CHAPMAN-KOLMOGOROV

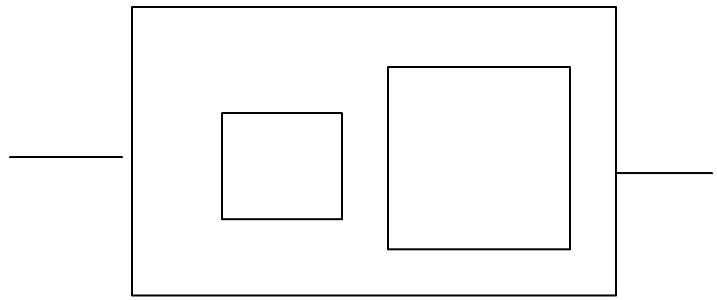
$$V^{i+1} = V^{i} \cdot P$$

EN RÉGIMEN PERMANENTE

$$V = V \cdot P$$

$$\sum p(n) = 1$$

$$\begin{cases} \begin{bmatrix} p(1) & p(2) & p(3) & \ldots\ldots & p(N) \end{bmatrix} \cdot P = \begin{bmatrix} p(1) & p(2) & p(3) & \ldots\ldots & p(N) \end{bmatrix} \\ p(1) + p(2) + p(3) + \ldots + p(N) = 1 \end{cases}$$



$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left[\begin{array}{ccc} 1-\lambda\Delta t & \lambda\Delta t & \\ \mu\Delta t & 1-\mu\Delta t-\lambda\Delta t & \lambda\Delta t \\ & \mu\Delta & 1-\mu\Delta t \end{array}\right] \end{array}$$

$$
\begin{bmatrix} p(0) & p(1) & p(2) \end{bmatrix} \cdot
\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}
\begin{array}{c}
\begin{array}{ccc} 0 & 1 & 2 \end{array} \\
\begin{bmatrix}
1-\lambda\Delta t & \lambda\Delta t & \\
\mu\Delta t & 1-\mu\Delta t-\lambda\Delta t & \lambda\Delta t \\
 & \mu\Delta t & 1-\mu\Delta t
\end{bmatrix}
\end{array}
= \begin{bmatrix} p(0) & p(1) & p(2) \end{bmatrix}
$$

$$p(0)\cdot[1-\lambda\Delta t]+p(1)\cdot\mu\Delta t = p(0)$$

$$p(0)-\lambda\Delta t\cdot p(0)+p(1)\cdot\mu\Delta t = p(0)$$

$$-\lambda\cdot p(0)+p(1)\cdot\mu = 0$$

$$p(1)\cdot\lambda\Delta t+p(2)\cdot[1-\mu\Delta t] = p(2)$$

$$p(1)\cdot\lambda\Delta t+p(2)-p(2)\cdot\mu\Delta t = p(2)$$

$$p(2) = p(1)\cdot\rho$$

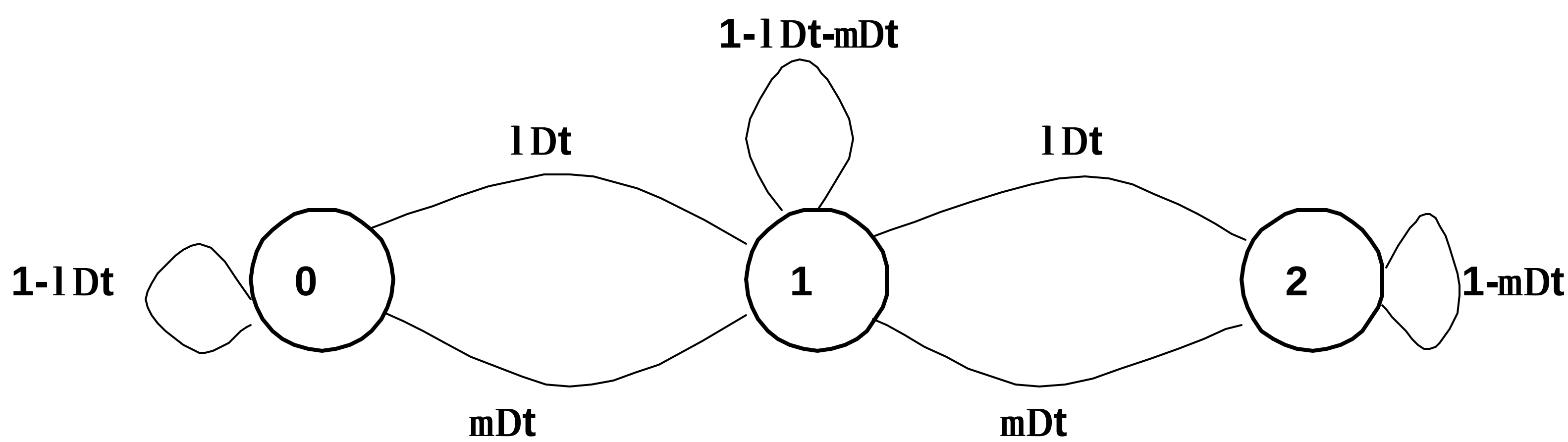
$$p(1) = \rho\cdot p(0)$$

$$p(2) = \rho^2\cdot p(0)$$

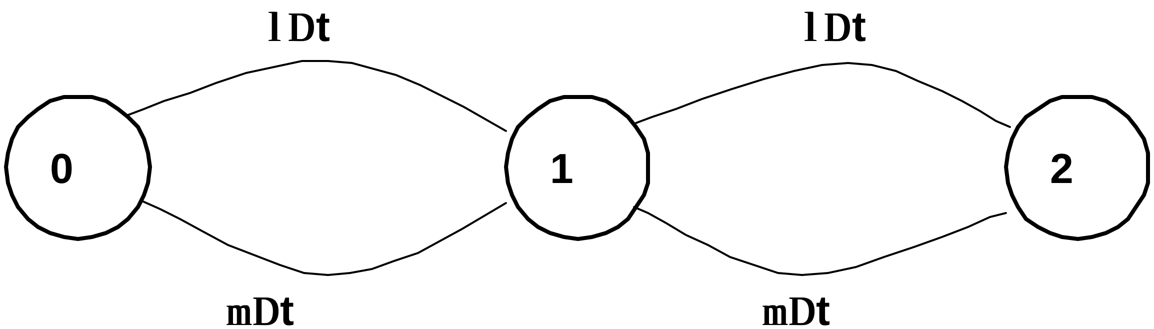
$$p(0)+p(1)+p(2) = 1$$

$$\begin{bmatrix} p(0) & p(1) & p(2) \end{bmatrix} \cdot \begin{bmatrix} 1-\lambda\Delta t & 1 & \\ \mu\Delta t & 1 & \lambda\Delta t \\ & 1 & 1-\mu\Delta t \end{bmatrix} = \begin{bmatrix} p(0) & 1 & p(2) \end{bmatrix}$$

$$\begin{bmatrix} p(0) & p(1) & p(2) \end{bmatrix} \cdot \begin{bmatrix} 1-\lambda & 1 & \\ \mu & 1 & \lambda \\ & 1 & 1-\mu \end{bmatrix} = \begin{bmatrix} p(0) & 1 & p(2) \end{bmatrix}$$



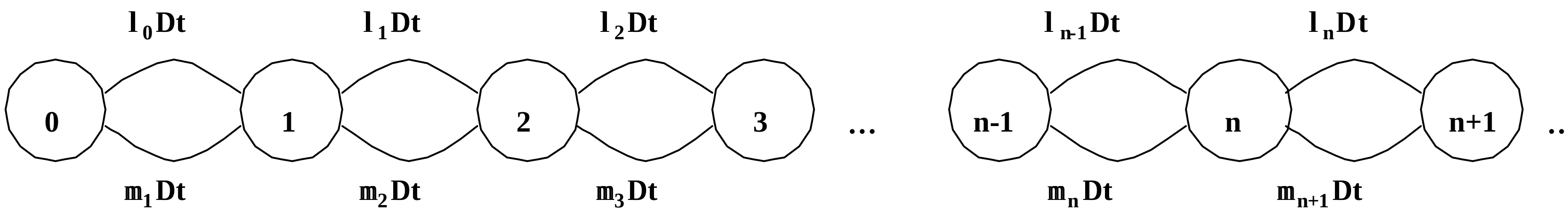
1-lDt-mDt
lDt
lDt
1-lDt
0
1
2
1-mDt
mDt
mDt



Nodo 0: $p(0)\cdot\lambda = p(1)\cdot\mu$ $\qquad$ $p(1) = \rho\cdot p(0)$

Nodo 2: $p(2)\cdot\mu = p(1)\cdot\lambda$ $\qquad$ $p(2) = \rho\cdot p(1) = \rho^2\cdot p(0)$

Suma: $p(0) + p(1) + p(2) = 1$



$$p(n)\cdot(\lambda_n + \mu_n)\Delta t = p(n-1)\cdot\lambda_{n-1}\Delta t + p(n+1)\cdot\mu_{n+1}\Delta t$$

$$0 = p(n-1)\cdot\lambda_{n-1} - p(n)\cdot(\lambda_n + \mu_n) + p(n+1)\cdot\mu_{n+1}$$

