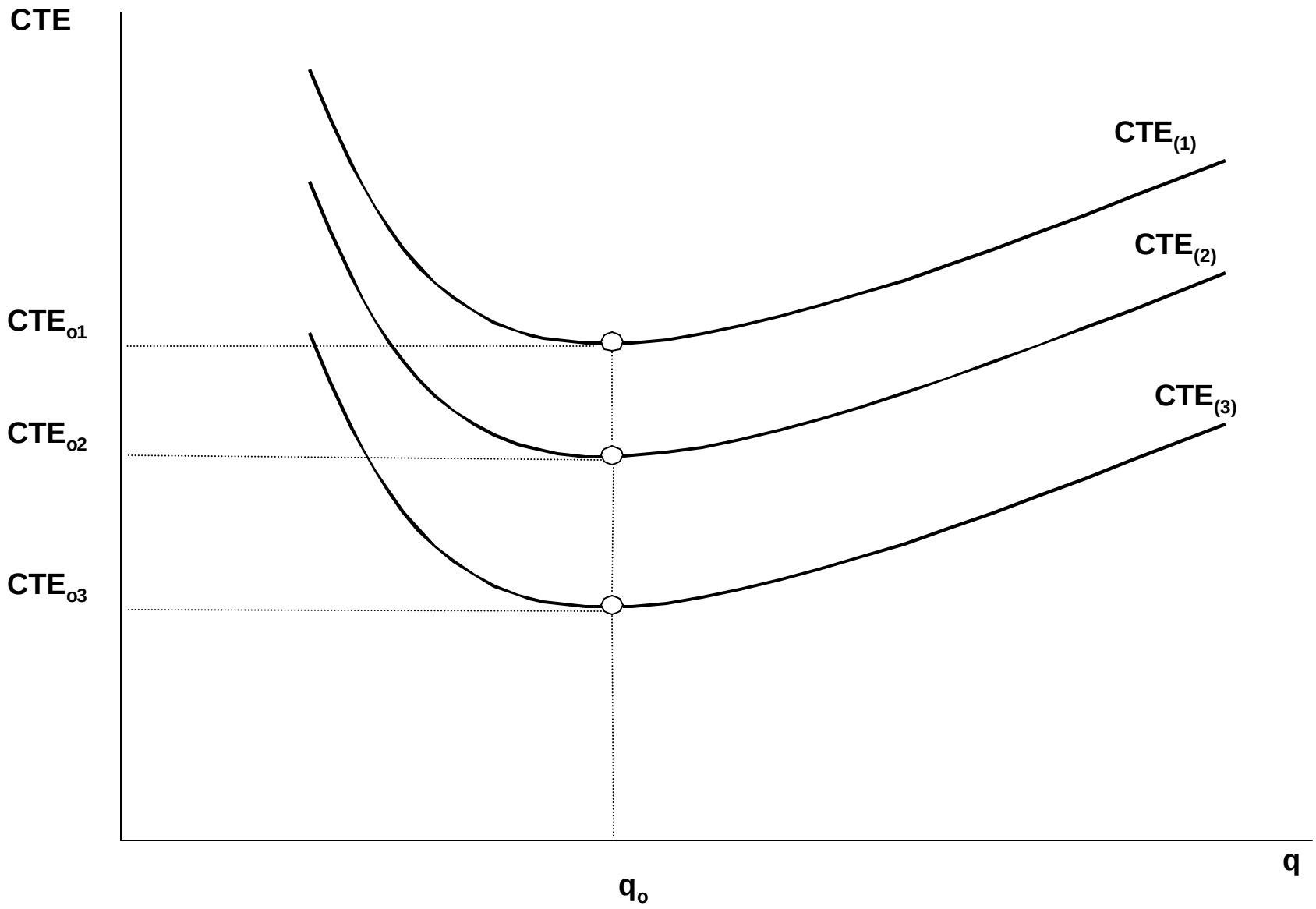

CTE
CTE(1)
CTE(2)
CTE(3)
CTEo1
CTEo2
CTEo3
qo1
qo2
qo3
q

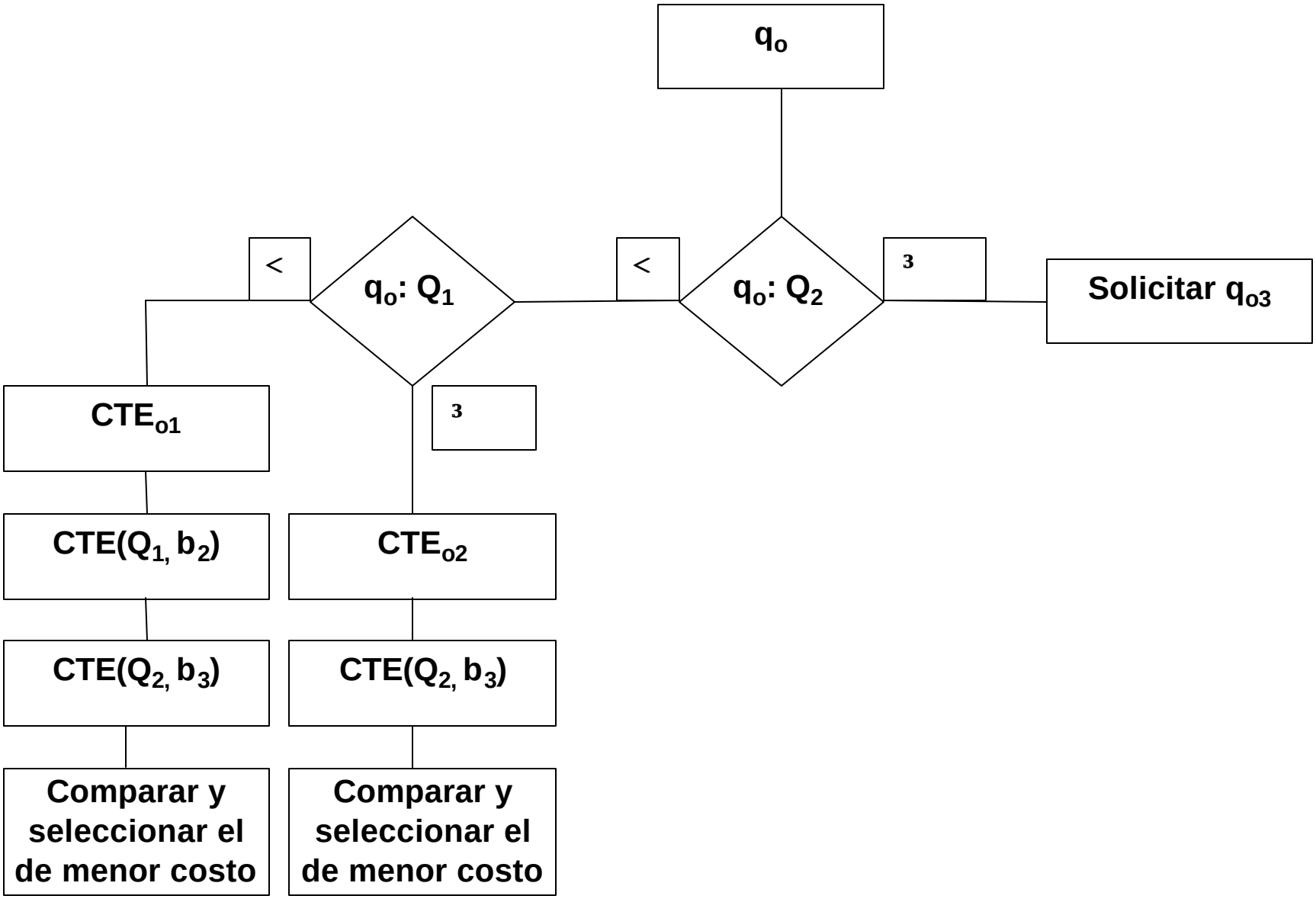

CTE
CTE(1)
CTE(2)
CTE(3)
CTEo1
CTEo2
CTEo3
qo1
Q1
qo2
Q2
qo3
q


CTE
CTE1
CTE2
CTE3
CTEo1
CTEo2
CTEo3
qo1
Q1
qo2
qo3
Q2
q
$CTE_{o2} = b_2 \cdot D + \sqrt{2 \cdot k \cdot D \cdot T \cdot (c'_1 + b_2 \cdot i)}$
$CTE(b_3, Q_2) = b_3 \cdot D + \frac{1}{2} \cdot Q_2 \cdot (c'_1 + b_3 \cdot i) \cdot T + k \cdot \frac{D}{Q_2}$


CTE
CTE1
CTE2
CTE3
CTEo1
CTEo2
CTEo3
CTE_{o1} = b_1 · D + √(2 · k · D · T · (c_1 + b_1 · i))
CTE(b_2, Q_1) = b_2 · D + 1/2 · Q_1 · (c'_1 + b_2 · i) · T + k · D/Q_1
CTE(b_3, Q_2) = b_3 · D + 1/2 · Q_2 · (c'_1 + b_3 · i) · T + k · D/Q_2
qo1 qo2
qo3
Q1
Q2
q


q_{o3}
Q_{o3}:Q_2
<
3
q_{o2}
Solicitar q_{o3}
q_{o2}: Q_1
3
<
CTE_{o1}
CTE_{o2}
$CTE(Q_1, b_2)$
$CTE(Q_2, b_3)$
$CTE(Q_2, b_3)$
Comparar y seleccionar el de menor costo
Comparar y seleccionar el de menor costo

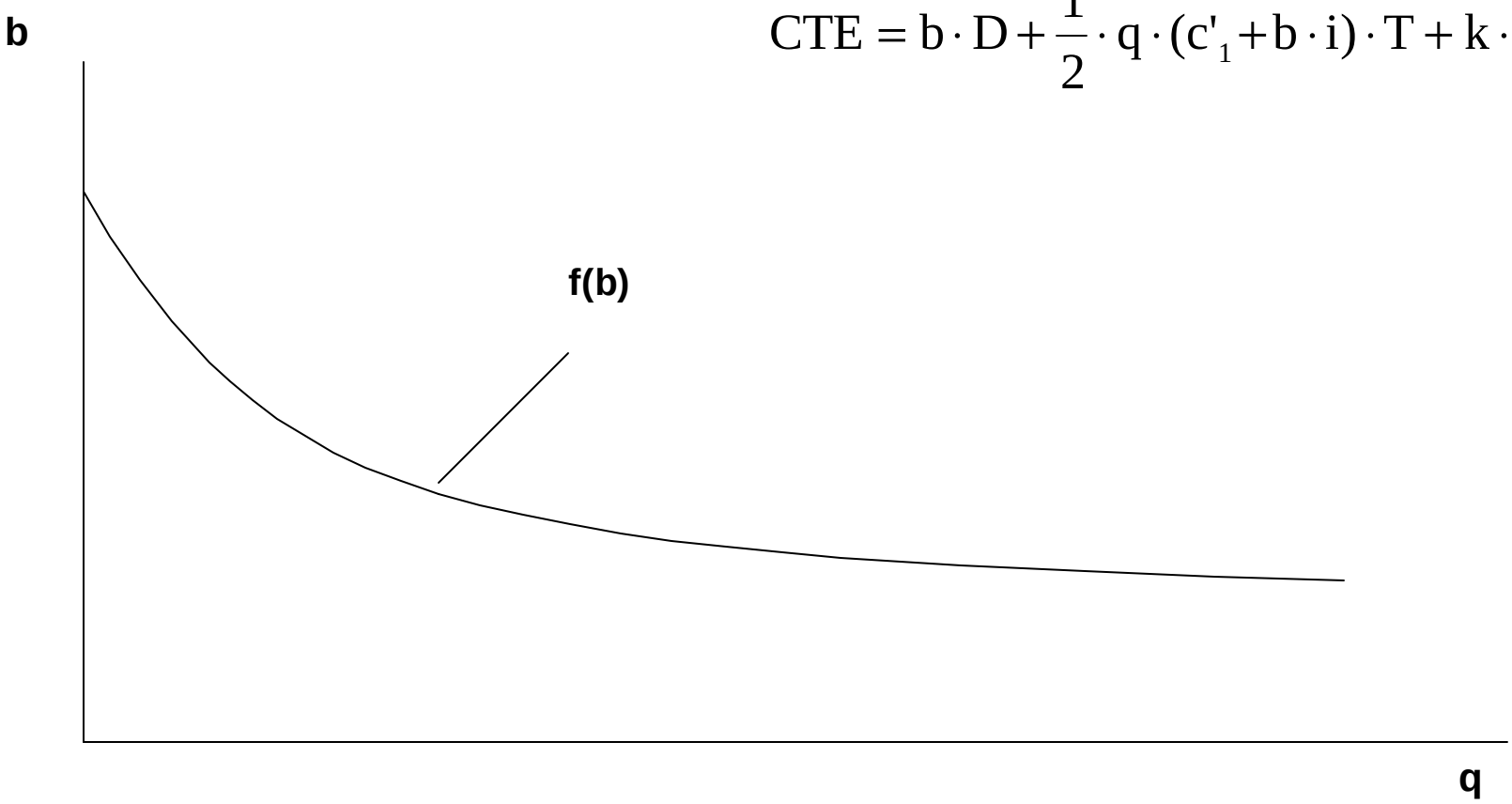

CTE
CTE(1)
CTE(2)
CTE(3)
CTEo1
CTEo2
CTEo3
q
qo


q_o
<
q_o: Q_1
<
q_o: Q_2
3
Solicitar q_{o3}
CTE_{o1}
3
$CTE(Q_1, b_2)$
CTE_{o2}
$CTE(Q_2, b_3)$
$CTE(Q_2, b_3)$
Comparar y seleccionar el de menor costo
Comparar y seleccionar el de menor costo

```
MIN = CTE;
CTE = + b*D + 1/2 *(q*(1 - d/p) - Sa)^2/(q*(1 - d/p))*c1*T + 1/2 * Sa^2 / (q*(1 - d/p))*c2*T
+ k * D / q + Sa * f2 * D/q + F * D / q + Sp * c1 * T;
c1 = c1op + i*b;
T = 1;
n = D / q;
S = q * (1 - d/p) - Sa;
Smax = S + Sp;
ti = t1 + t2;
t1 = t1d + t1p;
t2 = (t2d + t2p);
ti = q/D * T;
t1p = S / (p - d);
t1d = S / d;
t2p = Sa / (p - d);
t2d = Sa / d;
tp = t1p + t2p;
td = t1d + t2d;
c1 = c1op + i * b;
! Descuento por cantidad;
q = q1 + q2 + q3;
b = b1 * I1 + b2 * I2 + b3 * I3;
q1 < Ql * I1;
q2 > Ql * I2;
q2 < QII * I2;
q3 > QII * I3;
q3 < QIII * I3;
I1 + I2 + I3 = 1;
@BIN( I1);
@BIN( I2);
@BIN( I3);
```

