P/P/1

- Arribos Po
- Servicio Po
- Régimen permanente
- 1 canal
- 1 cola
- FIFO
- Capacidad infinita
- Sin impaciencia
- Población infinita

Parámetros

- λ

$$T_a = \frac{1}{\lambda}$$

- μ

$$T_s = \frac{1}{\mu}$$

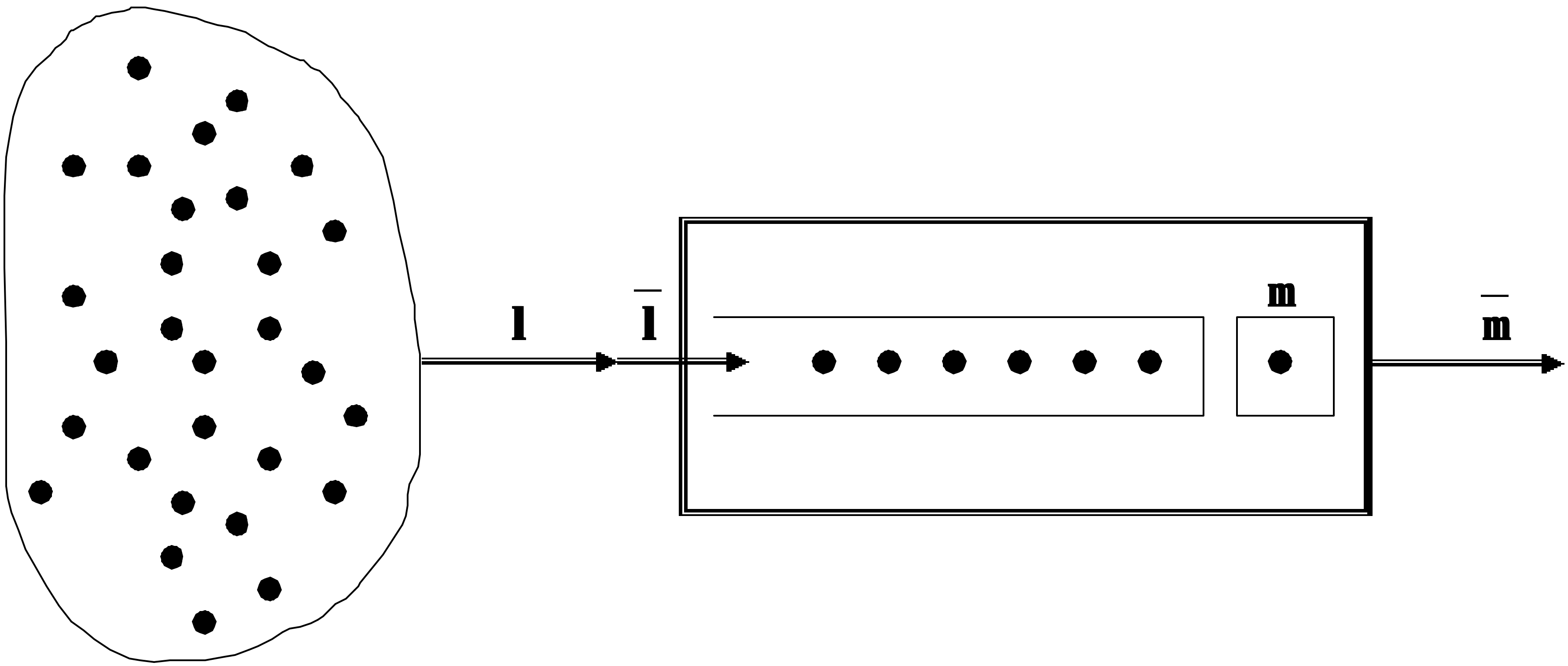
- M = 1

Variables

- $p(n)$
- L_c
- L
- H
- W_c
- W
- $\varphi(W_c)$
- $\varphi(W)$

$\bar{\lambda}$

$\bar{\mu}$



l
l̄
m
m̄

$$\lambda_n = \lambda \qquad n = 0, 1, 2, 3, \ldots \qquad \lambda_n = \lambda \cdot p(i/n)$$

$$\mu_n = \begin{cases} 0 & n = 0 \\ \mu & n = 1, 2, 3, \ldots \end{cases}$$

$$\boxed{p(n) = \frac{\lambda_{n-1}}{\mu_n} \cdot p(n-1)}$$

$$p(1) = \frac{\lambda_0}{\mu_1} \cdot p(0) = \frac{\lambda}{\mu} \cdot p(0) \quad \Rightarrow \quad p(1) = \rho \cdot p(0)$$