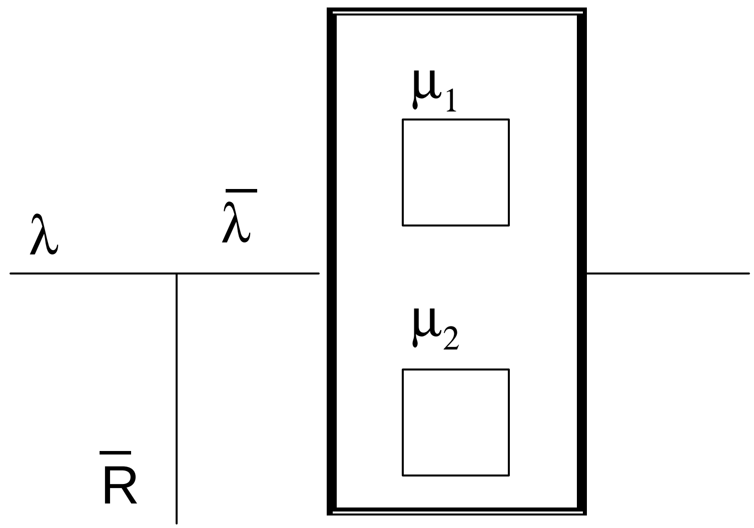
En un sistema de dos canales en paralelo con velocidades medias de atención m1 = 2 cl/h y m2 = 1 cl/h no se admite formación de cola. La tasa media de arribos (l) es igual a 2 cl/h. Los clientes tienen experiencia en el funcionamiento del sistema, de manera que:

- *si los dos canales están vacíos cuando arriba un cliente, éste se hace atender en el más rápido*

- *cuando uno de los dos canales está ocupado, el cliente se atiende en el que está libre*

Calcular las probabilidades asociadas a cada uno de los estados que puede asumir el sistema.

¿Cómo se resolvería el problema si los clientes no tuvieran experiencia?

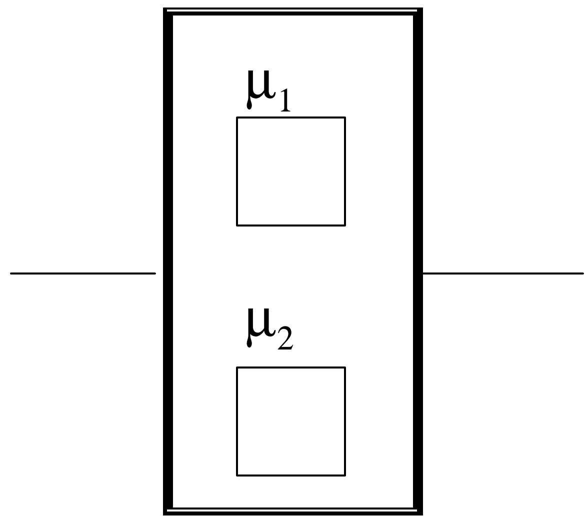


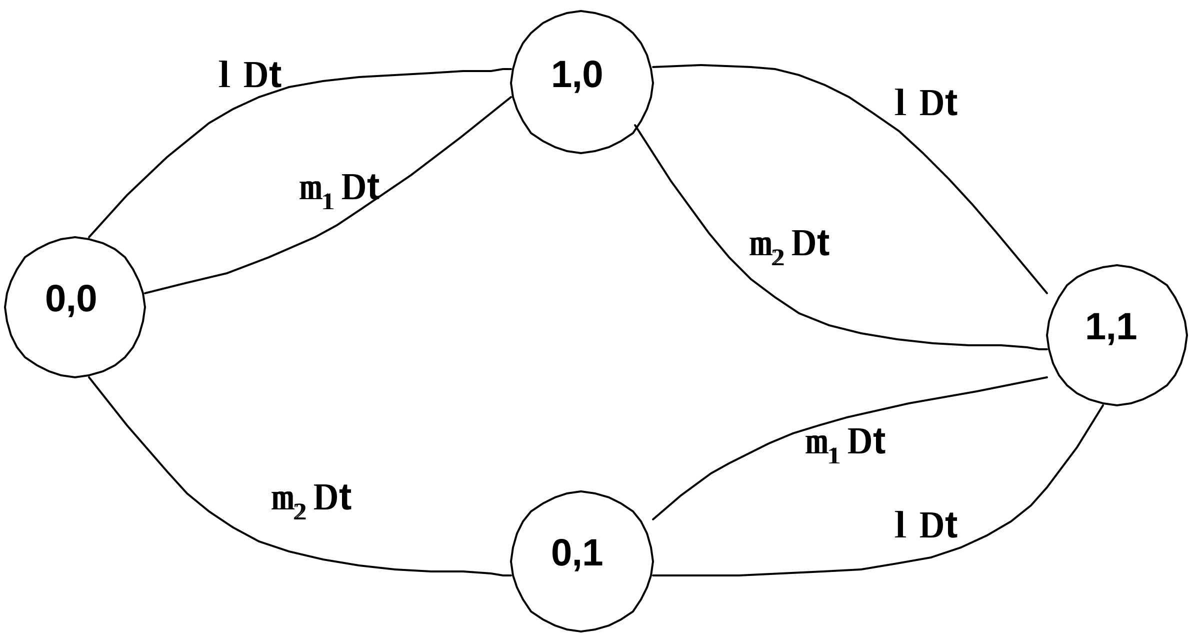
0,0: ningún cliente en el sistema

1,0: un cliente en el canal 1, ninguno en el canal 2

0,1: ningún cliente en el canal 1, un cliente en el canal 2

1,1: un cliente en cada canal.