$$b = \frac{A}{B + q}$$

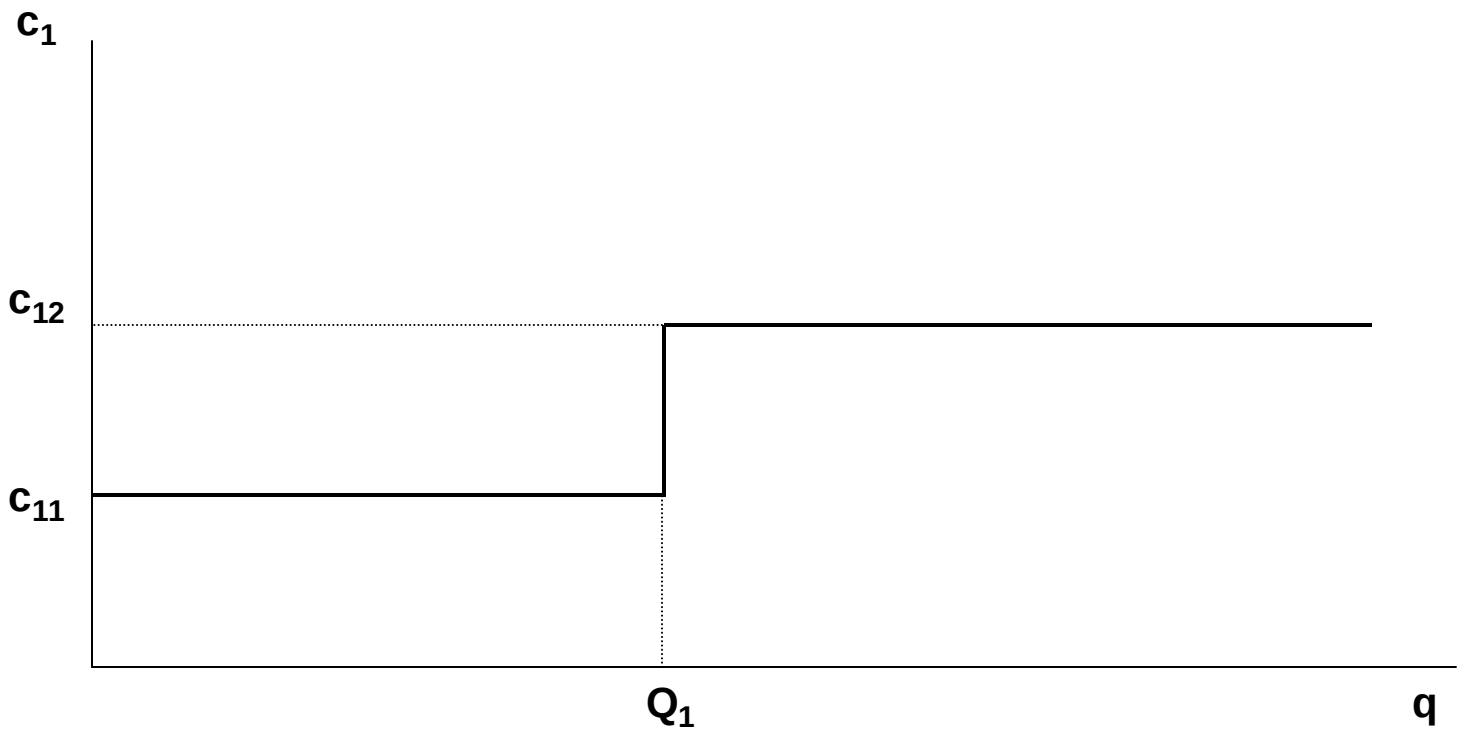
$$CTE = b \cdot D + \frac{1}{2} \cdot q \cdot (c'_1 + b \cdot i) \cdot T + k \cdot \frac{D}{q}$$

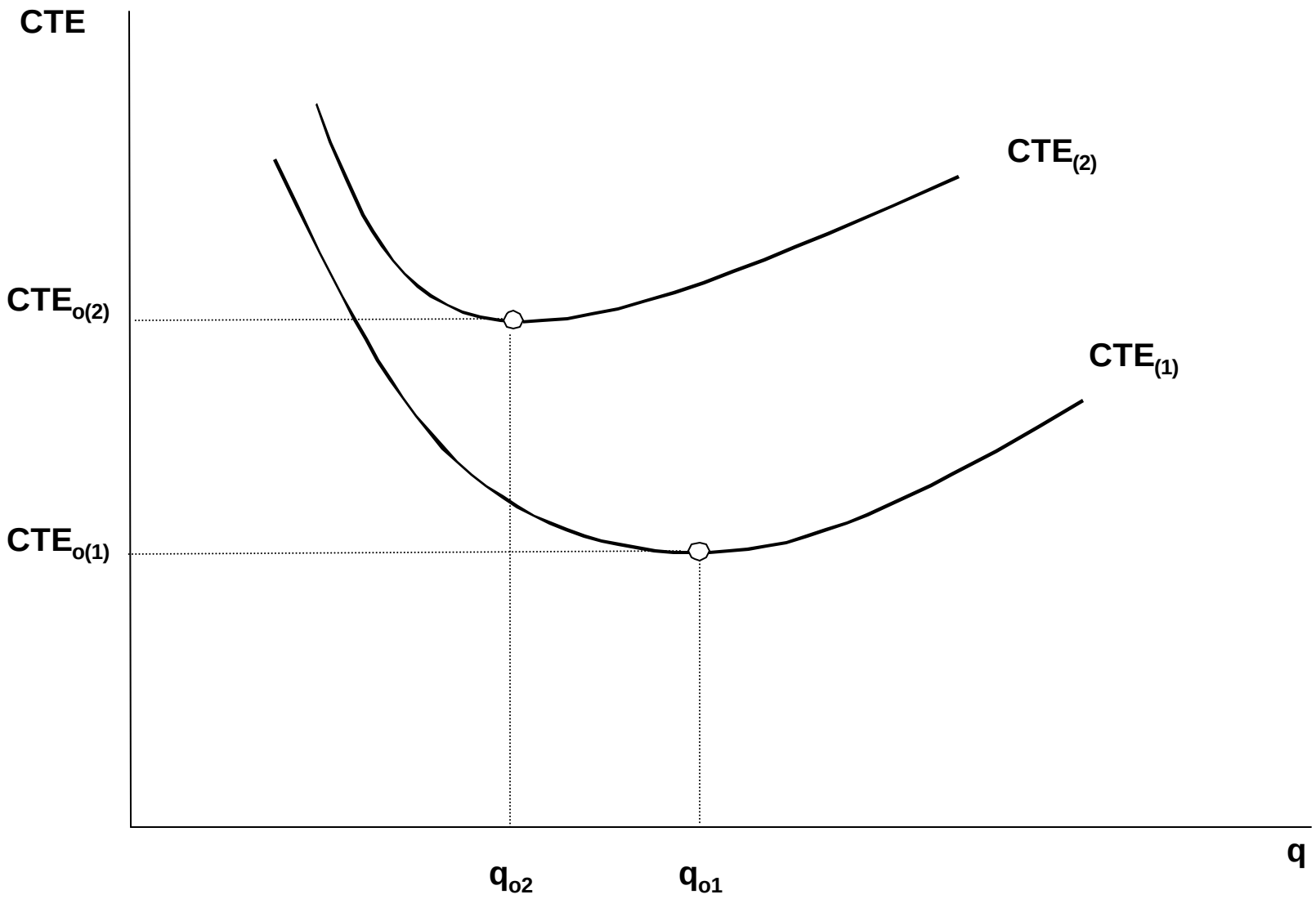


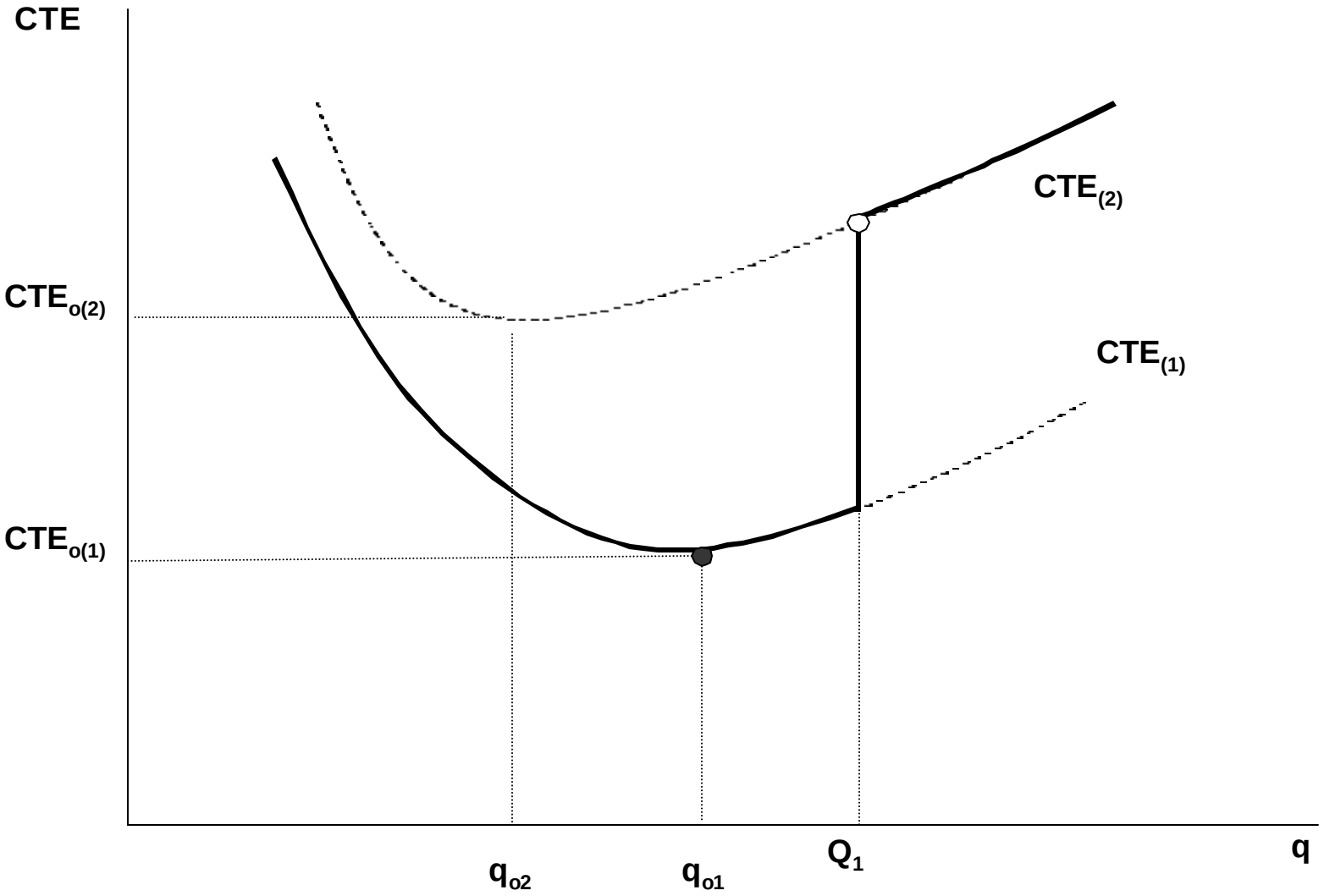
$$CTE_{(i)} = b \cdot D + \frac{1}{2} \cdot q \cdot c_{1i} \cdot T + k \cdot \frac{D}{q} \quad \Rightarrow \quad CTE_{(1)} < CTE_{(2)}$$

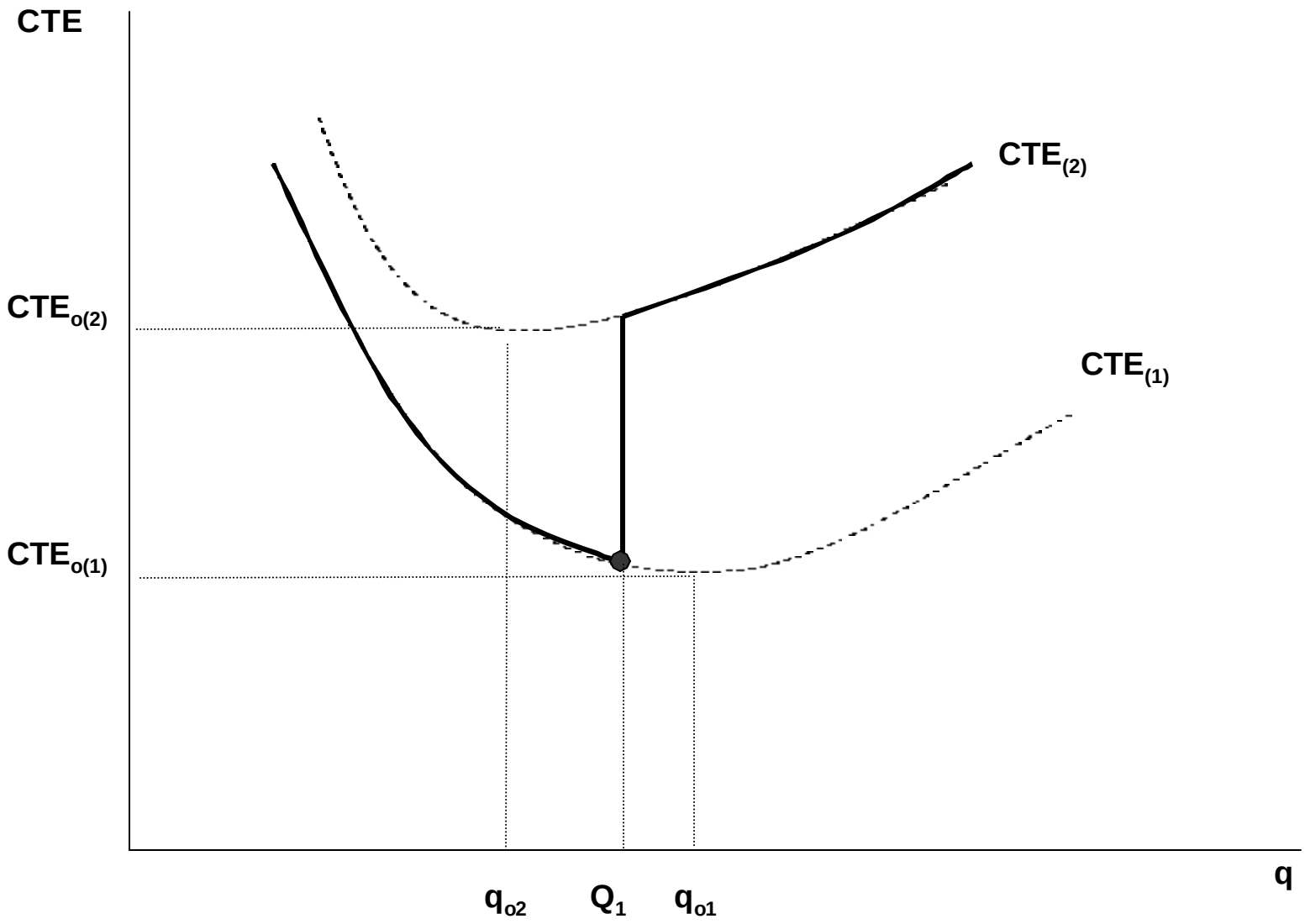
$$q_{oi} = \sqrt{\frac{2 \cdot k \cdot D}{T \cdot c_{1i}}} \quad \Rightarrow \quad q_{o1} > q_{o2}$$