$$p(2) = \frac{\lambda_1}{\mu_2} \cdot p(1) = \frac{\lambda}{\mu} \cdot p(0) \cdot \frac{\lambda}{\mu} \quad \Rightarrow \quad p(2) = \rho^2 \cdot p(0)$$

$$p(3) = \frac{\lambda_2}{\mu_3} \cdot p(2) = \frac{\lambda}{\mu} \cdot p(0) \cdot \frac{\lambda^2}{\mu^2} \quad \Rightarrow \quad p(3) = \rho^3 \cdot p(0)$$

$$\boxed{p(n) = \rho^n \cdot p(0)}$$

$$\sum_{0}^{\infty} p(n) = 1$$

$$\sum_{0}^{\infty} \rho^{n} \cdot p(0) = 1 \qquad \therefore \qquad p(0) \cdot \sum_{0}^{\infty} \rho^{n} = 1$$

$$p(0) = \frac{1}{\sum_{0}^{\infty} \rho^{n}} \qquad \frac{\lambda}{\mu} < 1 \qquad \lambda < \mu$$

$$\sum_{0}^{\infty} \rho^{n} = \frac{\rho^{0} \cdot \left(\rho^{\infty} - 1\right)}{\rho - 1} = \frac{1}{1-\rho}$$

$$p(0) = \frac{1}{1/(1-\rho)}$$

$$p(0) = 1 - \rho$$

$$p(n > k) = \sum_{k+1}^{\infty} p(n)$$

$$p(n > k) = \sum_{k+1}^{\infty} \rho^{n} \cdot (1-\rho) = (1-\rho) \cdot \sum_{k+1}^{\infty} \rho^{n} = (1-\rho) \cdot \frac{\rho^{k+1} \cdot (\rho^{\infty} - 1)}{\rho - 1} = (1-\rho) \cdot \frac{\rho^{k+1}}{(1-\rho)}$$

$$p(n > k) = \rho^{k+1}$$

$$p(n \geq k) = \rho^{k}$$

$$L = \sum_{0}^{\infty} n \cdot p(n)$$

$$L = \sum_{0}^{\infty} n \cdot \rho^{n} \cdot (1-\rho) = (1-\rho) \cdot \sum_{0}^{\infty} n \cdot \rho^{n}$$

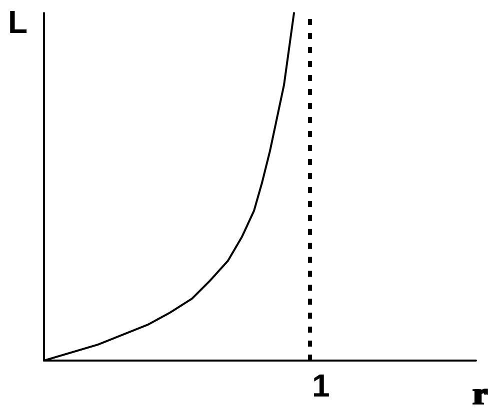
$$\sum_{0}^{\infty} \rho^{n} = \frac{1}{1-\rho}$$

$$\sum_{0}^{\infty} n \cdot \rho^{n-1} = \frac{1}{(1-\rho)^{2}}$$

$$\sum_{0}^{\infty} n \cdot \rho^{n} = \frac{\rho}{(1-\rho)^{2}}$$

$$L = (1-\rho) \cdot \frac{\rho}{(1-\rho)^{2}} = \frac{\rho}{1-\rho} = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} = \frac{\lambda}{\mu \cdot \left(1-\frac{\lambda}{\mu}\right)}$$

$$L = \frac{\lambda}{\mu - \lambda}$$



$$L_C = \sum_{1}^{\infty}(n-1)\cdot p(n) = \sum_{1}^{\infty} n\cdot p(n) - \sum_{1}^{\infty} p(n) = \sum_{0}^{\infty} n\cdot p(n) - [1 - p(0)]$$

$$L_C = L - [1 - p(0)]$$

$$L_C = L - \rho$$

$$L_C = \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu} = \frac{\lambda\cdot\mu - \lambda\cdot(\mu-\lambda)}{(\mu-\lambda)\cdot\mu}$$

$$L_C = \frac{\lambda^2}{(\mu-\lambda)\cdot\mu}$$

$$H = \sum_{1}^{\infty} p(n) = 1 - p(0)$$

$$H = \rho$$

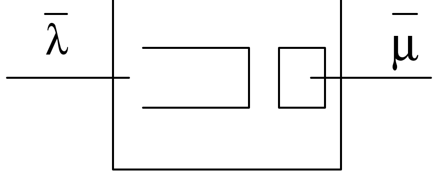
$$L = L_C + H$$

$$PA = \frac{H}{M} = \frac{\rho}{1} = \rho$$

$$\bar{\lambda} = \sum_{0}^{\infty} \lambda_n p(n)$$

$$\bar{\lambda} = \sum_{0}^{\infty} \lambda \cdot p(n) = \lambda \cdot \sum_{0}^{\infty} p(n) = \lambda$$