μ1
μ2

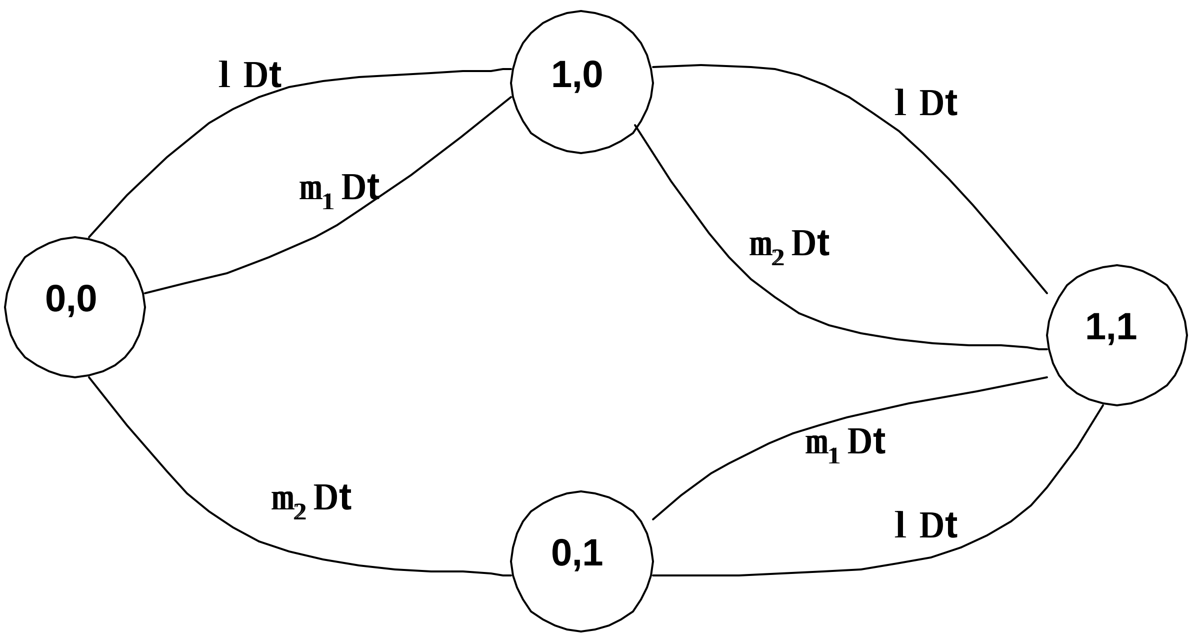

0,0
1,0
0,1
1,1
l Dt
m1 Dt
l Dt
m2 Dt
m2 Dt
m1 Dt
l Dt

$$p(0,0)\cdot\lambda = p(1,0)\cdot\mu_1 + p(0,1)\cdot\mu_2$$

$$p(1,0)\cdot(\mu_1+\lambda) = p(0,0)\cdot\lambda + p(1,1)\cdot\mu_2$$

$$p(0,1)\cdot(\mu_2+\lambda) = p(1,1)\cdot\mu_1$$

$$p(0,0)+p(1,0)+p(0,1)+p(1,1) = 1$$



p(0,0)	0.3182
p(1,0)	0.2273
p(0,1)	0.1818
p(1,1)	0.2727

$$\bar{\lambda} = \lambda \cdot [1 - p(1,1)] = 1{,}4546$$

$$\bar{\mu}_1 = \mu_1 \cdot [p(1,0) + p(1,1)] = 1$$

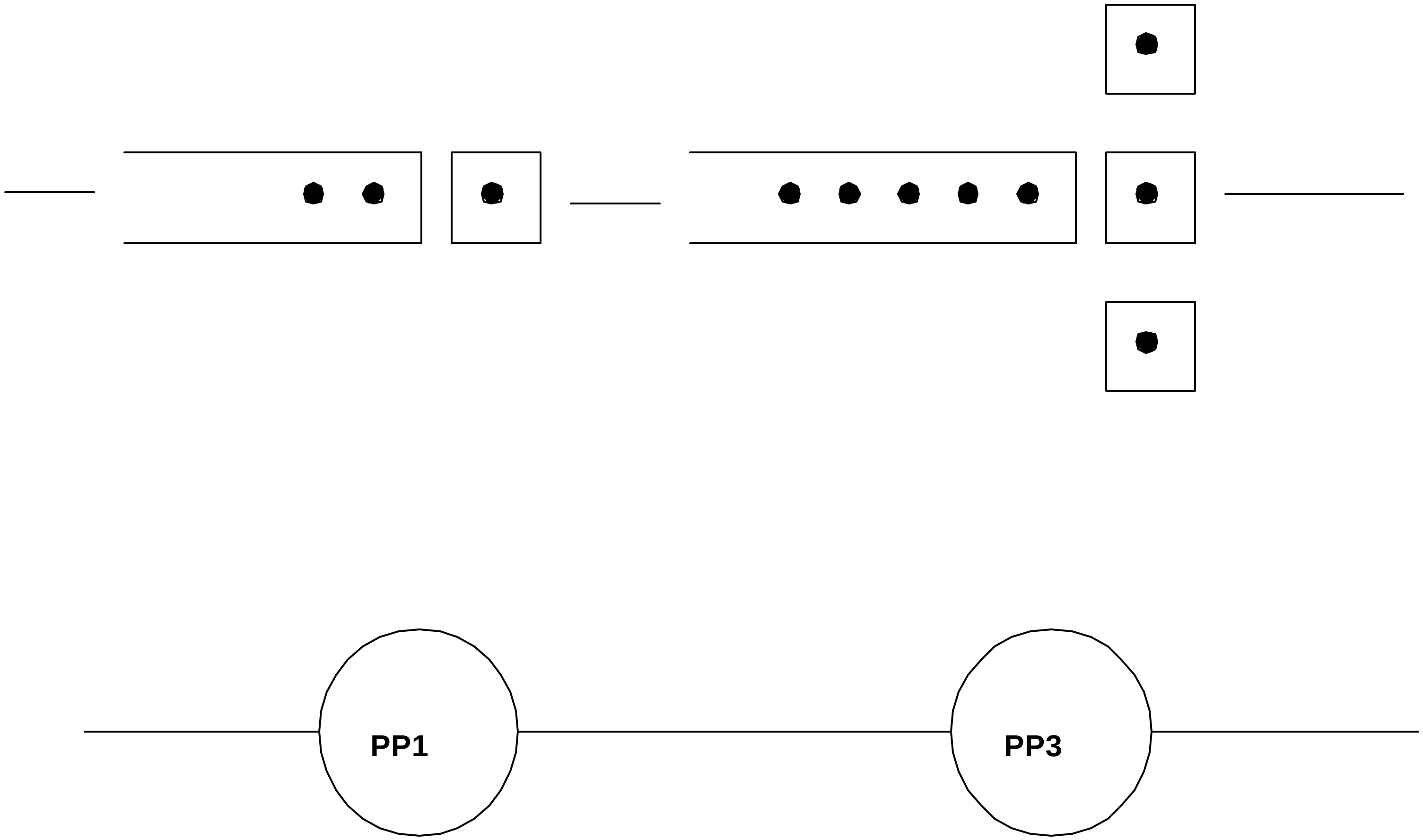
$$\bar{\mu}_2 = \mu_2 \cdot [p(0,1) + p(1,1)] = 0{,}4545$$

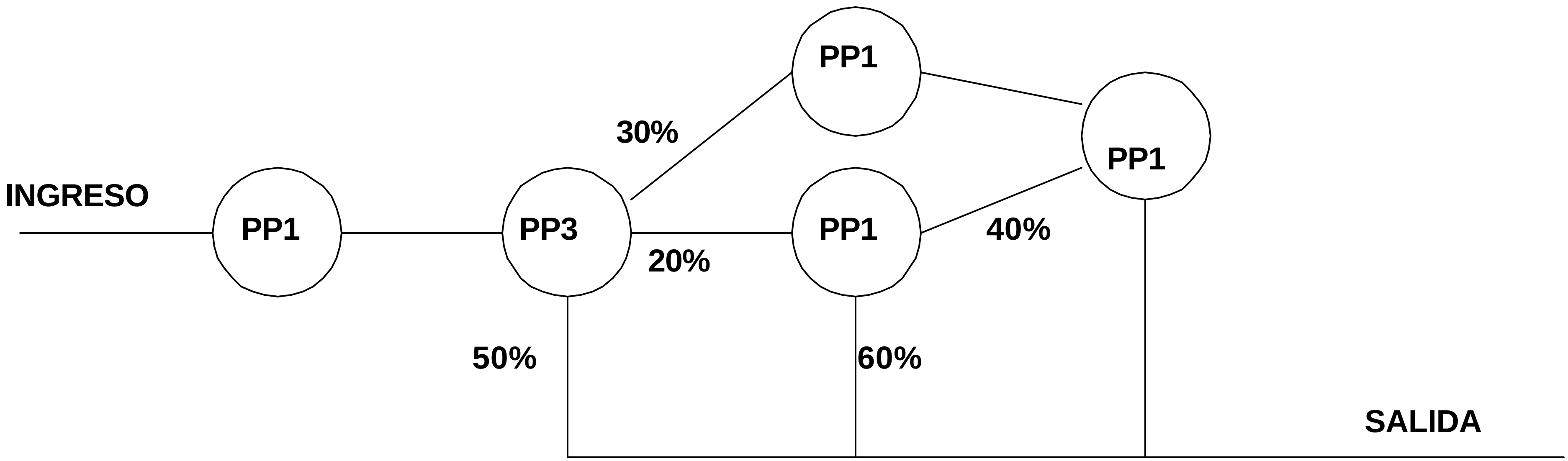
$$P_1 = \frac{\bar{\mu}_1}{\bar{\mu}} = 0{,}6875$$

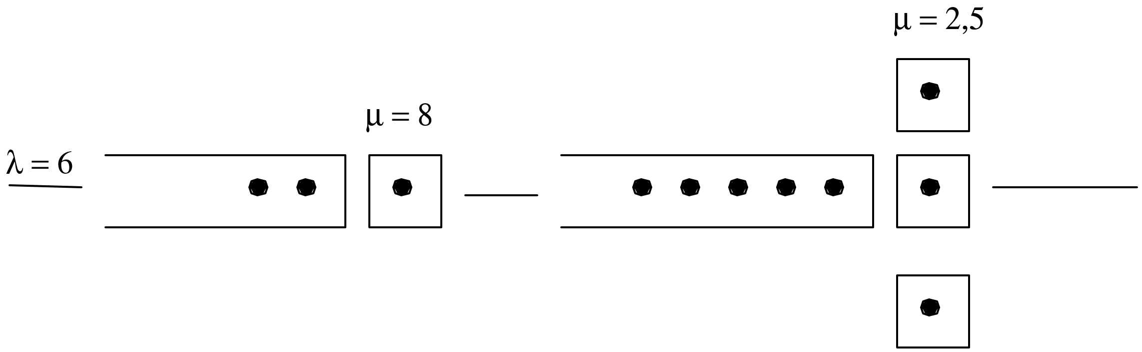
$$P_2 = \frac{\bar{\mu}_2}{\bar{\mu}} = 0{,}3125$$

$$L = 1 \cdot [p(1,0) + p(0,1)] + 2 \cdot p(1,1) = 0{,}9545$$

$$W = \frac{1}{\mu_1} \cdot P_1 + \frac{1}{\mu_2} \cdot P_2 = 0{,}6562$$


PP1
PP3


INGRESO
PP1
PP3
30%
PP1
20%
PP1
40%
PP1
50%
60%
SALIDA



$r_1 = 0,75$
$y_1 = 0,75$
$H_1 = 0,75$
$p(0)_1 = 0,25$
$L_{C1} = 2,25$
$L_1 = 3$
$W_{C1} = 0,375$
$W_1 = 0,5$

$\bar{\mu}_1 = 6$

$r_2 = 2,40$
$y_2 = 0,8$
$H_2 = 2,40$
$p(0)_2 = 0,0562$
$L_{C2} = 2,59$
$L_2 = 4,99$
$W_{C2} = 0,4315$
$W_2 = 0,8315$

$\bar{\mu}_2 = 6$

$H = 0,75 + 2,40 = 3,15$

$L_C = 2,25 + 2,59 = 4,84$

$L = 3 + 4,99 = 7,99$

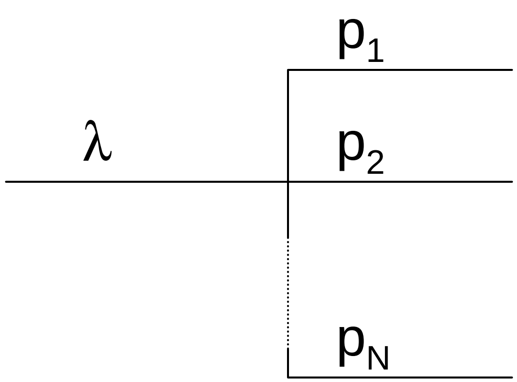
$W_C = 0,375 + 0,4315 = 0,8065$

$W = 0,5 + 0,8315 = 1,3315$

$p(0) = 0,25 \,.\, 0,0562 = 0,01405$

TEOREMA DE BURKE

La división de un flujo de clientes de un proceso Poisson, probabilísticamente lleva a procesos que también son de naturaleza poissoniana