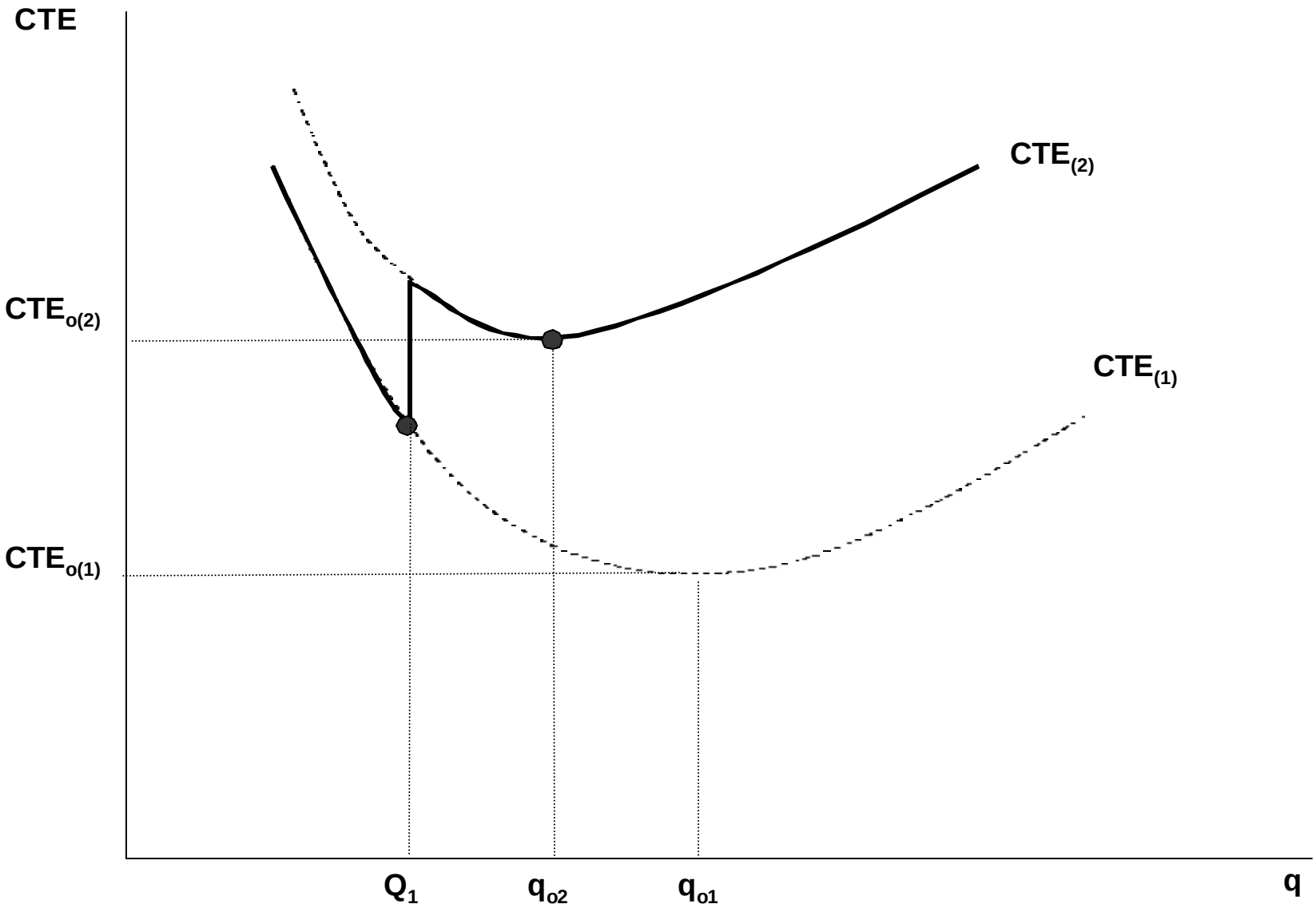
$$CTE_{o(i)} = b \cdot D + \sqrt{2 \cdot k \cdot D \cdot T \cdot c_{1i}} \quad \Rightarrow \quad CTE_{o(1)} < CTE_{o(2)}$$




CTE
CTE(2)
CTEo(2)
CTE(1)
CTEo(1)
q
qo2
qo1


CTE
CTE(2)
CTEo(2)
CTE(1)
CTEo(1)
qo2
qo1
Q1
q


CTE
CTE(2)
CTEo(2)
CTE(1)
CTEo(1)
q
qo2
Q1
qo1


CTE
CTE(2)
CTEo(2)
CTE(1)
CTEo(1)
Q1
qo2
qo1
q

$$CTE_{(i)} = b \cdot D + \frac{1}{2} \cdot q \cdot c_1 \cdot T + k_i \cdot \frac{D}{q}$$

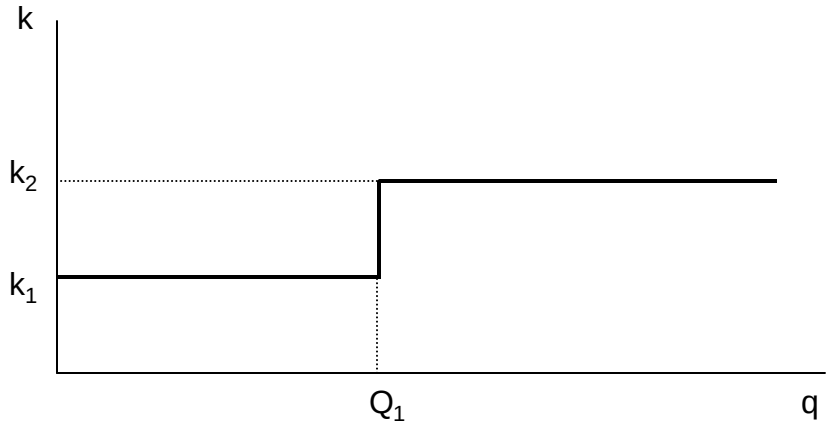
CTE(1) < CTE(2)

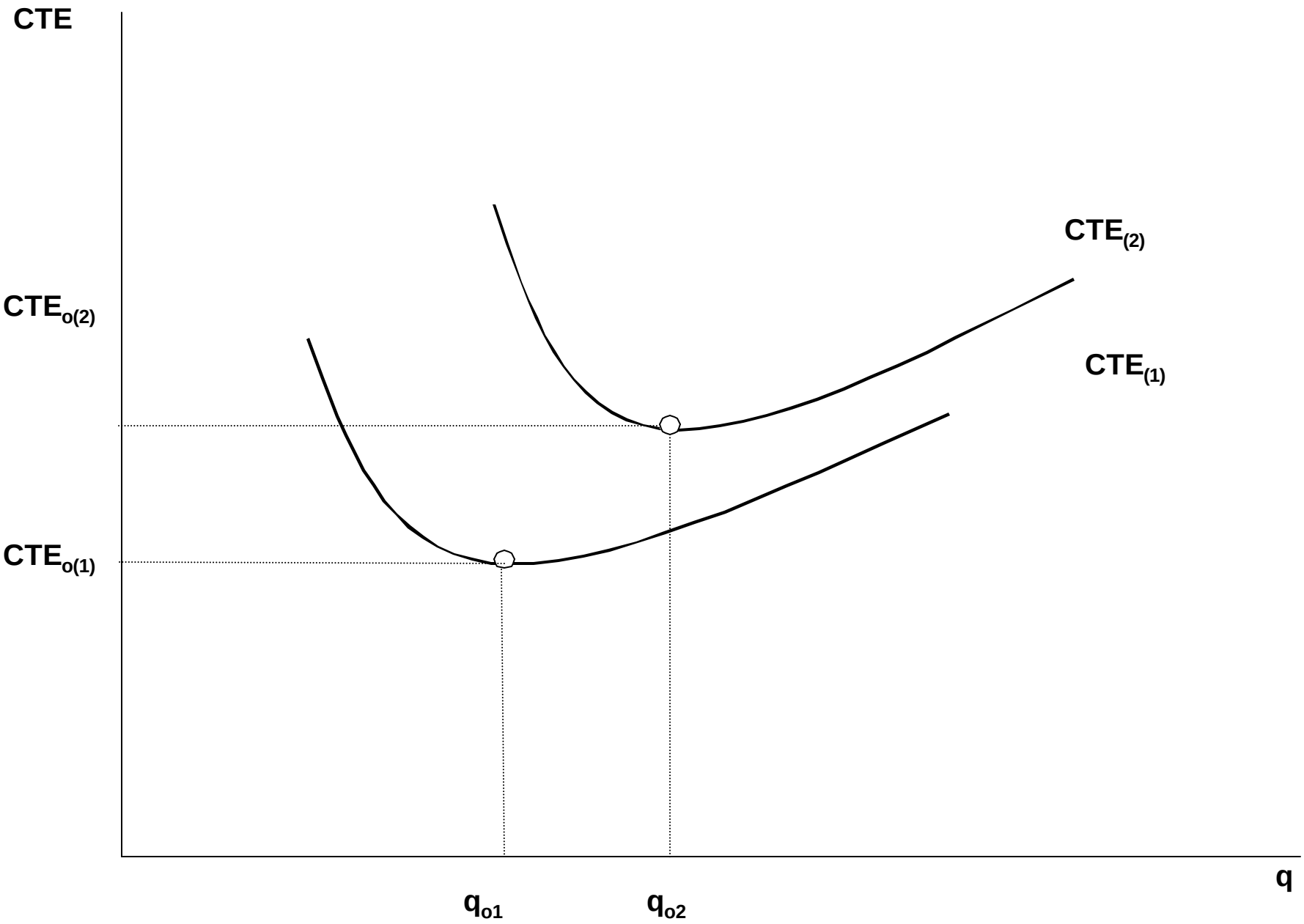
$$q_{oi} = \sqrt{\frac{2 \cdot k_i \cdot D}{T \cdot c_i}}$$

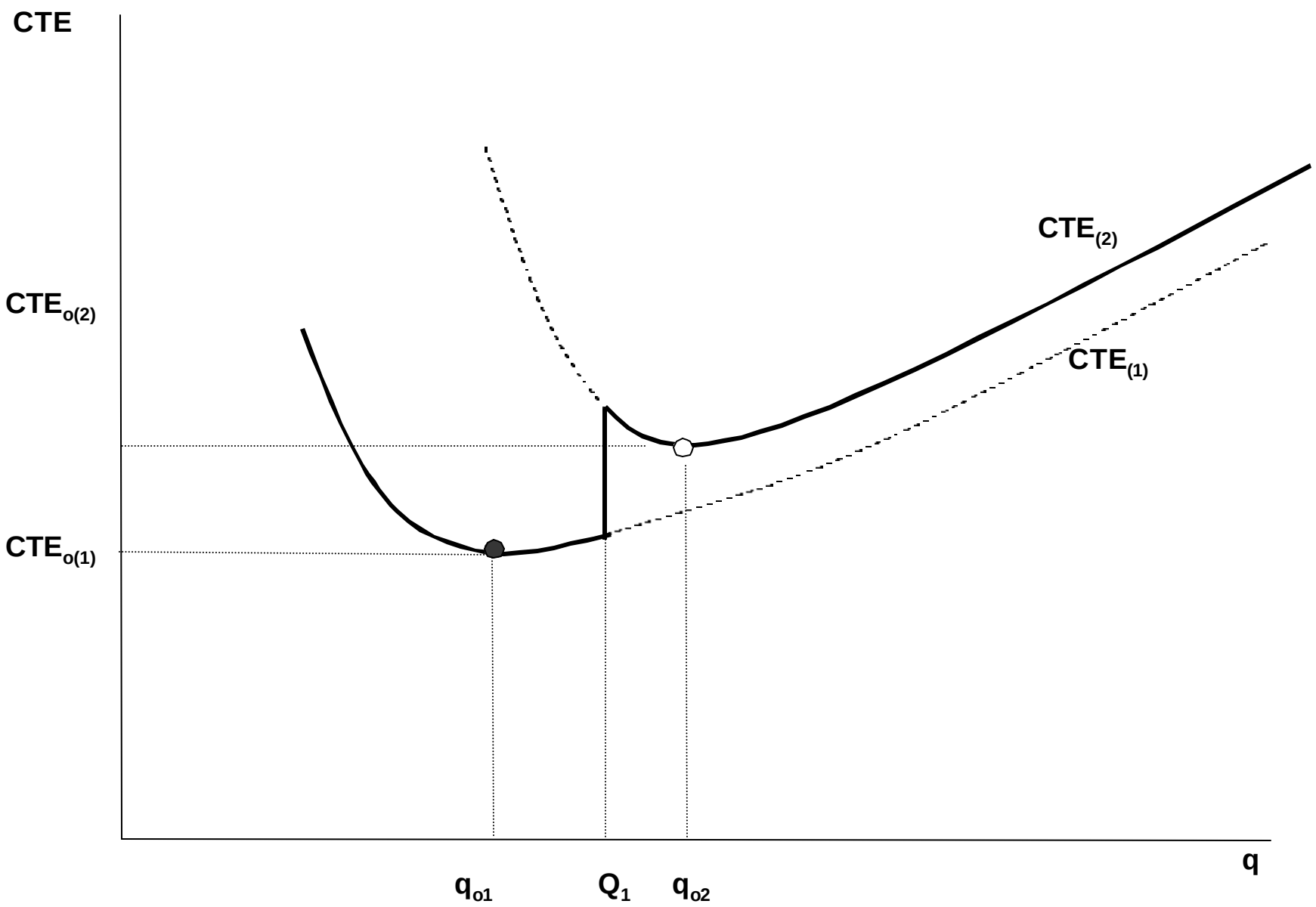
$q_o1 < q_o2$

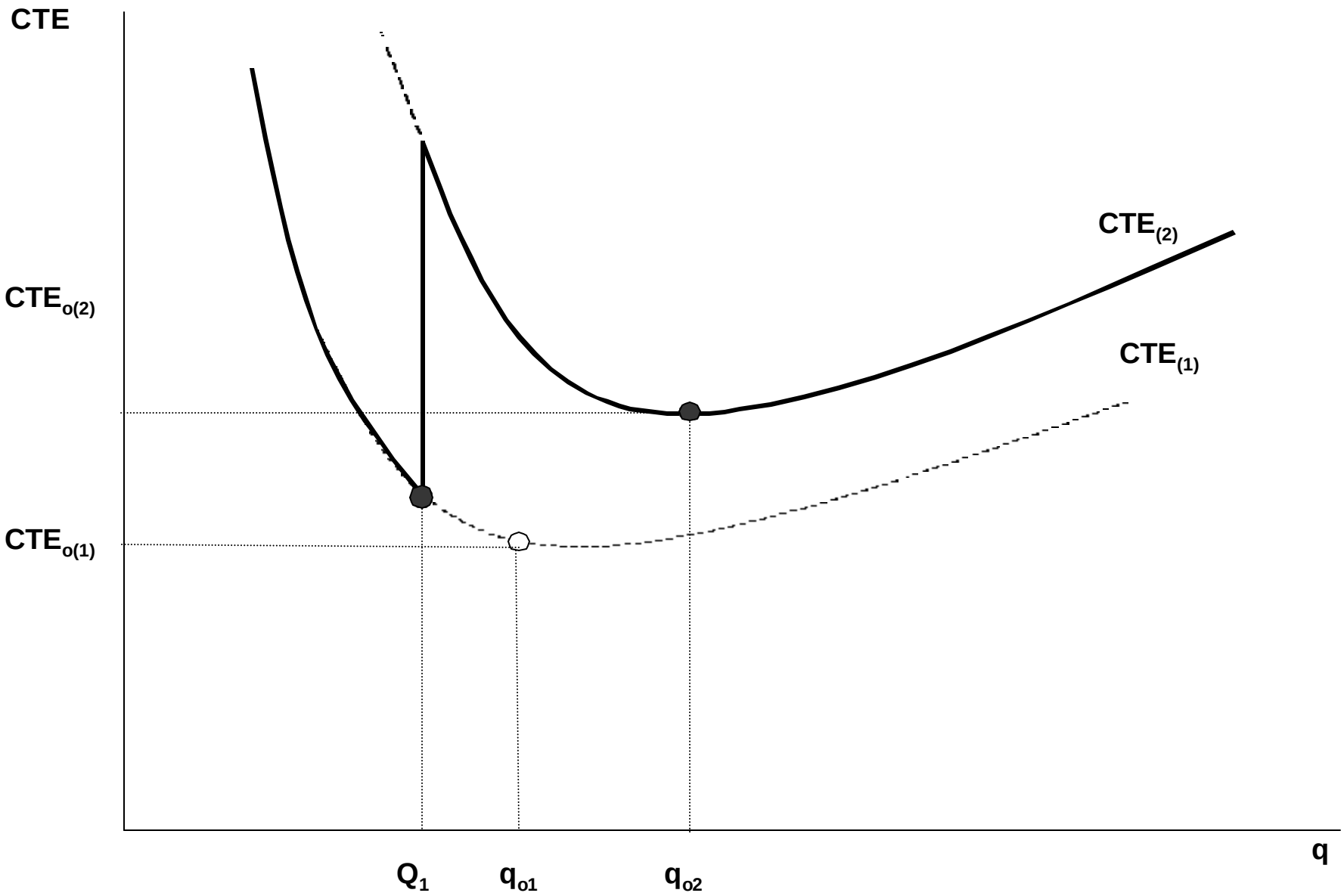
$$CTE_{o(i)} = b \cdot D + \sqrt{2 \cdot k_i \cdot D \cdot T \cdot c_1}$$