$$\bar{\lambda} = \bar{\mu}$$

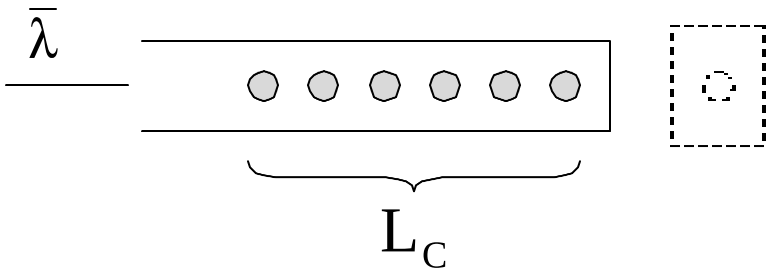


$$\bar{\mu} = \sum_{0}^{\infty} \mu_n \cdot p(n)$$

$$\bar{\mu} = \sum_{1}^{\infty} \mu \cdot p(n) = \mu \cdot [1 - p(0)] = \mu \cdot H$$

$$\lambda = \mu \cdot [1 - p(0)]$$

$$\rho = 1 - p(0)$$

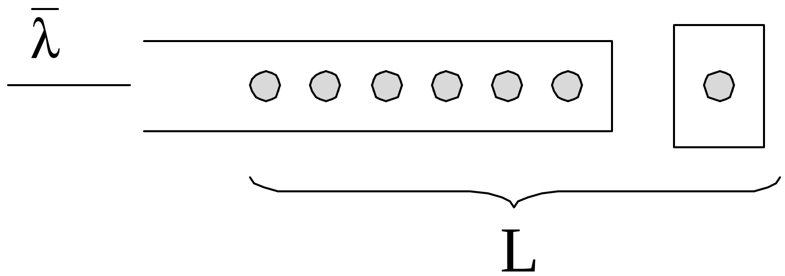


$$W_C = \frac{L_C}{\bar{\lambda}}$$

LITTLE

$$\boxed{W_C = \frac{L_C}{\lambda}}$$

$$W_C = \frac{\lambda^2}{\mu \cdot (\mu - \lambda) \cdot \lambda} = \frac{\lambda}{\mu \cdot (\mu - \lambda)}$$



$$W = W_C + T_S$$

$$W = \frac{\lambda}{\mu \cdot (\mu - \lambda)} + \frac{1}{\mu} = \frac{\lambda + (\mu - \lambda)}{\mu \cdot (\mu - \lambda)} = \frac{1}{\mu - \lambda}$$

$$W = \frac{L}{\bar{\lambda}}$$ LITTLE

$$W = \frac{L}{\lambda}$$

$$\varpi(t) = p(W > t) = e^{-(\mu-\lambda).t}$$

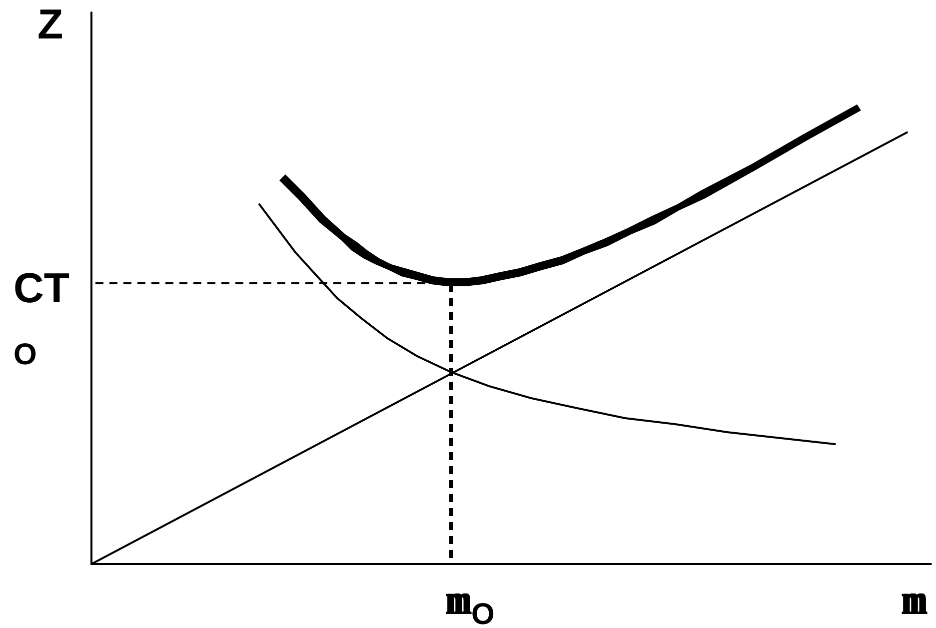
$$\varpi_C(t) = p(W_C > t) = \rho \cdot e^{-(\mu-\lambda)\cdot t}$$