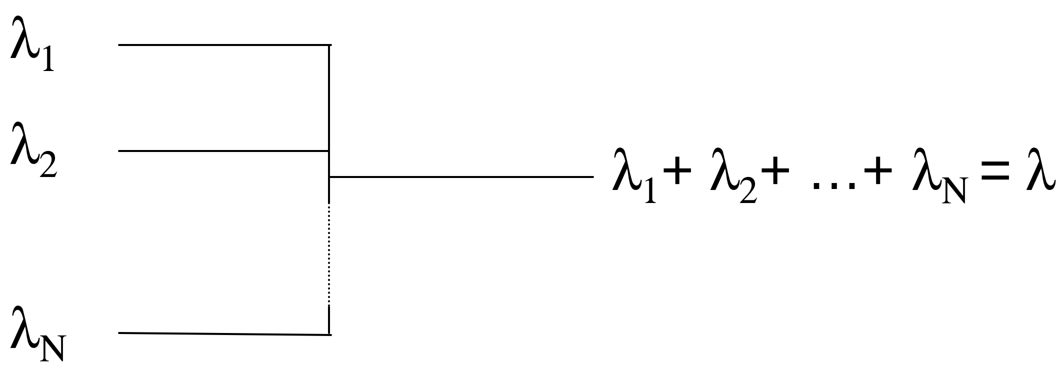
$\lambda_1 = p_1 \cdot \lambda$

$\lambda_2 = p_2 \cdot \lambda$

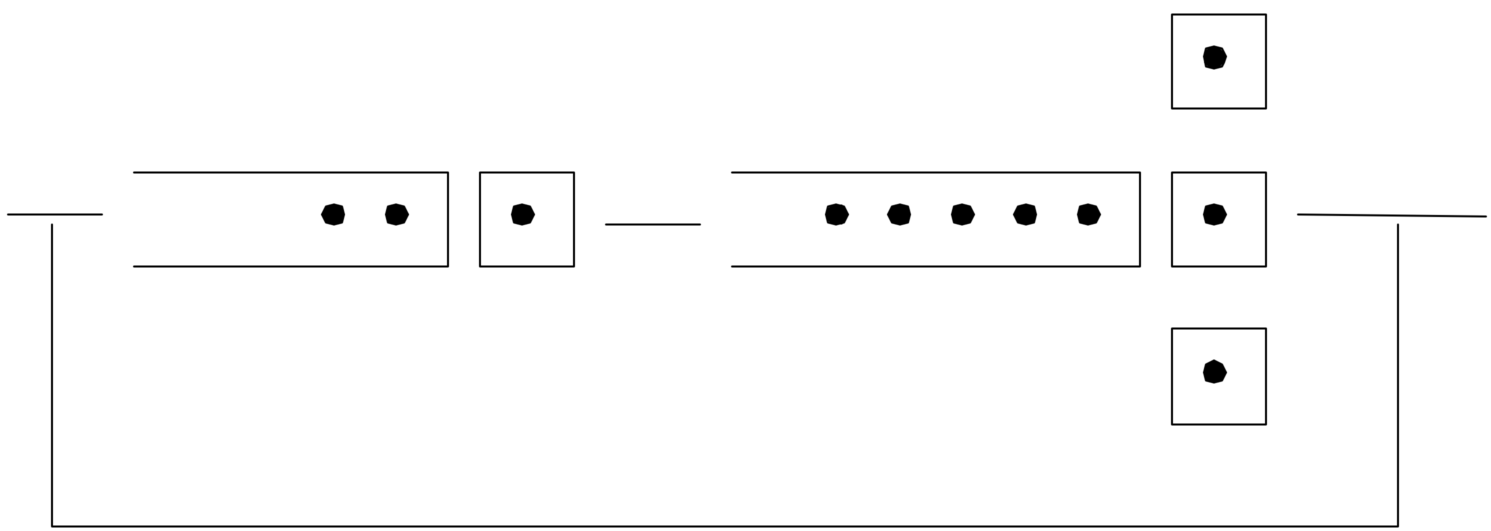
$\lambda_N = p_N \cdot \lambda$

$$\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_N$$

La combinación de flujos independientes de clientes de procesos Poisson lleva a un Proceso que también es de naturaleza poissoniana



En redes con reciclaje, siendo el proceso de arribos externos de tipo Poisson, el flujo entrante en una estación se comporta como si fuera Poisson, a pesar de que no es estrictamente de naturaleza Poissoniana

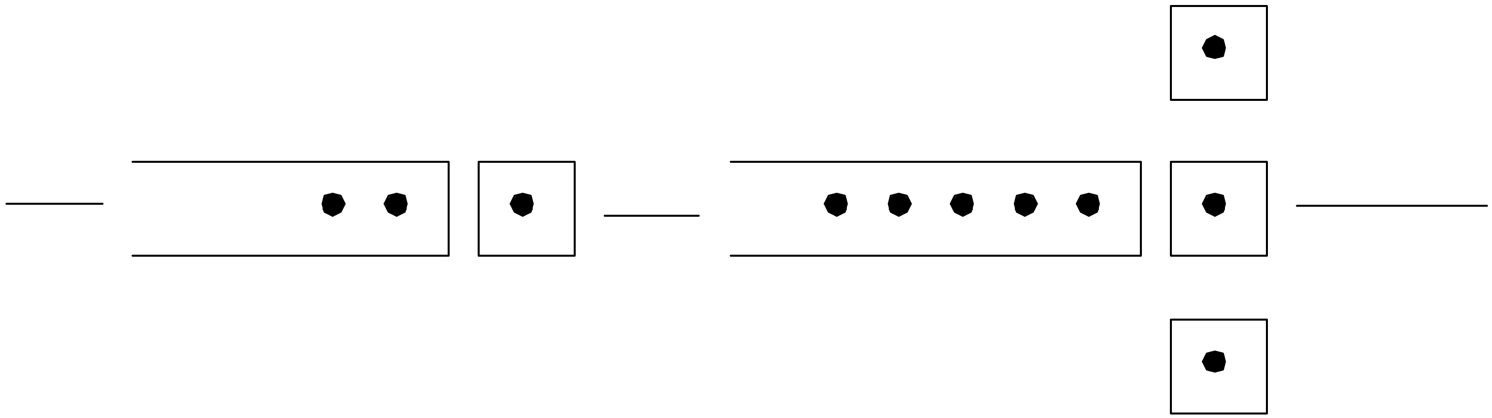


TEOREMA DE JACKSON

Para un sistema de “K” centros de atención, la probabilidad de que el sistema se encuentre en el estado n_1, n_2, ..., n_K está dada por

$$p(n_1, n_2, \cdots, n_K) = \prod_{i=1}^{K} p_i(n_i)$$

TEOREMA DE JACKSON



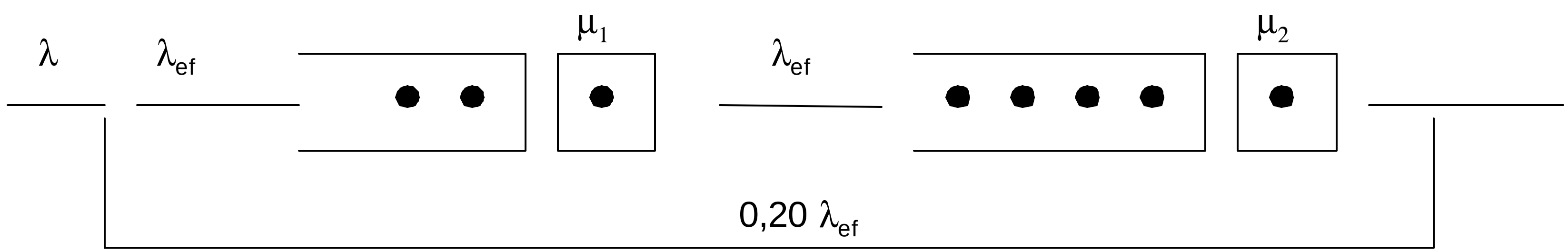
$$p(n_1, n_2) = p(n_1) \cdot p(n_2)$$

En condiciones de equilibrio, la tasa de ingreso al sistema es igual a la tasa de egreso