$CTE_o(1) < CTE_o(2)$




CTE
CTE(2)
CTEo(2)
CTE(1)
CTEo(1)
q
qo1
qo2


CTE
CTE(o(2))
CTE(o(1))
CTE(2)
CTE(1)
q
q(o1)
Q(1)
q(o2)


CTE
CTE(2)
CTEo(2)
CTE(1)
CTEo(1)
q
Q1
qo1
qo2

DESCUENTOS INCREMENTALES DE COSTOS DE ADQUISICIÓN

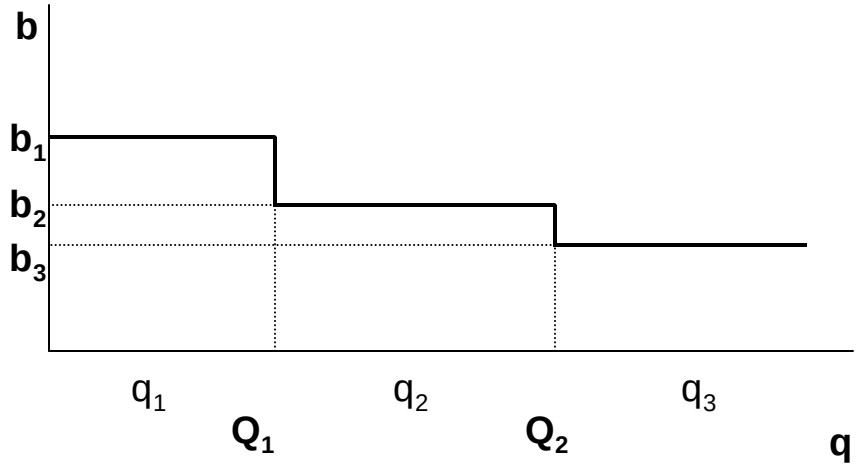
- Para una cantidad a adquirir comprendida entre 0 y Q_1, el precio de adquisición es "b_1".
- Para un lote de adquisición comprendido entre Q_1 y Q_2, el precio de adquisición es "b_1" para las primeras Q_1 unidades, y "b_2" para el resto.
- Para un lote mayor a Q_2, el precio de adquisición es b_1" para las primeras Q_1 unidades, "b_2" para las unidades comprendidas entre Q_1 y Q_2, y "b_3" para el resto.

$$q = q_1 + q_2 + q_3$$

$$\begin{cases} q_1 \le Q_1 \cdot I_1 \\ q_2 \le (Q_2 - Q_1) \cdot I_2 \\ q_3 \le (M - Q_2) \cdot I_3 \end{cases}$$

$$\begin{cases} I_1 \ge I_2 \\ q_1 \ge Q_1 \cdot I_2 \end{cases}$$

$$\begin{cases} I_2 \ge I_3 \\ q_2 \ge (Q_2 - Q_1) \cdot I_3 \end{cases}$$



$$CTE = b_1 \cdot q_1 \cdot \frac{D}{q} + b_2 \cdot q_2 \cdot \frac{D}{q} + b_3 \cdot q_3 \cdot \frac{D}{q} +$$
$$+ \frac{1}{2} \cdot q_1 \cdot (c'_1 + b_1 \cdot i) \cdot T +$$
$$+ \frac{1}{2} \cdot q_2 \cdot (c'_1 + b_2 \cdot i) \cdot T +$$
$$+ \frac{1}{2} \cdot q_3 \cdot (c'_1 + b_3 \cdot i) \cdot T + k \cdot \frac{D}{q}$$

```
MIN = CTE;
CTE = b1*q1*D/q + b2*q2*D/q + b3*q3*D/q + 0.5*q1*c11*T + 0.5*q2*c12*T +
      + 0.5*q3*c13*T + k*D/q;
c11 = c1op + i * b1;
c12 = c1op + i * b2;
c13 = c1op + i * b3;
q = q1 + q2 + q3;
q1 < QI * I1;
q1 > QI * I2;
q2 < (QII-QI) * I2;
q2 > (QII-QI) * I3;
q3 < (QIII-QII) * I3;
@BIN(I1);
@BIN(I2);
@BIN(I3);
I1>I2;
I2>I3;
n = D / q;
ti = q/D * T;
T = 1;
```

MULTI-ÍTEMS SIN RESTRICCIONES

2 PRODUCTOS “A” y “B”

$$CTE_A = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A . \frac{D_A}{q_A}$$

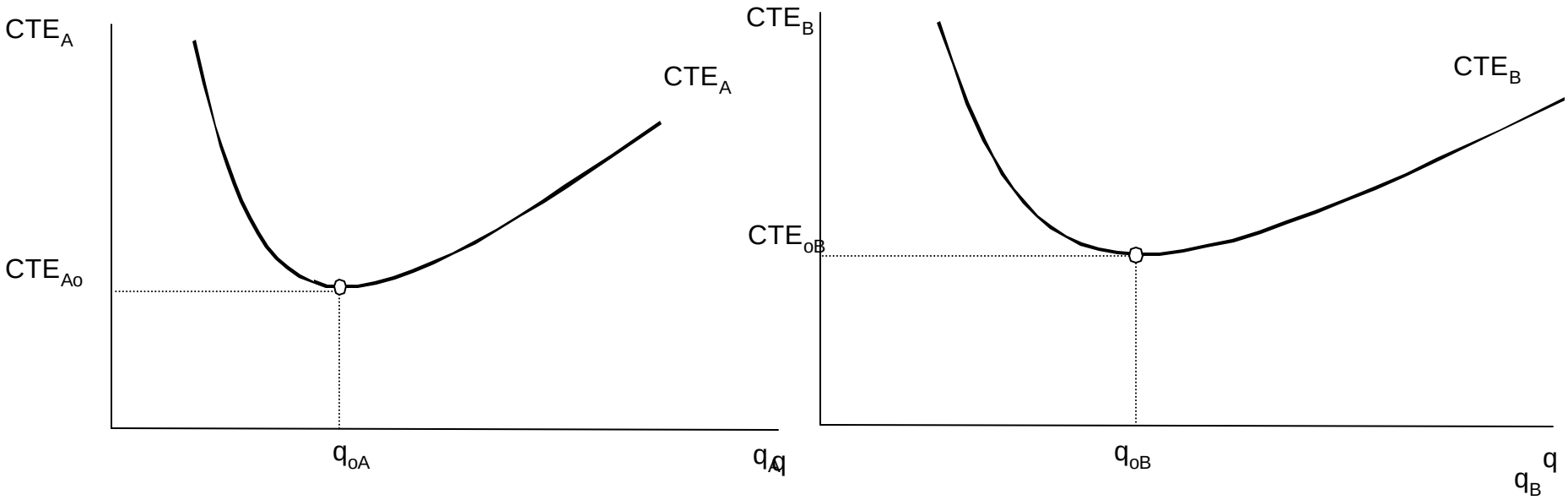
$$CTE_B = b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B}$$

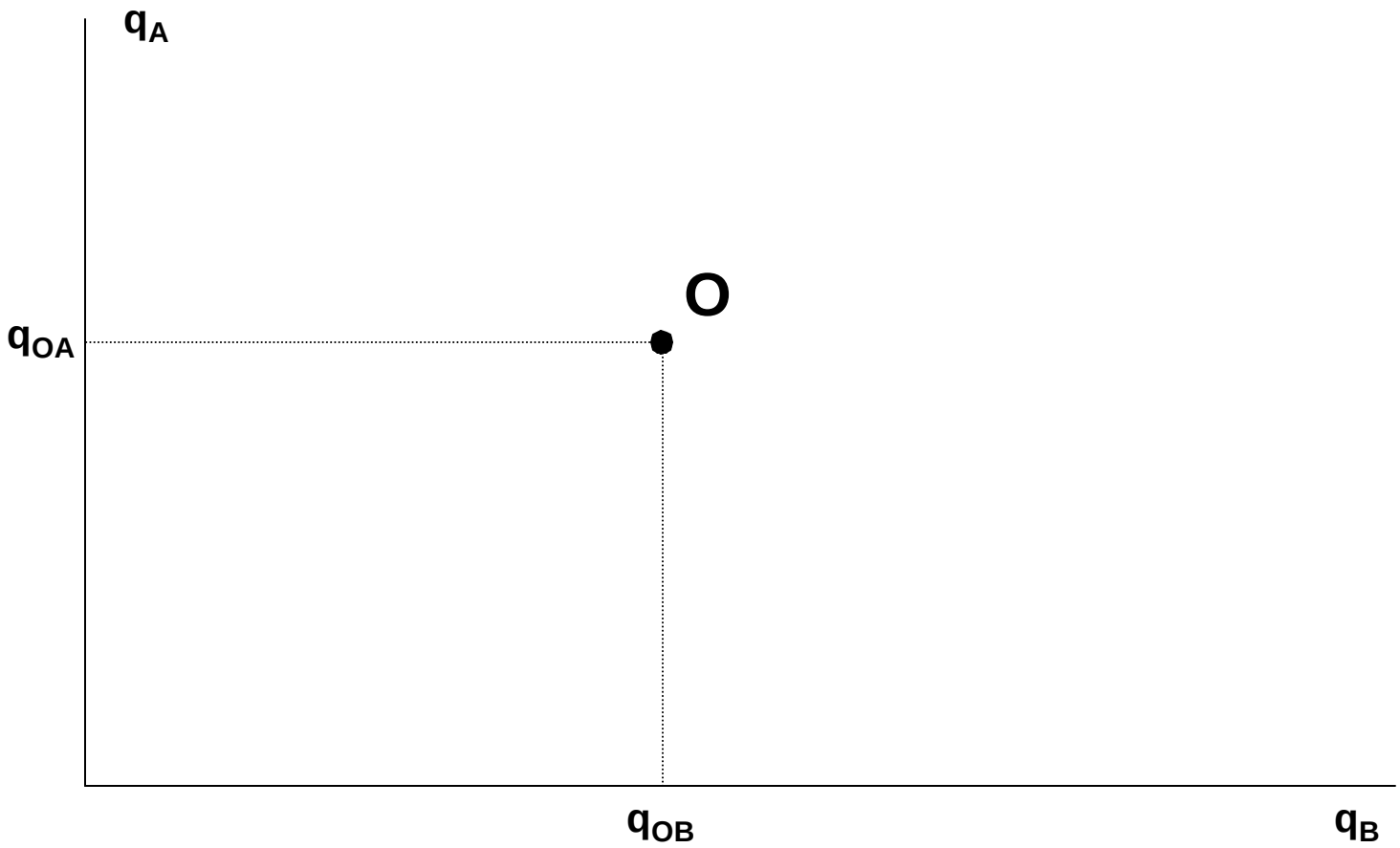
$$CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B}$$

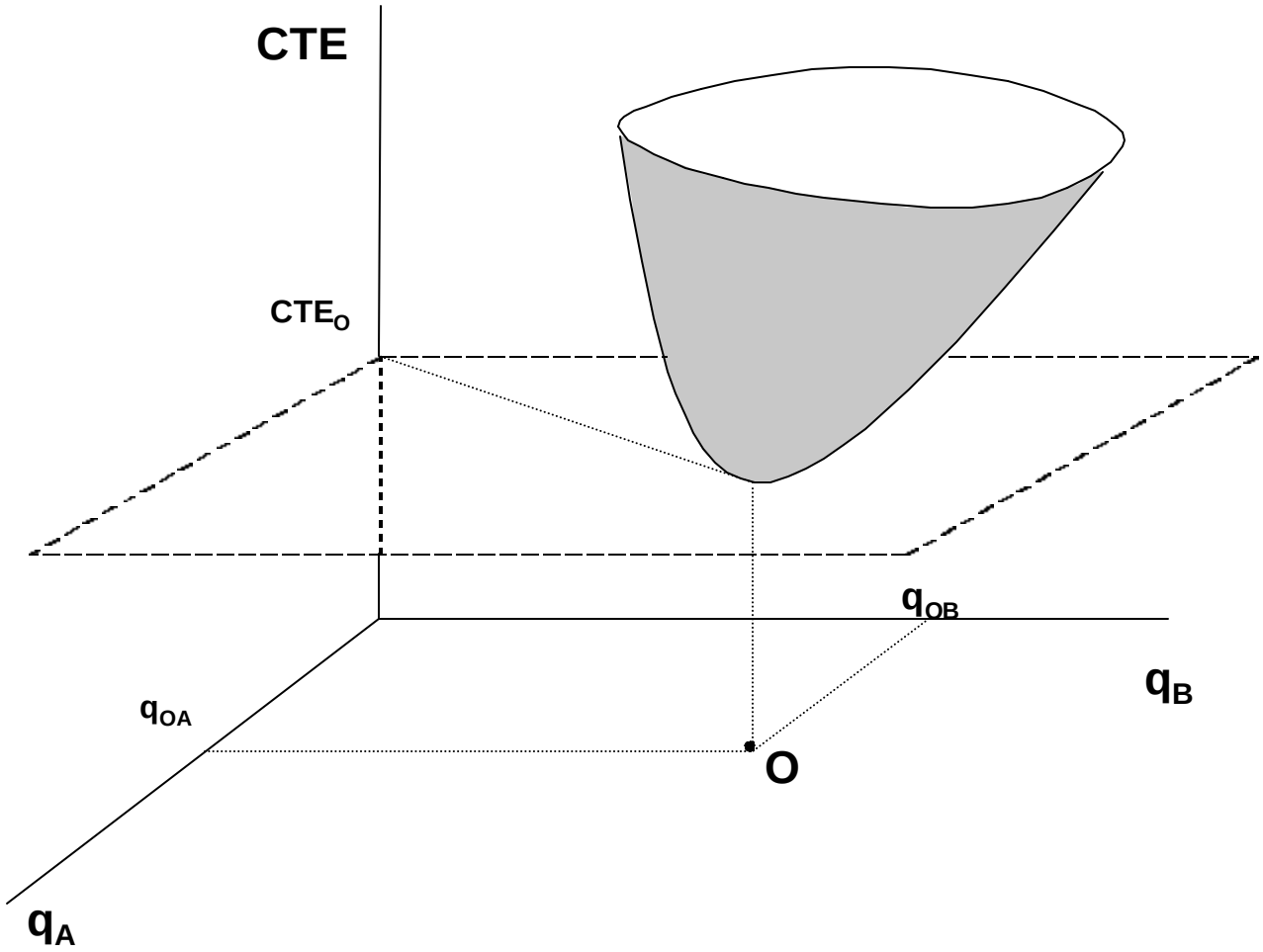
$$\frac{\partial CTE}{\partial q_A} = \frac{1}{2} \cdot c_{1A} \cdot T - k_A \cdot \frac{D_A}{q_A^2} = 0 \qquad \Rightarrow \qquad q_{oA} = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A}}}$$

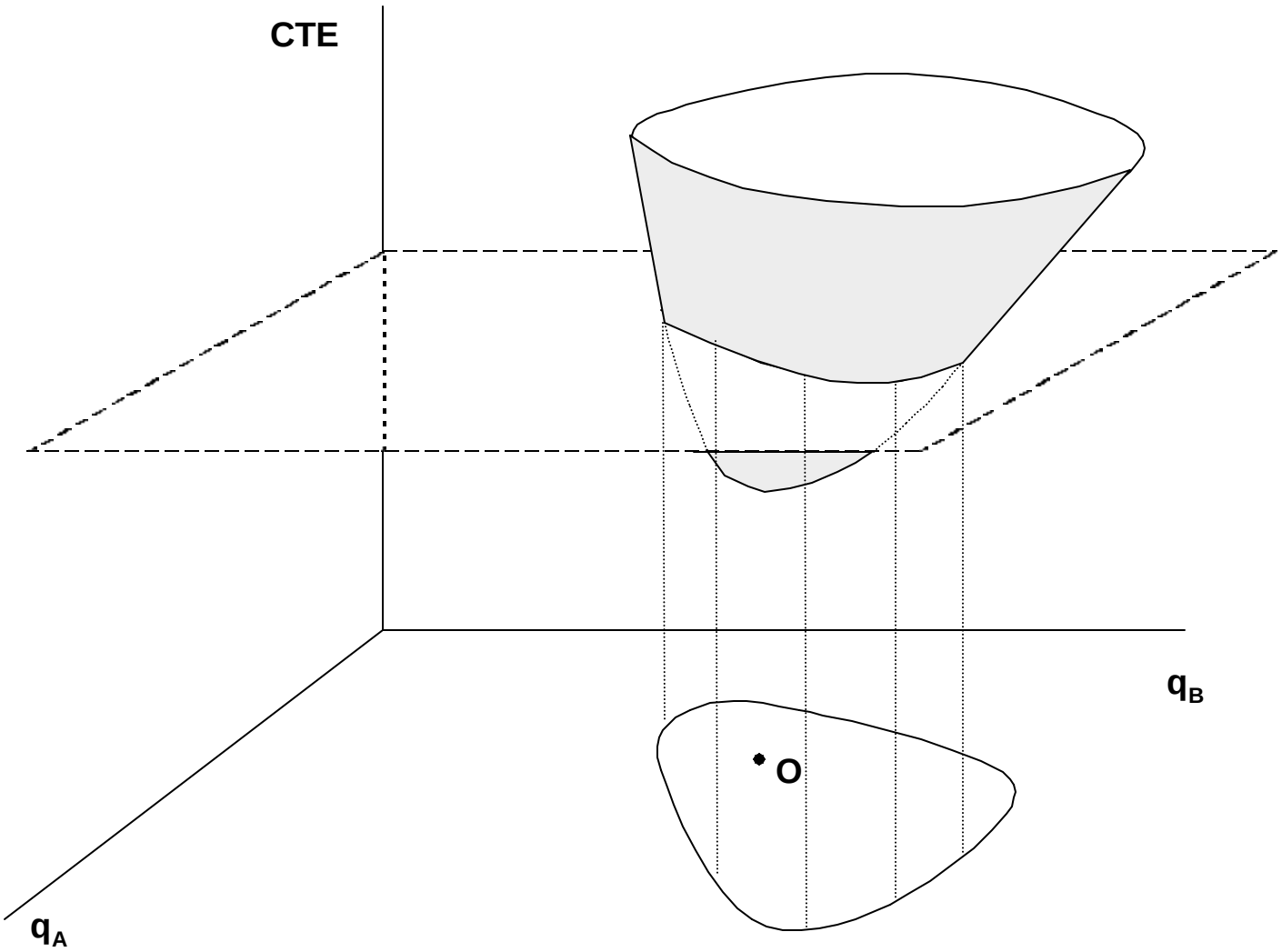
$$\frac{\partial CTE}{\partial q_B} = \frac{1}{2} \cdot c_{1B} \cdot T - k_B \cdot \frac{D_B}{q_B^2} = 0 \qquad \Rightarrow \qquad q_{oB} = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B}}}$$

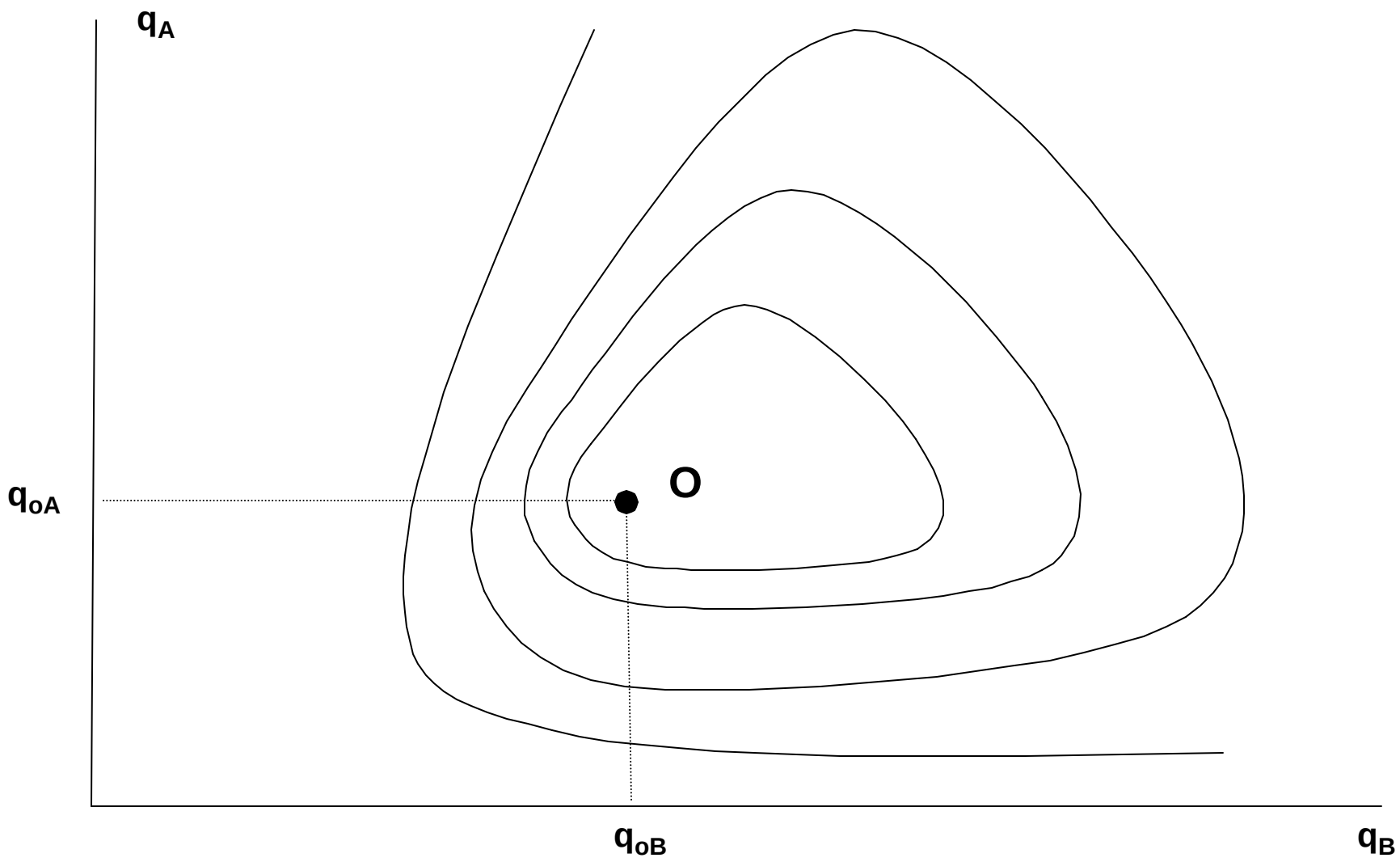
$$CTE_o = b_A \cdot D_A + \sqrt{2 \cdot k_A \cdot D_A \cdot T \cdot c_{1A}} + b_B \cdot D_B + \sqrt{2 \cdot k_B \cdot D_B \cdot T \cdot c_{1B}}$$


CTE_A
CTE_A
CTE_Ao
q_oA
q_A
q
CTE_B
CTE_B
CTE_oB
q_oB
q_B
q


q_A
O
q_{OA}
q_{OB}
q_B


CTE
CTE$_O$
q$_{OB}$
q$_B$
q$_{OA}$
O
q$_A$


CTE
q_B
O
q_A


q_A
q_{oA}
O
q_{oB}
q_B

SOLUCIÓN DE PROGRAMAS MATEMÁTICOS CON RESTRICCIONES

- RESTRICCIONES DE =
  - LAGRANGIANO
- RESTRICCIONES DE £
  - Karush-Kuhn-Tucker (KKT)

MIN (o MAX) = $f(x_1, x_2, ..., x_n)$

sujeto a:

$g_1 (x_1, x_2,..., x_n) = b_1$

$g_2 (x_1, x_2,..., x_n) = b_2$

.....

$g_m (x_1, x_2,..., x_n) = b_m$

x_i no negativas

$$L = f(x_1, x_2, ..., x_N) + \sum_{1}^{m} \lambda_i \cdot \left[g_i(x_1, x_2, ..., x_n) - b_i \right]$$

$$\frac{\partial L}{\partial x_i} = 0$$

$$\frac{\partial L}{\partial \lambda_i} = 0$$

MIN (o MAX) = $f(x_1, x_2, ..., x_n)$

sujeto a:

$g_1 (x_1, x_2,..., x_n) \leq b_1$

$g_2 (x_1, x_2,..., x_n) \leq b_2$

.....

$g_m (x_1, x_2,..., x_n) \leq b_m$

x_i no negativas

$$L = f(x_1, x_2, ..., x_N) \pm \sum_1^m \lambda_i \cdot \left[g_i(x_1, x_2, ..., x_n) - b_i \right]$$

$$\frac{\partial L}{\partial x_i} = 0$$

$$\lambda_i \cdot \left[g_i(x_1, x_2, ..., x_n) - b_i \right] = 0$$

$$\lambda_i \geq 0$$

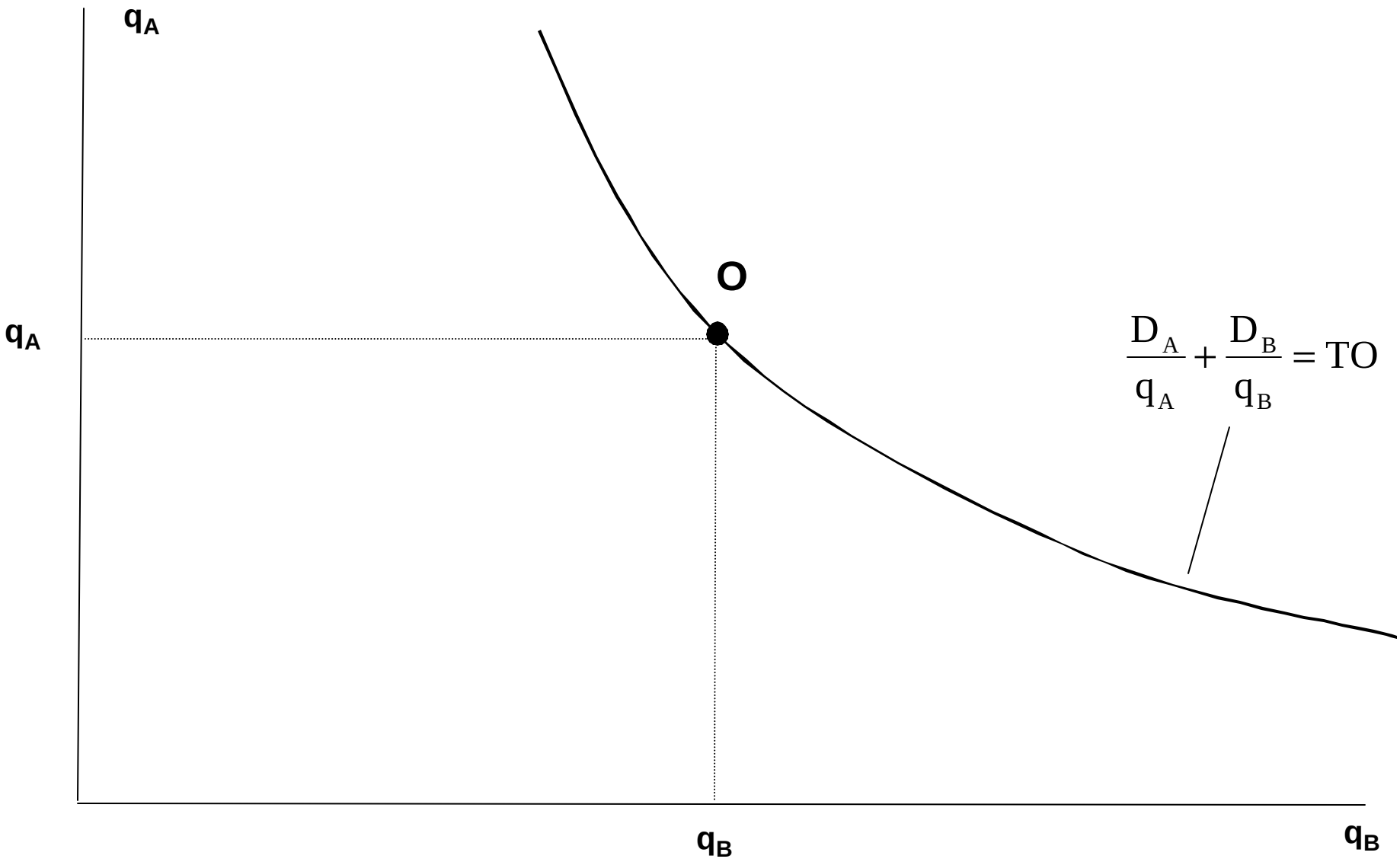
MULTI-ÍTEMS CON UNA RESTRICCIÓN DE IGUAL