$$\overline{\lambda} = \overline{\mu}$$

$$\overline{\lambda} = \mu \cdot H = \frac{1}{T_S} \cdot H$$

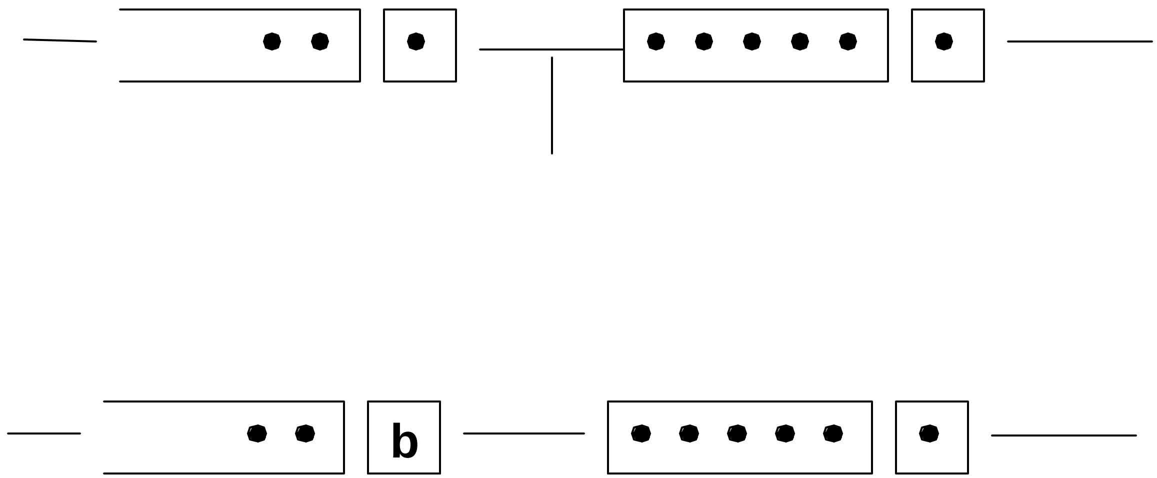
$$T_S = \frac{H}{\overline{\lambda}}$$

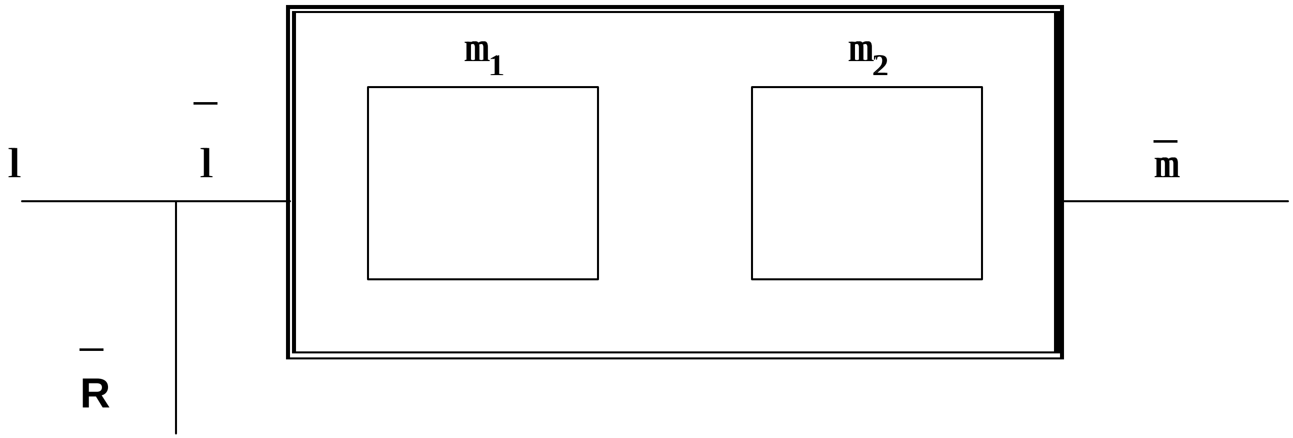


$$\lambda + 0{,}20 \cdot \lambda_{ef} = \lambda_{ef}$$

$$\lambda_{ef} = \frac{\lambda}{0{,}8}$$

BLOQUEO





Estados posibles:

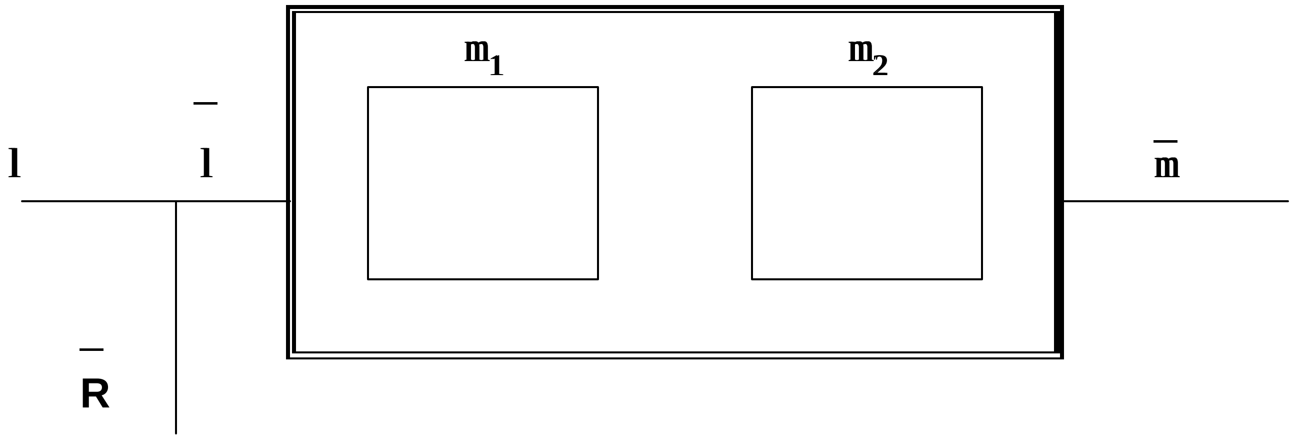
0,0

1,0

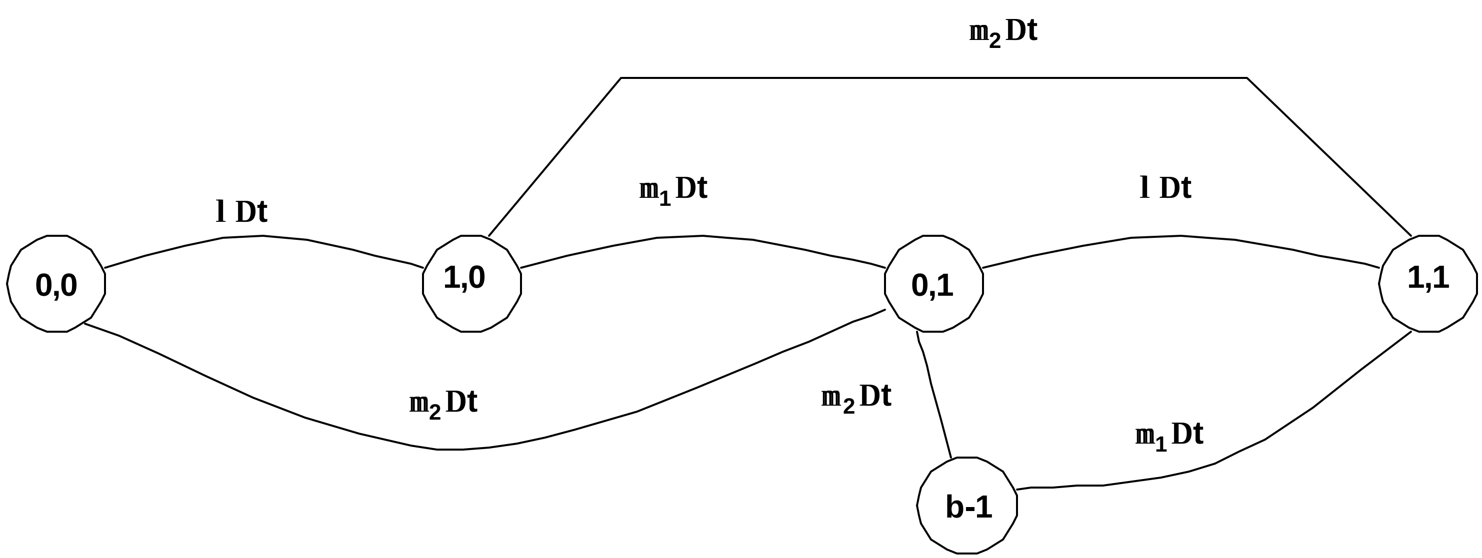
0,1

1,1

b,1



l
l
m1
m2
m
R



m2 Dt
l Dt
m1 Dt
l Dt
0,0
1,0
0,1
1,1
m2 Dt
m2 Dt
m1 Dt
b-1

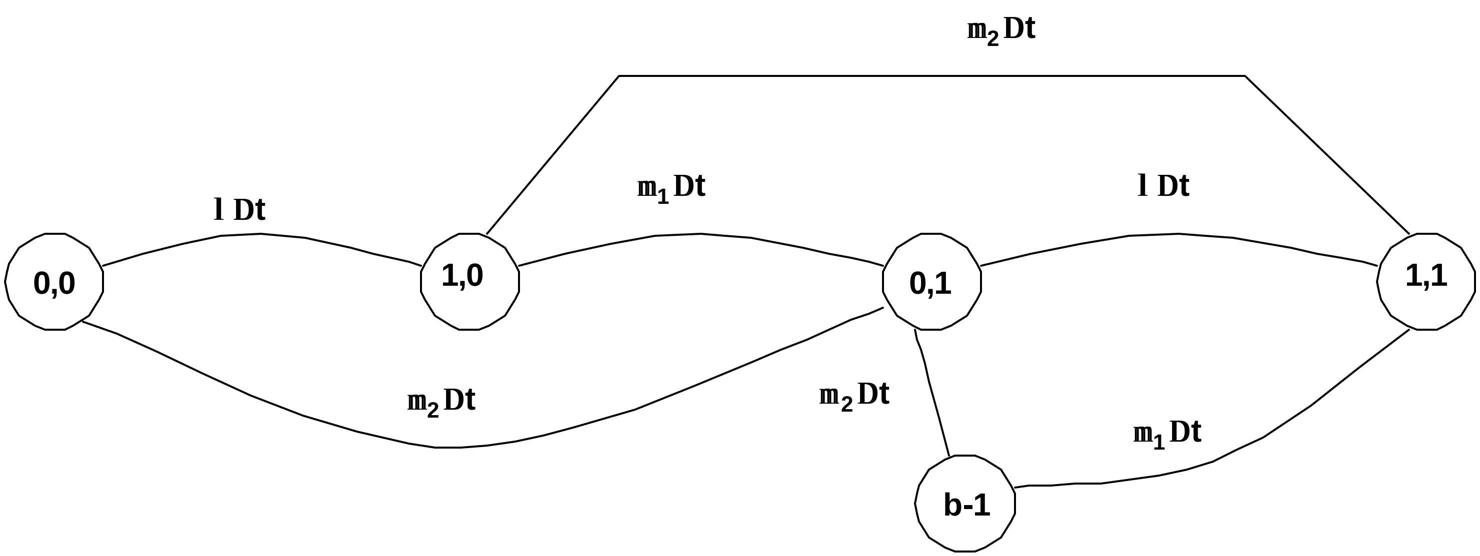
NODO (0,0): $$p(0,0)\cdot\lambda = p(0,1)\cdot\mu_2$$

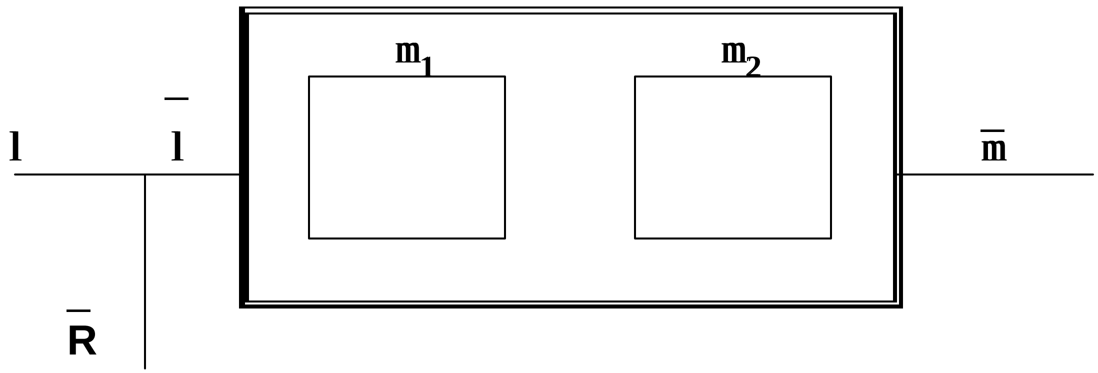
NODO (1,0): $$p(1,0)\cdot\mu_1 = p(0,0)\cdot\lambda + p(1,1)\cdot\mu_2$$

NODO (1,1): $$p(1,1)\cdot(\mu_1+\mu_2) = p(0,1)\cdot\lambda$$

NODO (b,1): $$p(b,1)\cdot\mu_2 = p(1,1)\cdot\mu_1$$

SUMA p(n1,n2): $$p(0,0)+p(1,0)+p(0,1)+p(1,1)+p(b,1)=1$$





Estados posibles:

0,0

1,0

0,1

1,1

b,1

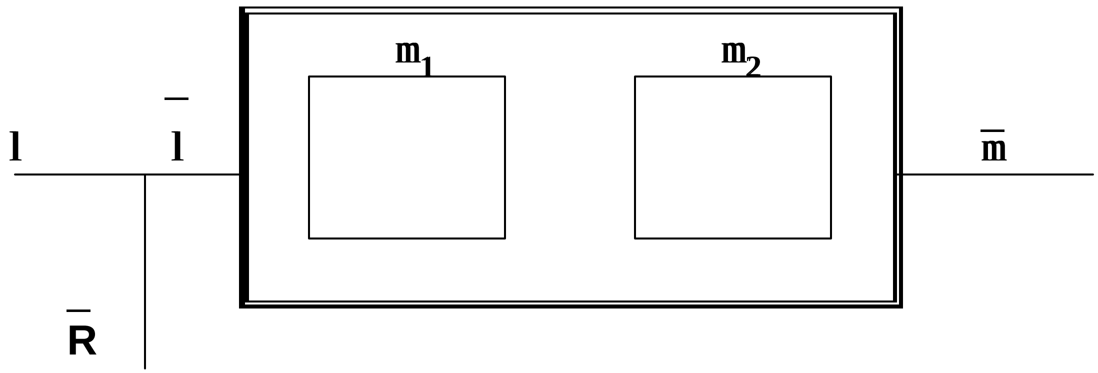
$$L = 1 \cdot [p(0,1) + p(1,0)] + 2 \cdot [p(1,1) + p(b,1)]$$

$$H = 1 \cdot [p(0,1) + p(1,0) + p(b,1)] + 2 \cdot p(1,1)$$

$$H_1 = p(1,0) + p(1,1)$$

$$H_2 = p(0,1) + p(1,1) + p(b,1)$$

$$B = p(b,1) \qquad L = H + B$$



Estados posibles:

0,0

1,0

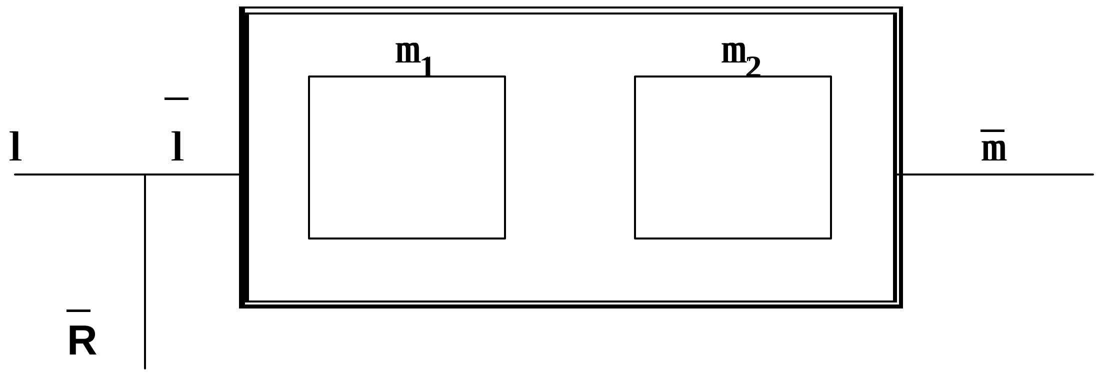
0,1

1,1

b,1

$$\bar{\lambda} = \lambda \cdot [p(0,0) + p(0,1)]$$

$$\bar{\mu} = \bar{\mu}_1 = \mu_1 \cdot H_1 = \bar{\mu}_2 = \mu_2 \cdot H_2$$



Estados posibles:

0,0

1,0

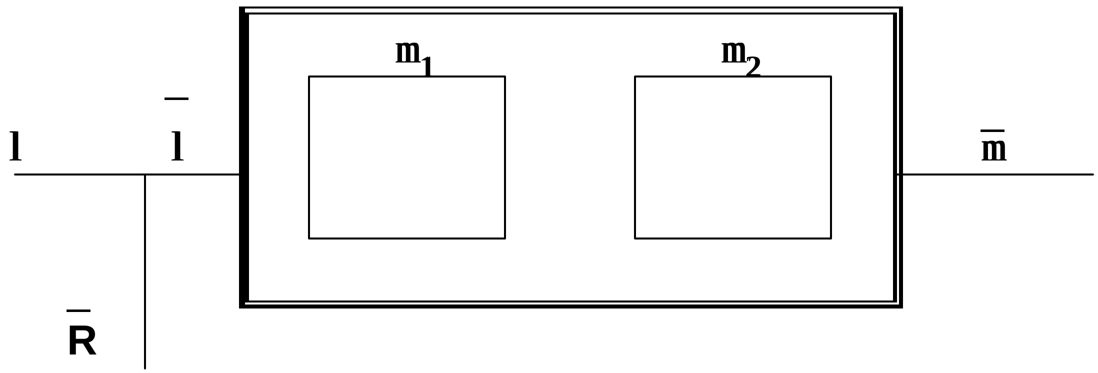
0,1

1,1

b,1

$$W = \frac{L}{\bar{\lambda}}$$

$$W_B = \frac{B}{\bar{\lambda}}$$



Estados posibles:

0,0

1,0

0,1

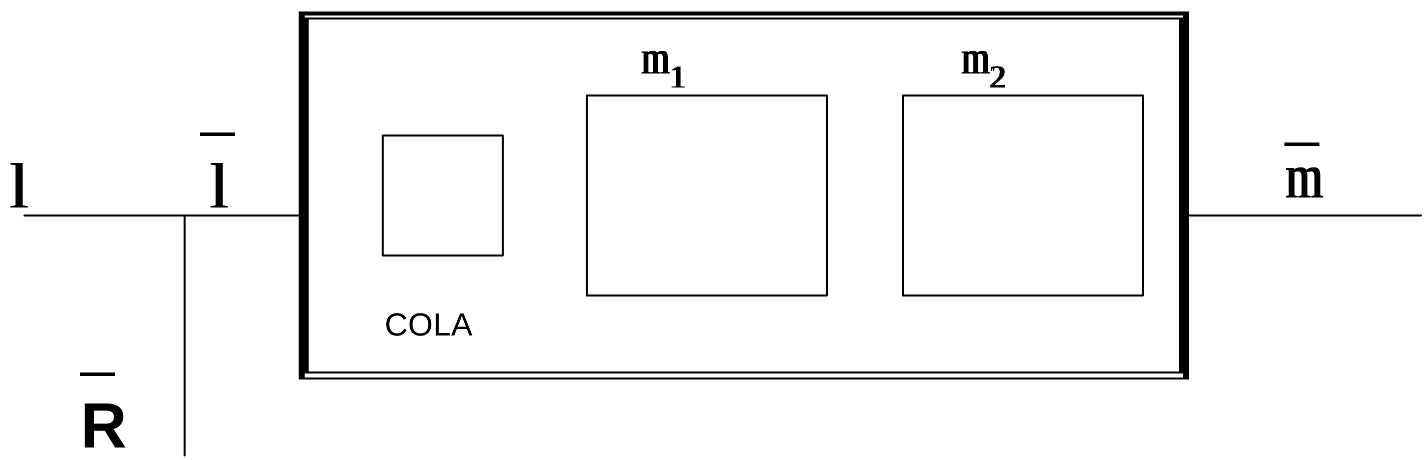
1,1

b,1

$$p(R) = p(1,0) + p(1,1) + p(b,1)$$

$$p(I) = p(0,0) + p(0,1)$$

$$\overline{R} = \lambda \cdot \left[p(1,0) + p(1,1) + p(b,1) \right]$$



0,0,0

0,1,0

0,0,1

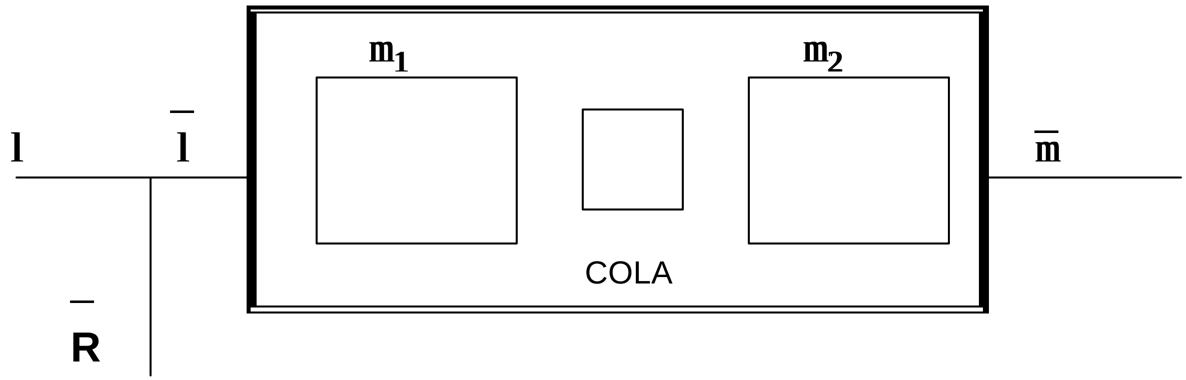
0,1,1

0,b,1

1,1,1

1,1,0

1,b,1



0,0,0

1,0,0

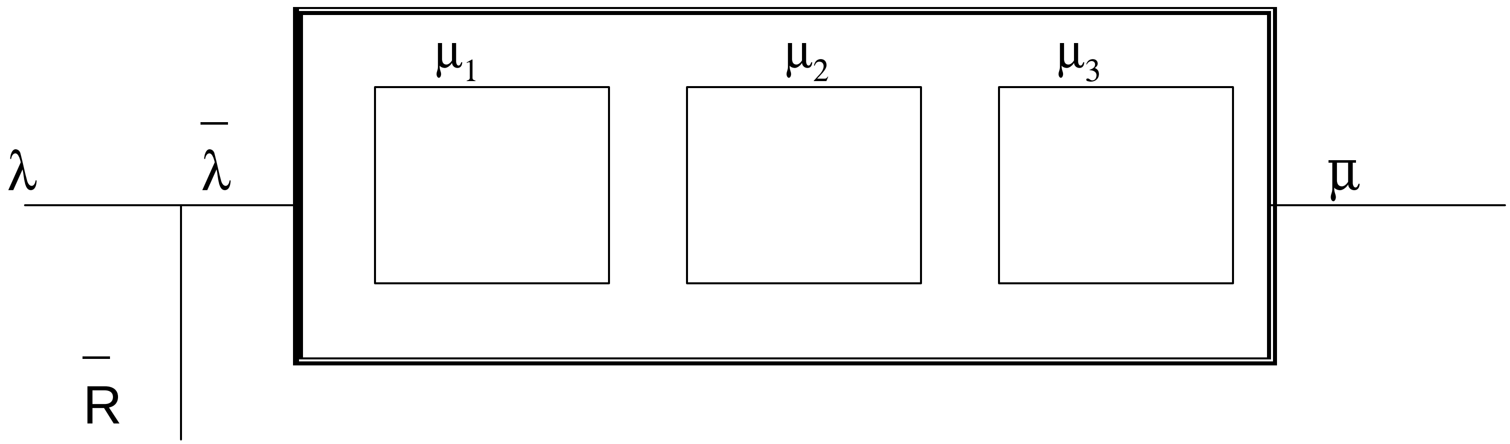
0,0,1

0,1,1

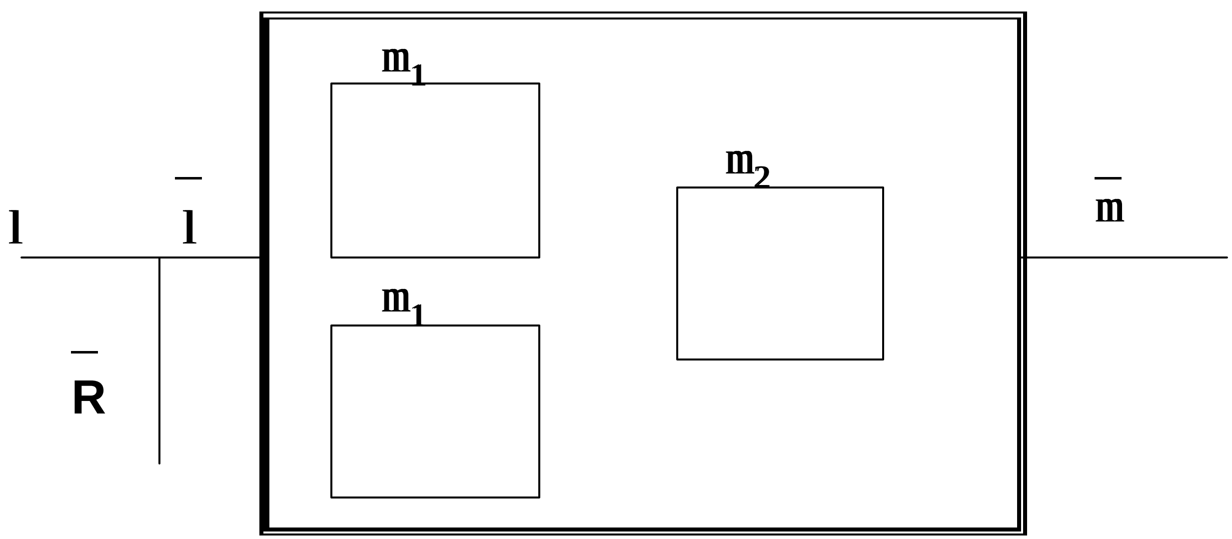
1,1,1

1,0,1

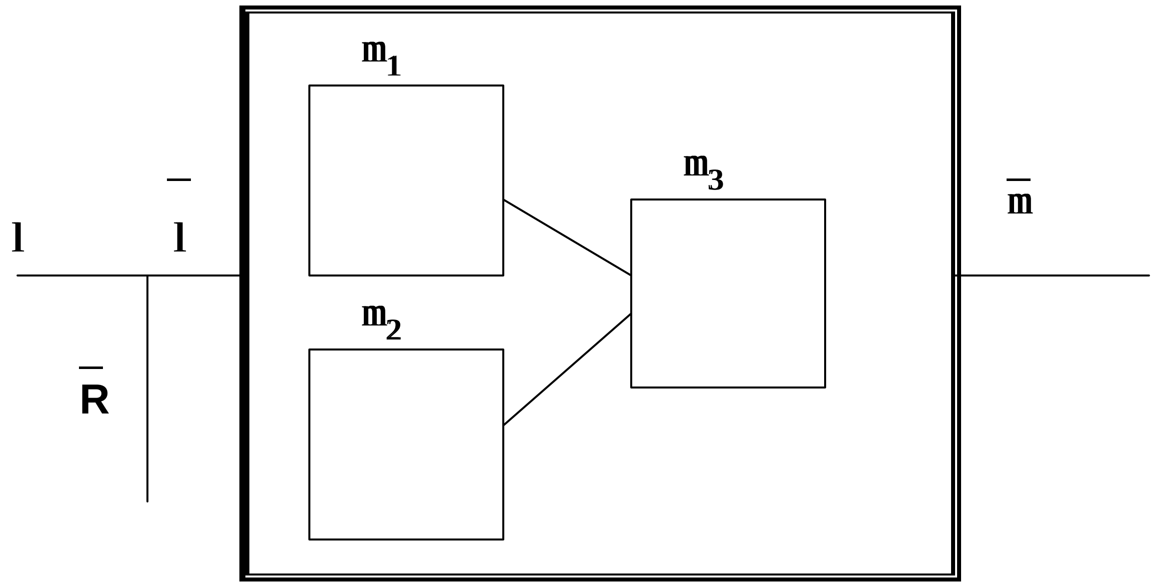
b,1,1



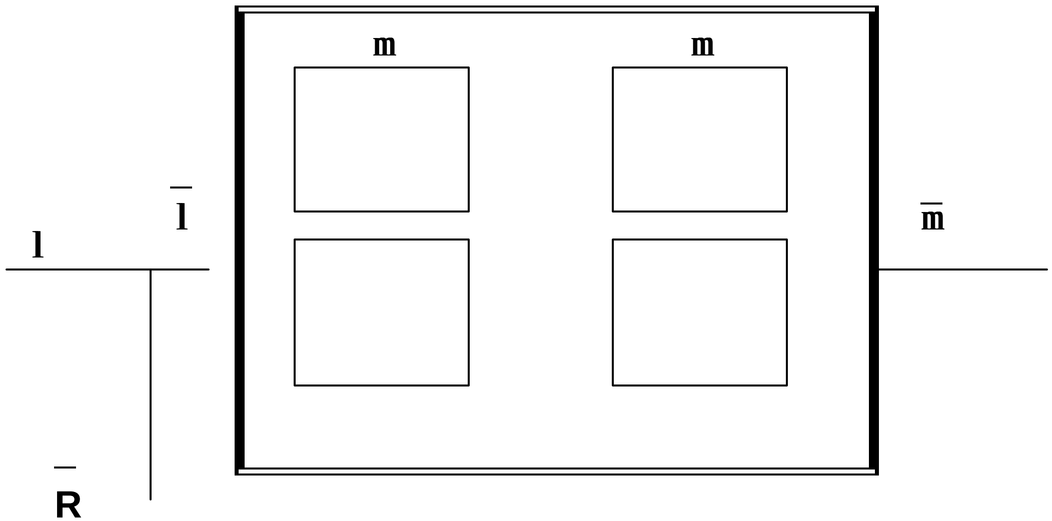
λ
$\bar{\lambda}$
$\bar{R}$
μ_1
μ_2
μ_3
μ



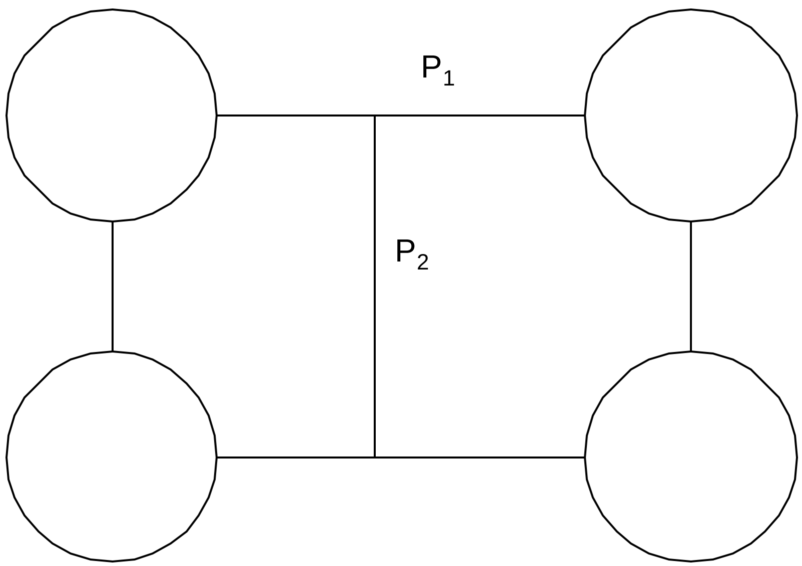
l
$\bar{l}$