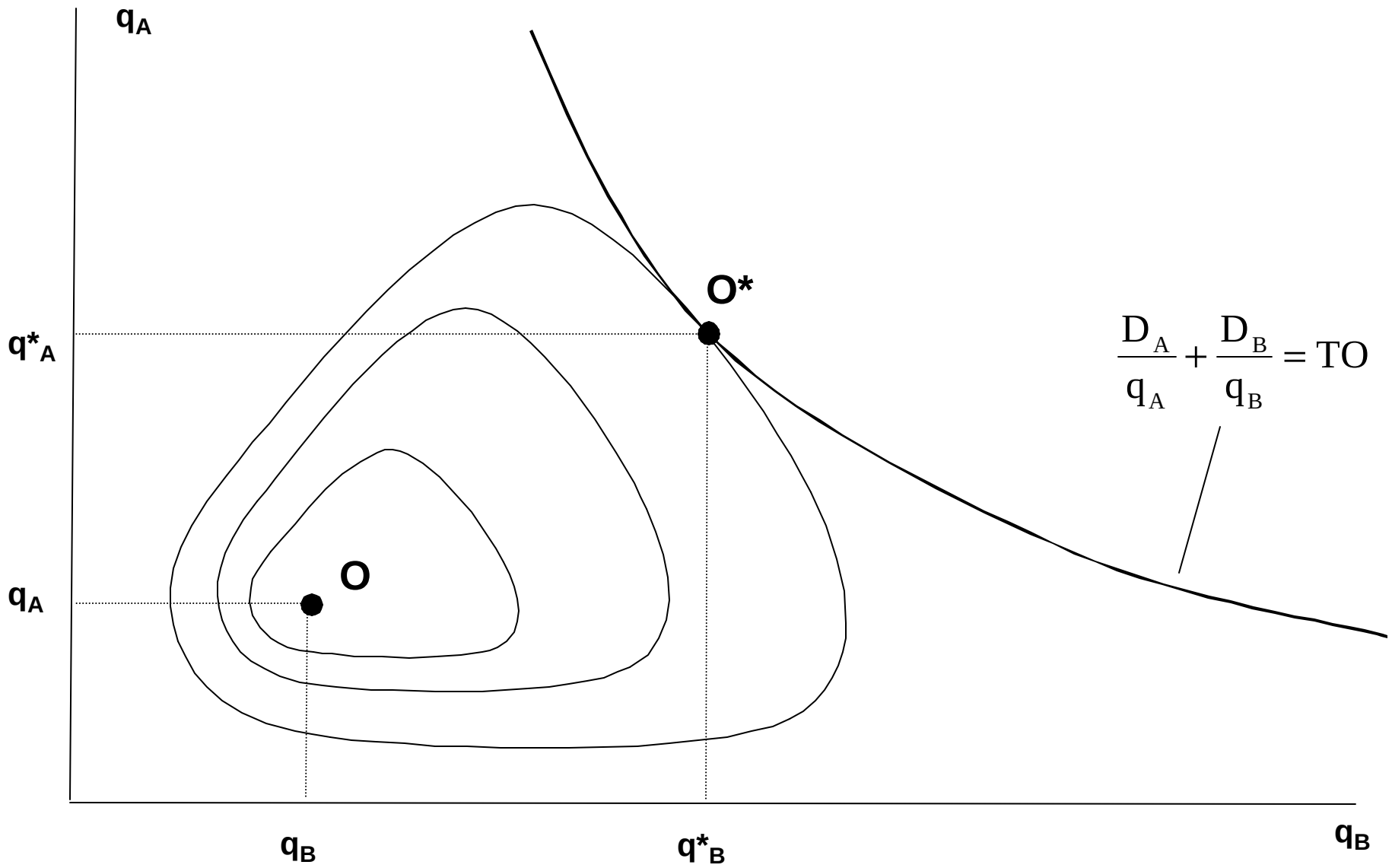
- TO: Total de órdenes a emitir entre “A” y “B” por año

$$\begin{cases} CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} \\ \frac{D_A}{q_A} + \frac{D_B}{q_B} = TO \end{cases}$$

$$q_{oA} = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A}}}$$

$$q_{oB} = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B}}}$$


q_A
O
q_A
$\frac{D_A}{q_A} + \frac{D_B}{q_B} = TO$
q_B
q_B


q_A
q^*_A
q_A
O*
O
$\frac{D_A}{q_A} + \frac{D_B}{q_B} = TO$
q_B
q^*_B
q_B

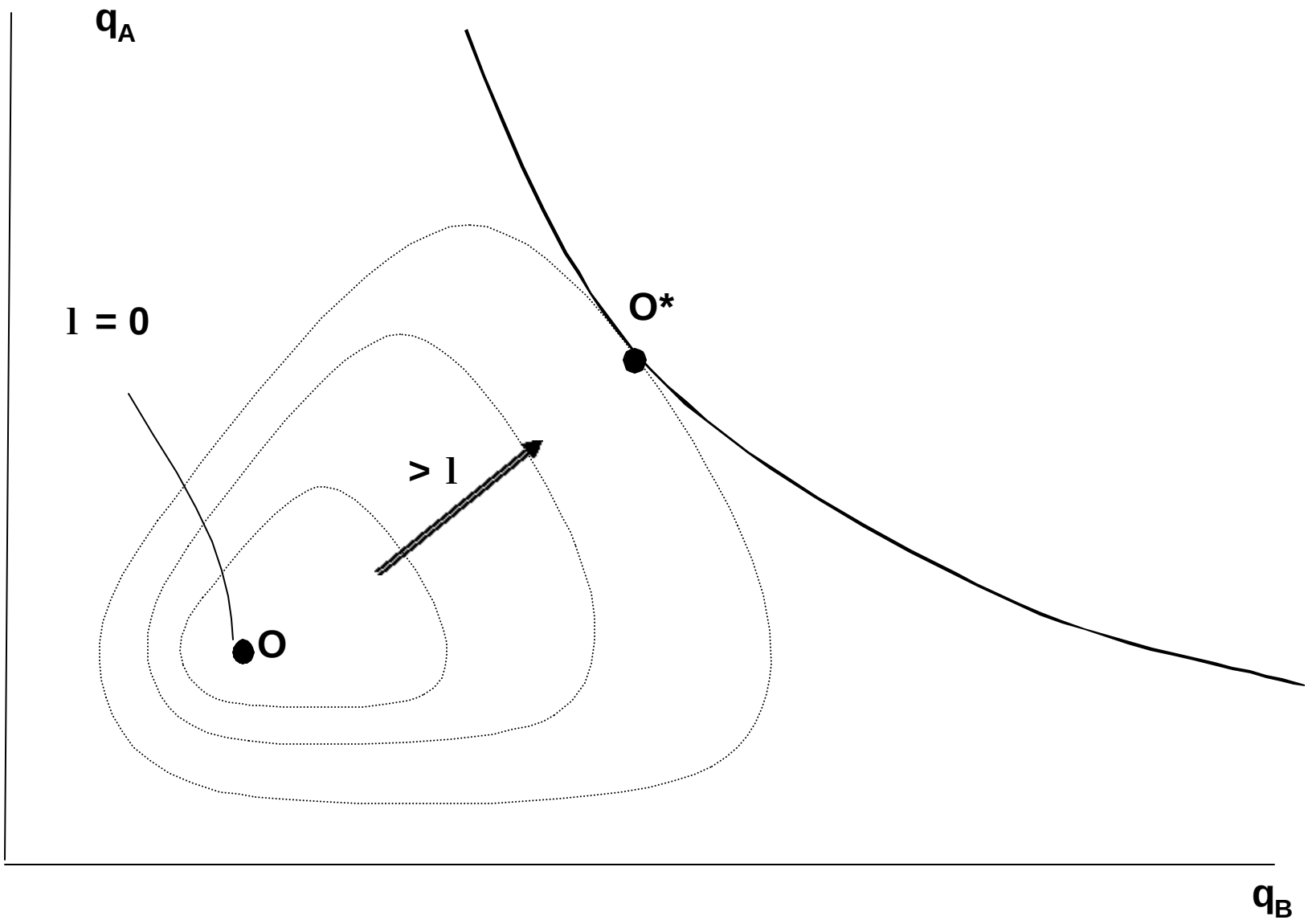
$$\begin{cases} CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} \\ \frac{D_A}{q_A} + \frac{D_B}{q_B} = TO \end{cases}$$

$$L = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} + \lambda \cdot \left(\frac{D_A}{q_A} + \frac{D_B}{q_B} - TO \right)$$

$$\frac{\partial L}{\partial q_A} = \frac{1}{2} \cdot c_{1A} \cdot T - \frac{k_A \cdot D_A}{q_A^2} - \lambda \cdot \frac{D_A}{q_A^2} = 0 \quad \Longrightarrow \quad q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda \cdot D_A}{T \cdot c_{1A}}}$$

$$\frac{\partial L}{\partial q_B} = \frac{1}{2} \cdot c_{1B} \cdot T - \frac{k_B \cdot D_B}{q_B^2} - \lambda \cdot \frac{D_A}{q_B^2} = 0 \quad \Longrightarrow \quad q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda \cdot D_B}{T \cdot c_{1B}}}$$

$$\frac{\partial L}{\partial \lambda} = \frac{D_A}{q_A} + \frac{D_B}{q_B} - TO = 0 \quad \Longrightarrow \quad \frac{D_A}{q_A} + \frac{D_B}{q_B} = TO$$

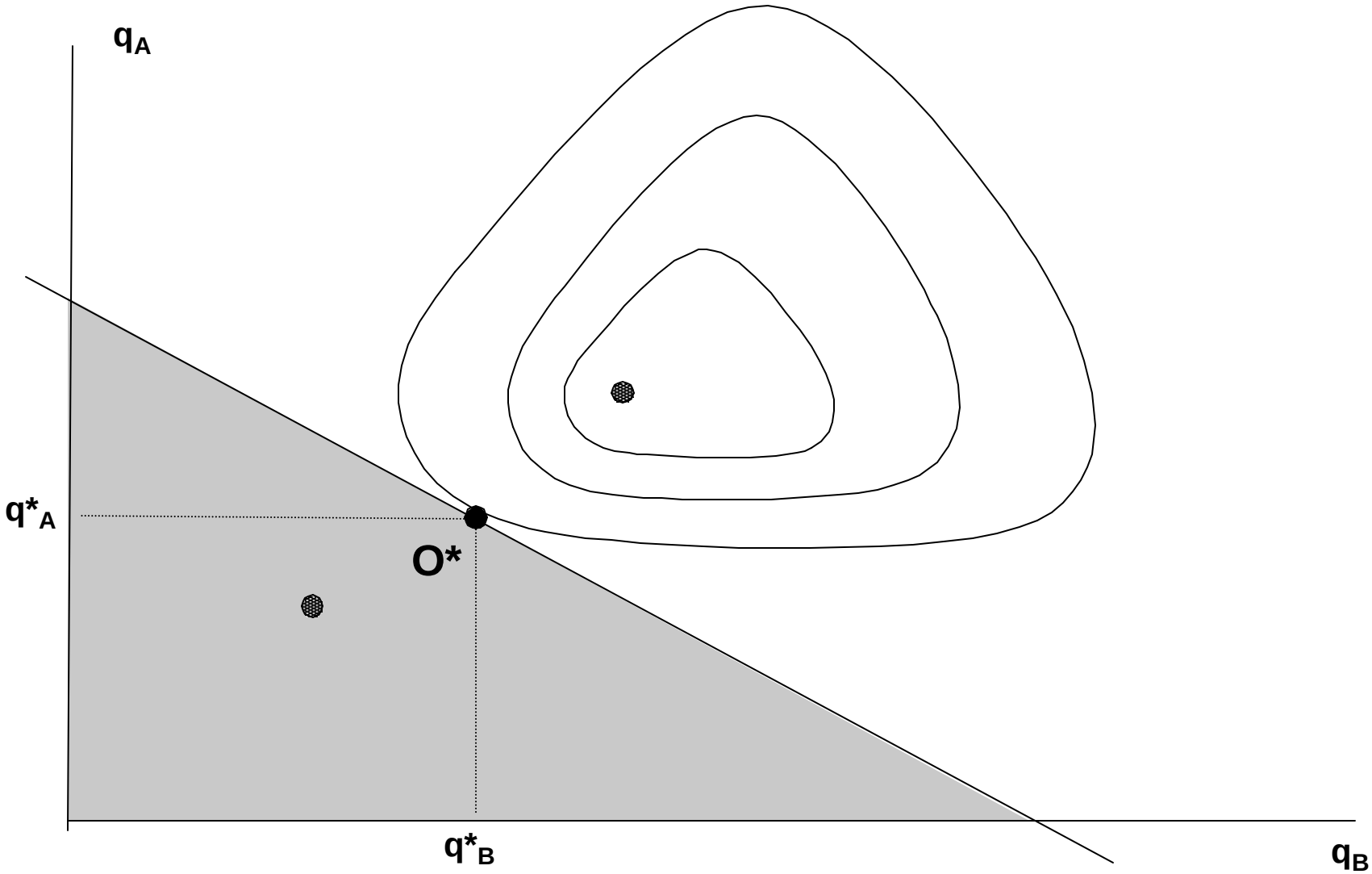

q_A
l = 0
O*
> l
O
q_B

MULTI-ÍTEMS CON UNA RESTRICCIÓN DE MENOR O IGUAL

$$\begin{cases} CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} \\ b_A \cdot q_A + b_B \cdot q_B \le TM \end{cases}$$

$$q_{oA} = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A}}}$$

$$q_{oB} = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B}}}$$


q_A
q^*_A
O*
q^*_B
q_B

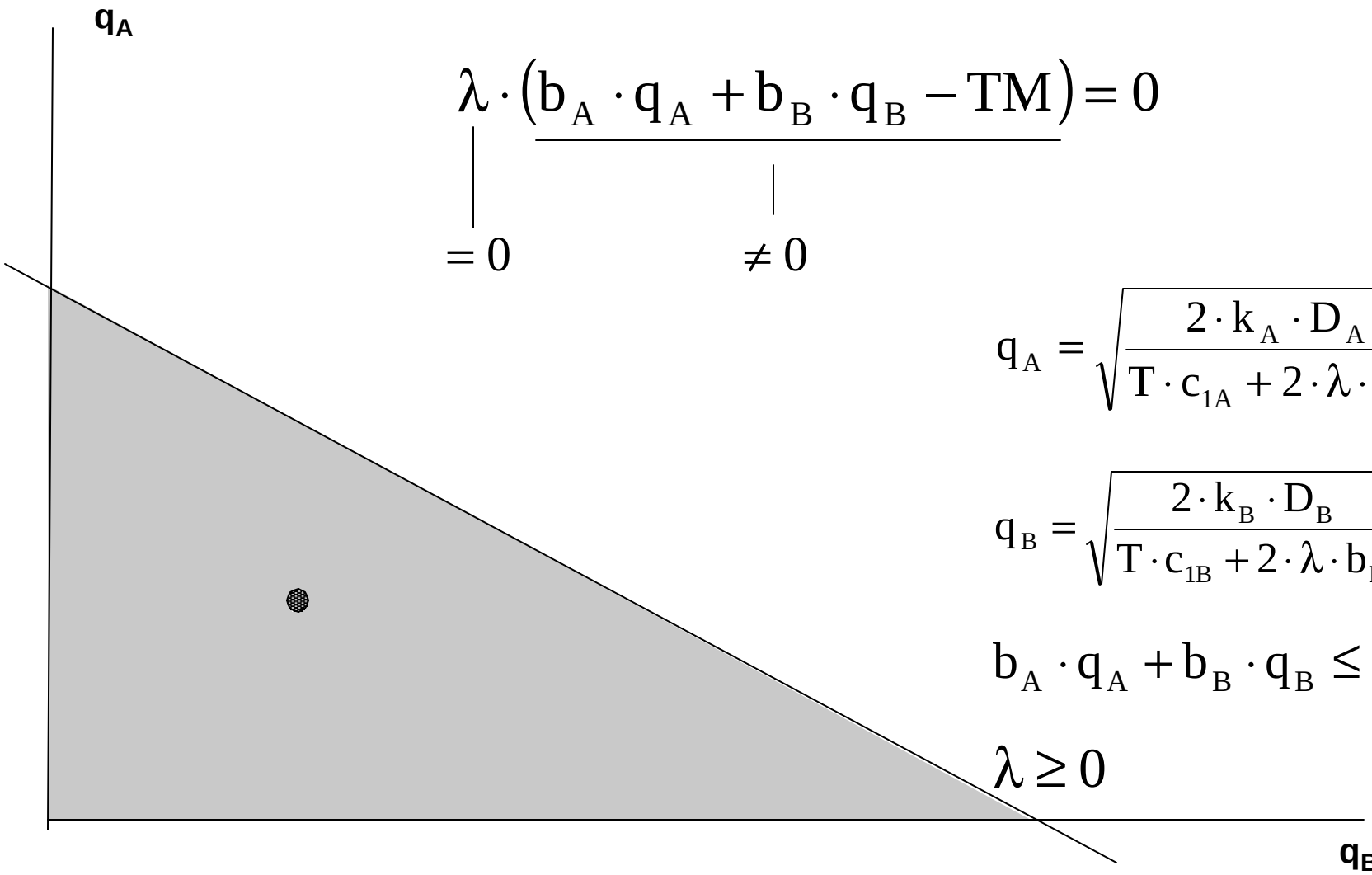
$$\begin{cases} CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} \\ b_A \cdot q_A + b_B \cdot q_B \le TM \end{cases}$$

$$\frac{\partial L}{\partial q_A} = \frac{1}{2} \cdot c_{1A} \cdot T - \frac{k_A \cdot D_A}{q_A^2} + \lambda \cdot b_A = 0 \qquad q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda \cdot b_A}}$$

$$\frac{\partial L}{\partial q_B} = \frac{1}{2} \cdot c_{1B} \cdot T - \frac{k_B \cdot D_B}{q_B^2} + \lambda \cdot b_B = 0 \qquad q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda \cdot b_B}}$$

$$\lambda \cdot \left(b_A \cdot q_A + b_B \cdot q_B - TM \right) = 0$$

$$\lambda \ge 0$$



$$\lambda \cdot \underbrace{\left(b_A \cdot q_A + b_B \cdot q_B - TM\right)}_{\neq 0} = \underbrace{0}_{=0}$$

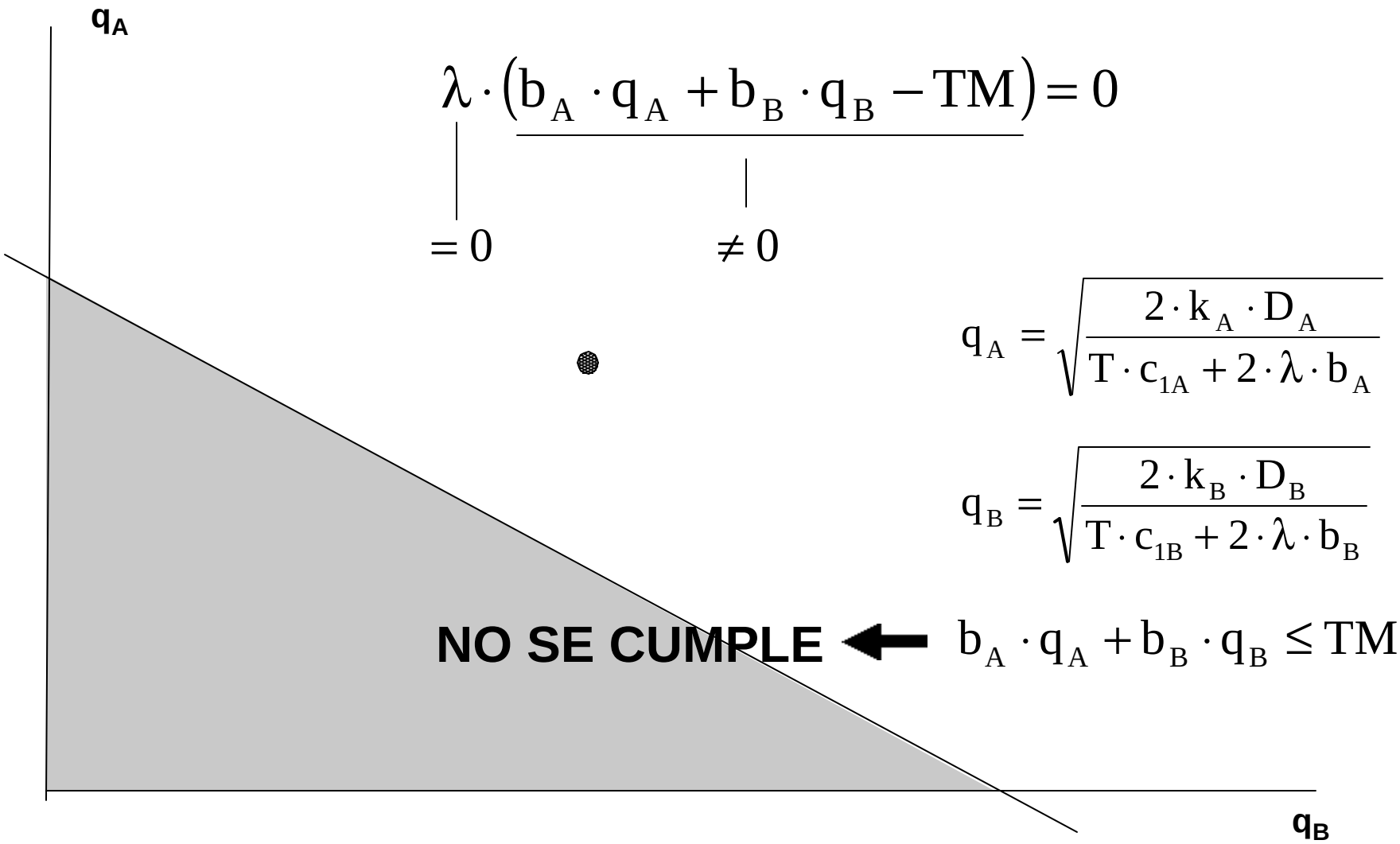
$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda \cdot b_A}}$$

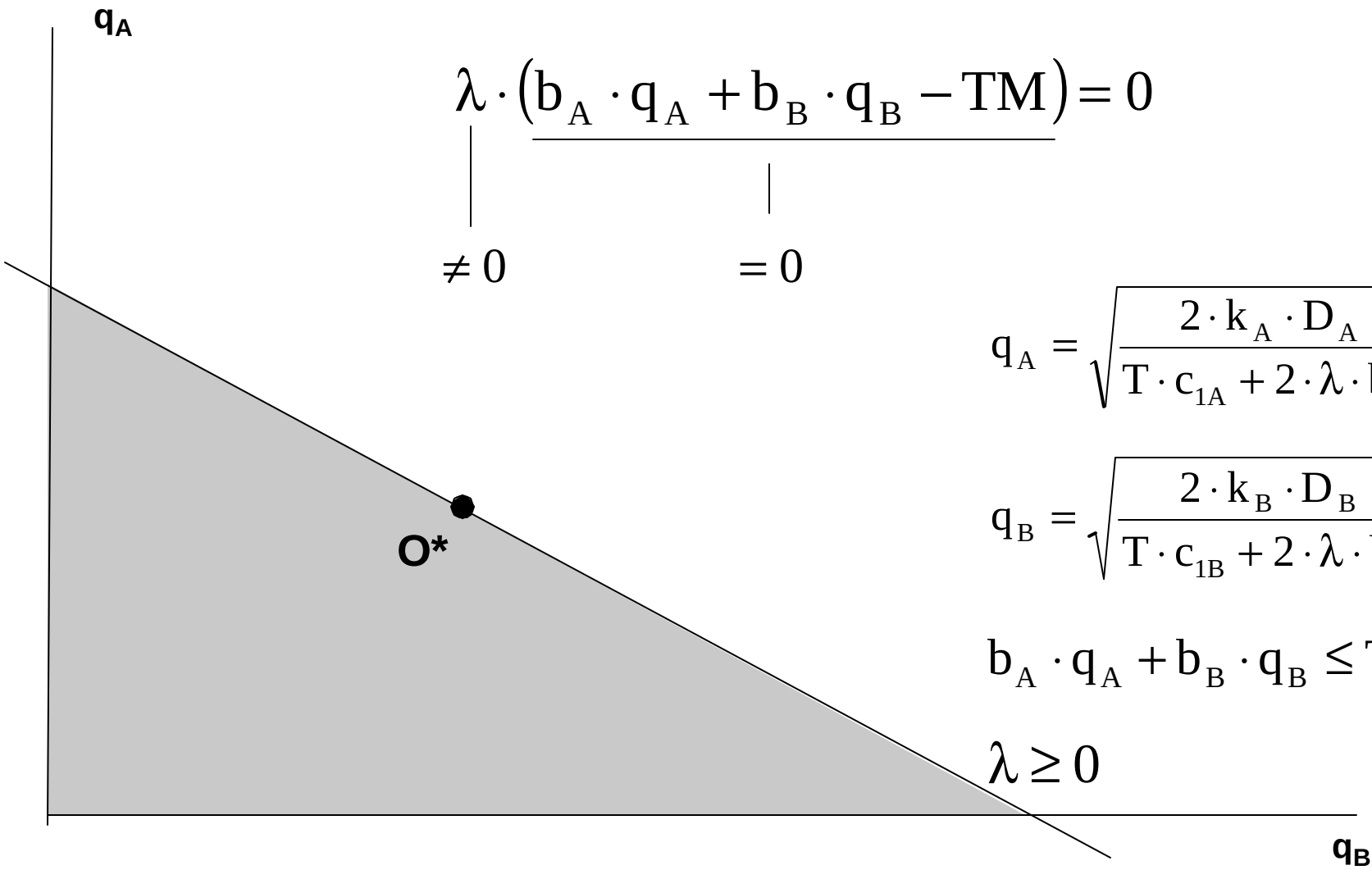
$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda \cdot b_B}}$$

$$b_A \cdot q_A + b_B \cdot q_B \leq TM$$

$$\lambda \geq 0$$

SE CUMPLEN TODAS LAS CONDICIONES DE KKT


q_A
$\lambda \cdot (b_A \cdot q_A + b_B \cdot q_B - TM) = 0$
$= 0$
$\neq 0$
$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda \cdot b_A}}$
$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda \cdot b_B}}$
NO SE CUMPLE
$b_A \cdot q_A + b_B \cdot q_B \leq TM$
q_B



$$\lambda \cdot \underbrace{\left(b_A \cdot q_A + b_B \cdot q_B - TM\right)}_{\neq 0} = \underbrace{0}_{= 0}$$

$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda \cdot b_A}}$$

$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda \cdot b_B}}$$

$$b_A \cdot q_A + b_B \cdot q_B \leq TM$$

$$\lambda \geq 0$$

SE CUMPLEN TODAS LAS CONDICIONES DE KKT

MULTI-ÍTEMS CON VARIAS RESTRICCIONES

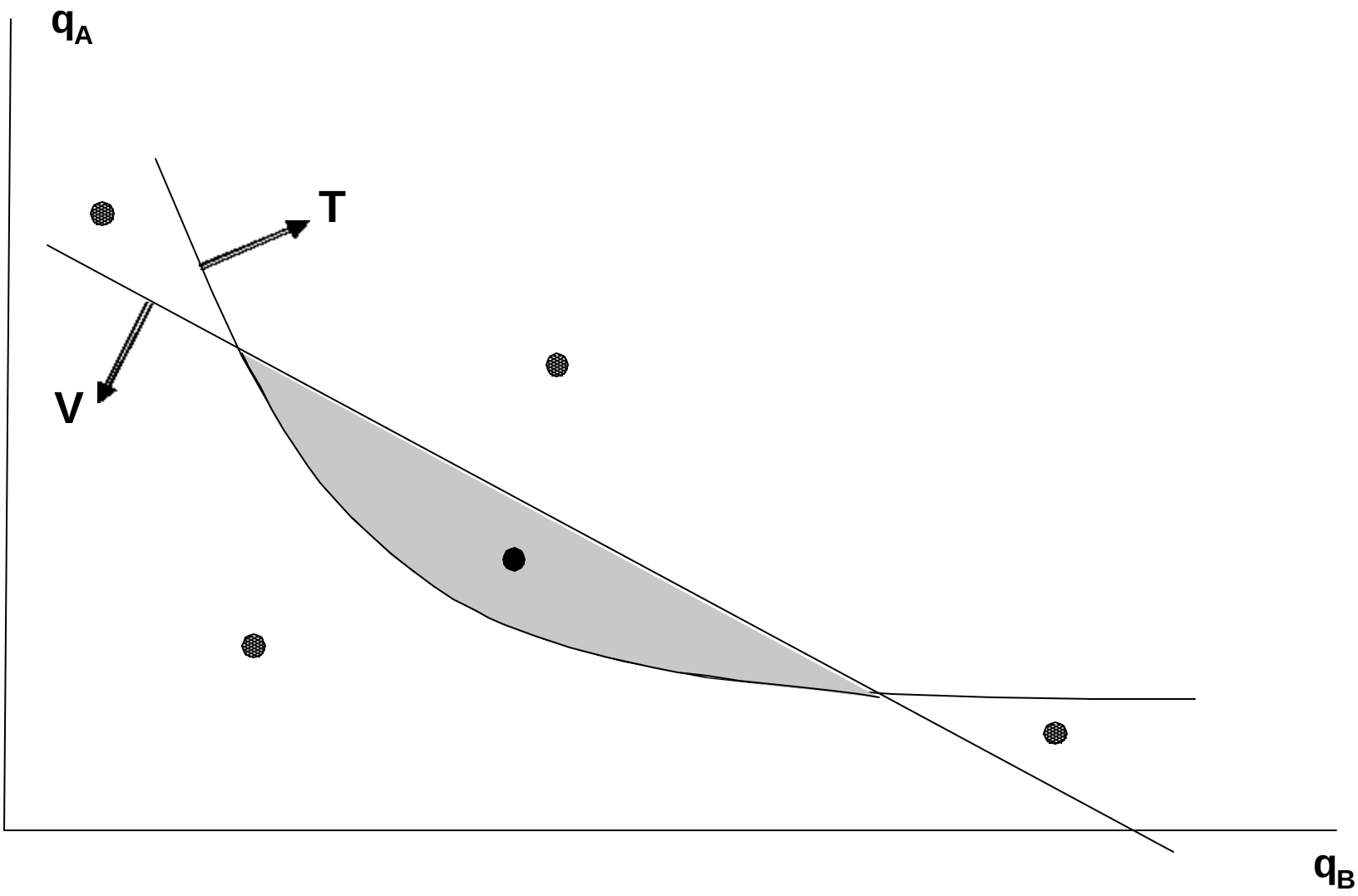
- v_A: volumen ocupado por una unidad de “A”
- v_B: volumen ocupado por una unidad de “B”
- V: volumen total disponible