$\bar{R}$
m_1
m_1
m_2
$\bar{m}$



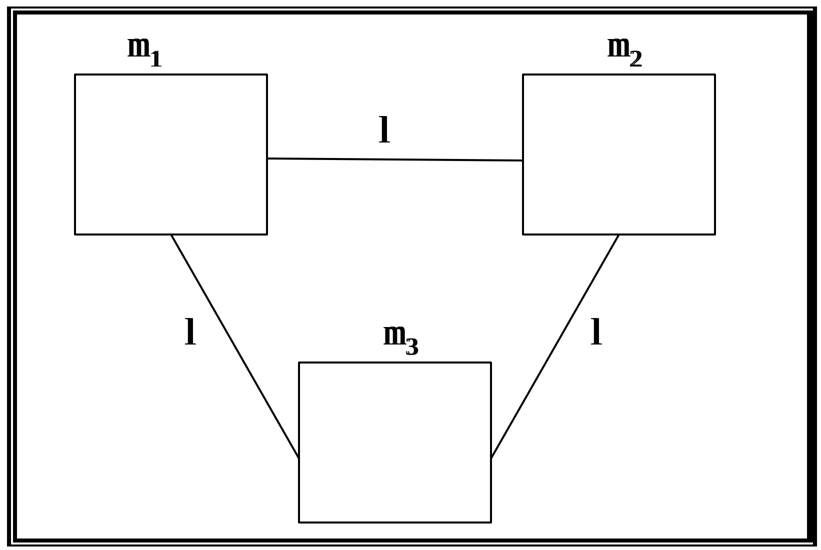
l
$\bar{l}$
$\bar{R}$
m_1
m_2
m_3
$\bar{m}$


m
m
$\bar{l}$
l
$\bar{m}$
$\bar{R}$

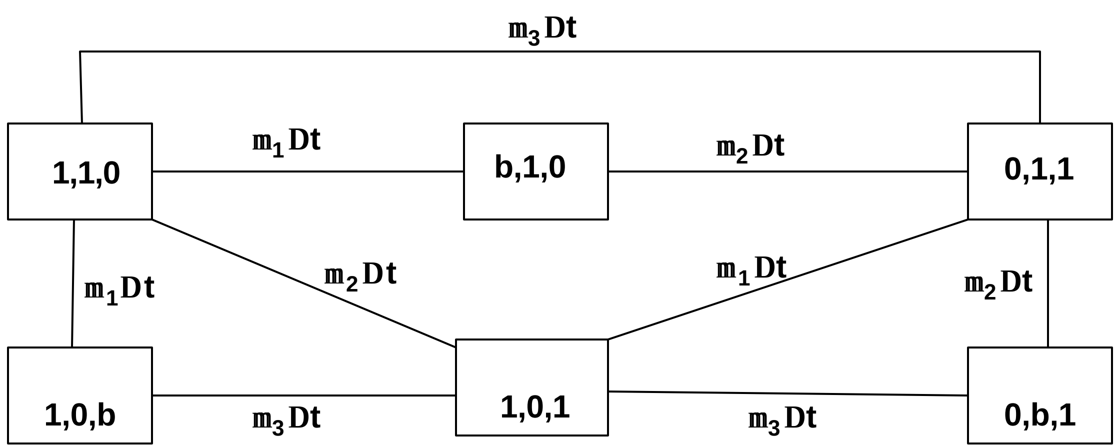
REDES CERRADAS



$\mu_1 = 10$
$\mu_2 = 9,6$



1,1,0
b,1,0
1,0,1
1,0,b
0,1,1
0,b,1



$$p(1,1,0)\cdot(\mu_1+\mu_2)=p(1,0,b)\cdot\mu_1+p(0,1,1)\cdot\mu_3$$

$$p(0,1,1)\cdot(\mu_2+\mu_3)=p(b,1,0)\cdot\mu_2+p(1,0,1)\cdot\mu_1$$

$$p(b,1,0)\cdot\mu_2=p(1,1,0)\cdot\mu_1$$

$$p(0,b,1)\cdot\mu_3=p(0,1,1)\cdot\mu_2$$

$$p(1,0,b)\cdot\mu_1=p(1,0,1)\cdot\mu_3$$

$$p(1,1,0)+p(1,0,1)+p(0,1,1)+p(b,1,0)+p(0,b,1)+p(1,0,b)=1$$