- t_A: tiempo requerido para preparar un lote de “A”
- t_B: tiempo requerido para preparar un lote de “B”
- TD: tiempo total disponible para armar lotes de “A” y de “B”

$$CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B}$$

$$v_A \cdot q_A + v_B \cdot q_B \leq V$$

$$t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} \leq TD$$


q_A
T
V
q_B

$$CTE = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B}$$

$$v_A \cdot q_A + v_B \cdot q_B \leq V$$

$$t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} \leq TD$$

$$L = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} +$$
$$+ \lambda_1 \cdot (v_A \cdot q_A + v_B \cdot q_B - V) + \lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right)$$

$$L = b_A \cdot D_A + \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + b_B \cdot D_B + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} +$$

$$+ \lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V\right) + \lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right)$$

$$\frac{\partial L}{\partial q_A} = \frac{1}{2} \cdot c_{1A} \cdot T - \frac{k_A \cdot D_A}{q_A^2} + \lambda_1 \cdot v_A - \frac{\lambda_2 \cdot t_A \cdot D_A}{q_A^2} = 0 \quad \Rightarrow \quad q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

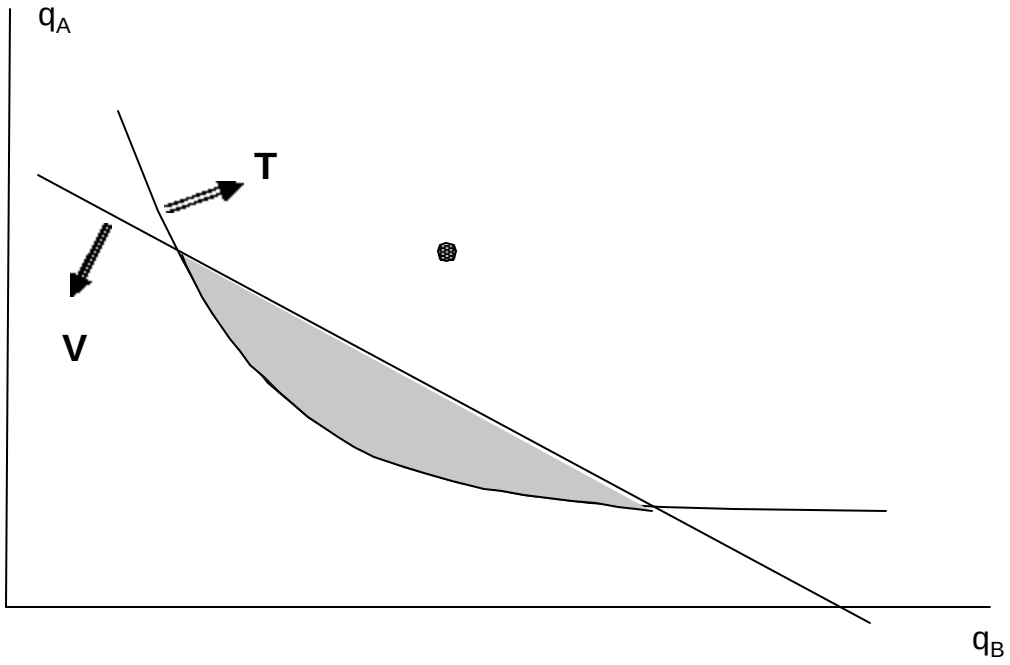
$$\frac{\partial L}{\partial q_B} = \frac{1}{2} \cdot c_{1B} \cdot T - \frac{k_B \cdot D_B}{q_B^2} + \lambda_1 \cdot v_B - \frac{\lambda_2 \cdot t_B \cdot D_B}{q_B^2} = 0 \quad \Rightarrow \quad q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V\right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

$$\lambda_1 \geq 0$$

$$\lambda_2 \geq 0$$



$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

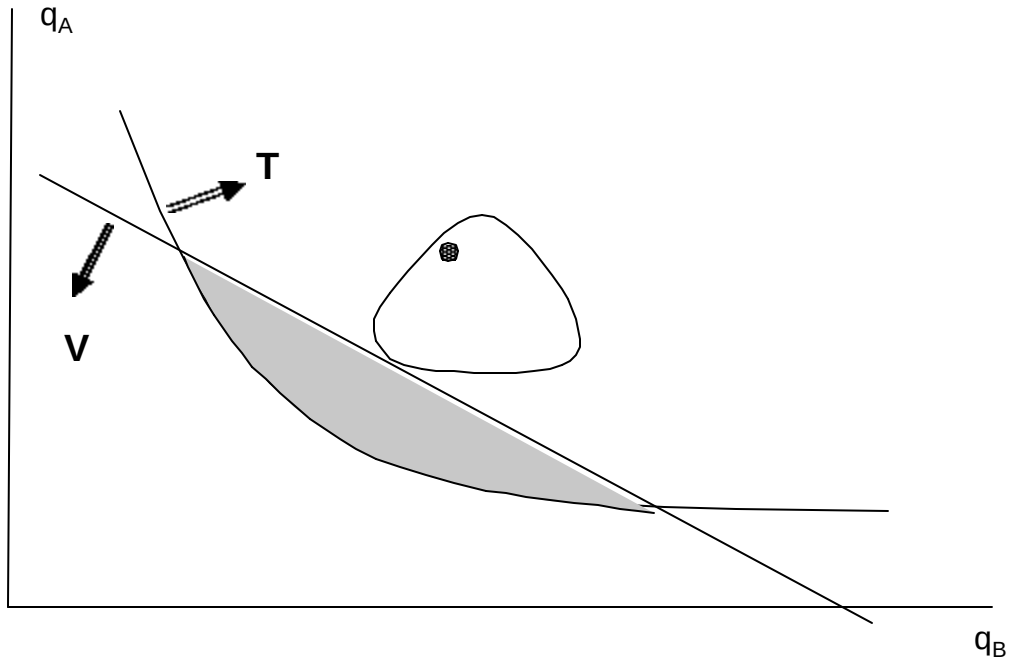
$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V\right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

NO SE CUMPLE

$$\lambda_1 = 0$$

$$\lambda_2 = 0$$



$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

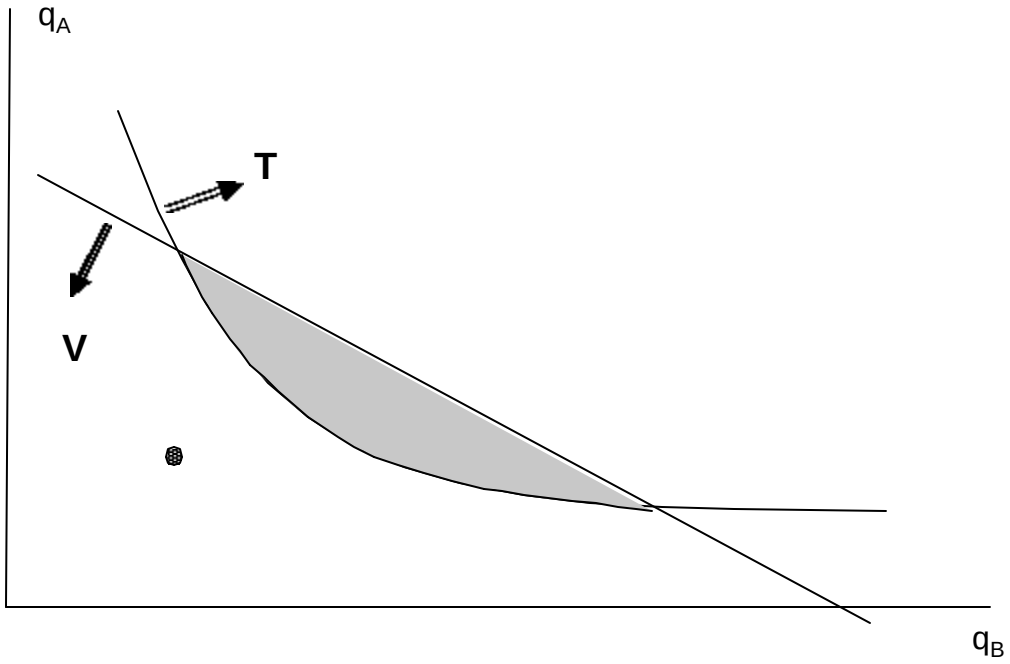
$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V \right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

$$\lambda_1 \geq 0$$

$$\lambda_2 = 0$$



$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

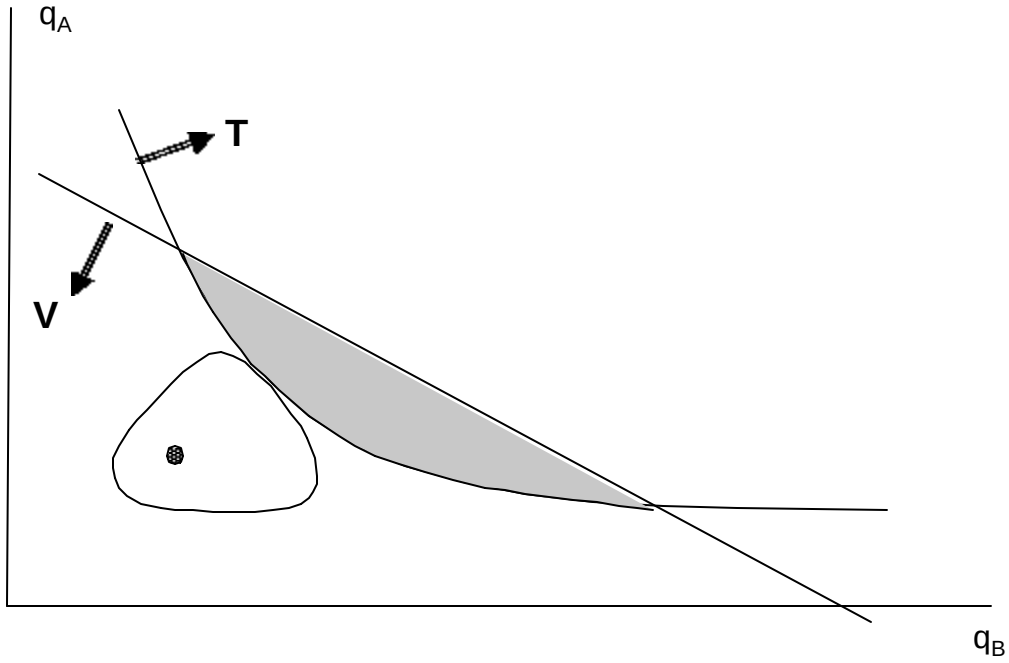
$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V\right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

NO SE CUMPLE

$$\lambda_1 = 0$$

$$\lambda_2 = 0$$



$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

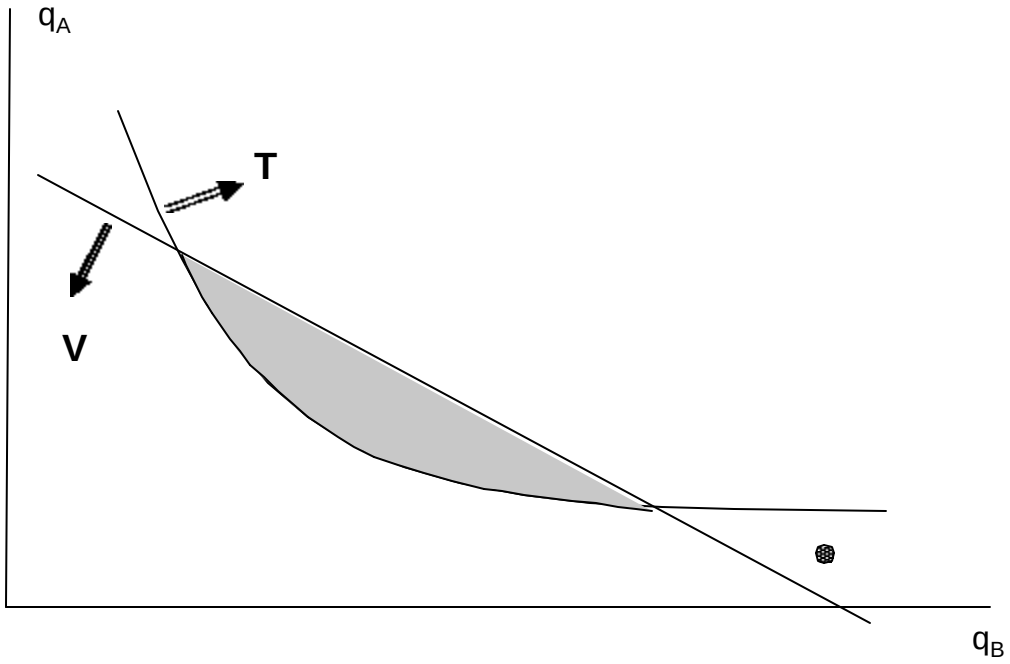
$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V \right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 \geq 0$$



$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

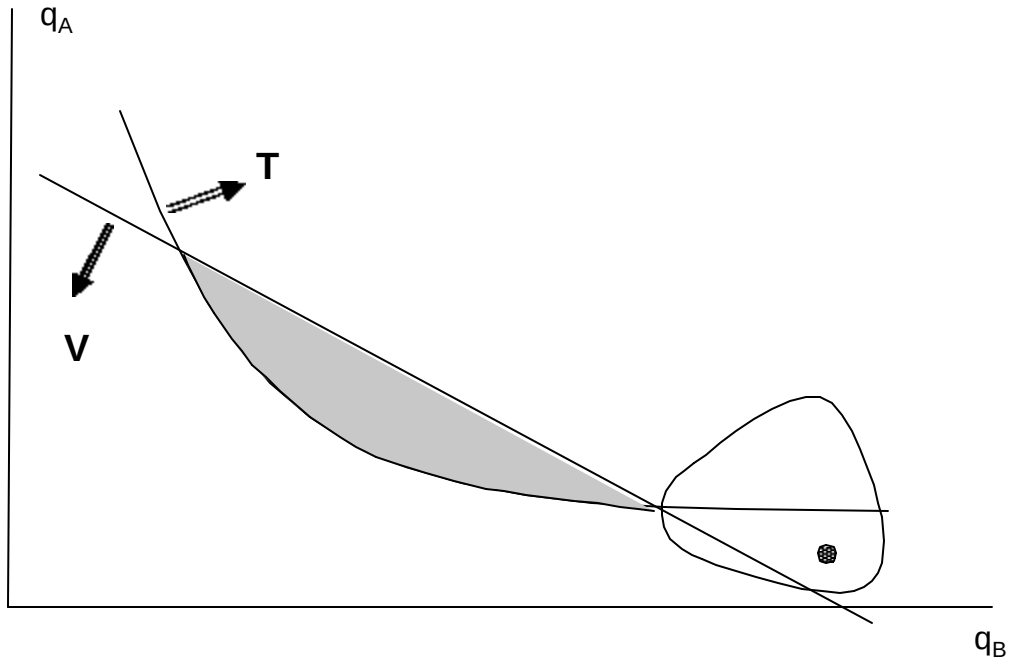
$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V\right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

NO SE CUMPLE

$$\lambda_1 = 0$$

$$\lambda_2 = 0$$



$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda_2 \cdot t_A \cdot D_A}{T \cdot c_{1A} + 2 \cdot \lambda_1 \cdot v_A}}$$

$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda_2 \cdot t_B \cdot D_B}{T \cdot c_{1B} + 2 \cdot \lambda_1 \cdot v_B}}$$

$$\lambda_1 \cdot \left(v_A \cdot q_A + v_B \cdot q_B - V \right) = 0$$

$$\lambda_2 \cdot \left(t_A \cdot \frac{D_A}{q_A} + t_B \cdot \frac{D_B}{q_B} - TD \right) = 0$$

$$\lambda_1 > 0$$

$$\lambda_2 > 0$$