$$\varpi_C(0) = p(W_C > 0) = \rho \cdot e^{-(\mu-\lambda)\cdot 0} = \rho$$

ANÁLISIS ECONÓMICO

$c_e \left(\frac{\$}{t \cdot cl} \right)$

$c_s \left(\frac{\$}{t \cdot \frac{cl}{t}} \right)$



$$Z = c_e \cdot L + c_s \cdot \mu \quad \rightarrow \quad Min$$

$$Z = c_e \cdot \frac{\lambda}{\mu - \lambda} + c_s \cdot \mu \quad \rightarrow \quad Min$$

$$Z = c_e \cdot \frac{\lambda}{\mu - \lambda} + c_s \cdot \mu \quad \rightarrow \quad \text{Min}$$

$$\frac{\partial Z}{\partial \mu} = -c_e \cdot \frac{\lambda}{(\mu - \lambda)^2} + c_s = 0$$

$$c_e \cdot \frac{\lambda}{(\mu - \lambda)^2} = c_s$$

$$(\mu - \lambda)^2 = \lambda \cdot \frac{c_e}{c_s}$$

$$(\mu - \lambda) = \pm\sqrt{\lambda \cdot \frac{c_e}{c_s}}$$

$$\boxed{\mu_o = \lambda + \sqrt{\lambda \cdot \frac{c_e}{c_s}}}$$

A un sistema P/P/1, sin impaciencia, arriba un cliente cada 12 minutos, en promedio. La duración media del servicio es de 10 minutos. Determinar:

- ***Probabilidad de que un cliente que arriba al sistema no tenga que esperar para recibir el servicio.***
- ***Porcentaje de ocupación del canal.***
- ***Probabilidad de que un cliente que llega al sistema encuentre que hay más de tres personas esperando en cola.***
- ***Probabilidad de que haya menos de dos personas en el sistema.***
- ***Número promedio de clientes dentro del sistema.***
- ***Número promedio de clientes esperando ser atendidos.***
- ***Ingreso de caja esperado, si cada servicio se cobra $50.***
- ***Tiempo de espera promedio de un cliente en cola.***
- ***Tiempo promedio de permanencia de un cliente dentro del sistema.***
- ***Probabilidad de que un cliente permanezca dentro del sistema más de 5 minutos.***
- ***Probabilidad de que un cliente tenga que esperar en cola más de 3 minutos.***
- ***La velocidad óptima de atención, suponiendo que el lucro cesante es de 20$/h por cada cliente que se encuentre dentro del sistema, y el costo por atender a una velocidad de 1 cl/h es de $15/h.***

$$\begin{cases} \lambda = \dfrac{1}{12}\dfrac{\text{cl}}{\text{min}} \cdot 60\dfrac{\text{cl}}{\text{h}} = 5\ \dfrac{\text{cl}}{\text{h}} \\ \mu = \dfrac{1}{10}\dfrac{\text{cl}}{\text{min}} \cdot 60\dfrac{\text{cl}}{\text{h}} = 6\ \dfrac{\text{cl}}{\text{h}} \\ \rho = \dfrac{\lambda}{\mu} = \dfrac{5}{6} = 0{,}8333 \end{cases}$$

$$p(0) = 1 - \rho = 0{,}1667$$

$$\rho = 0{,}8333$$

$$p(n \geq 4) = \rho^4 = 0{,}4823$$

$$\begin{cases} p(1) = \rho \cdot p(0) = 0{,}8333 \cdot 0.1667 = 0{,}1389 \\ p(n \leq 1) = p(0) + p(1) = 0.1667 + 0{,}1389 = 0{,}3056 \end{cases}$$

$$L = \frac{\lambda}{\mu - \lambda} = \frac{5}{6-5} = 5\ \text{cl}$$

$$L_C = L - \rho = 5 - 0{,}8333 = 4{,}1667\,\text{cl}$$

$$\begin{cases} \lambda = \frac{1}{12}\frac{\text{cl}}{\text{min}} \cdot 60\frac{\text{cl}}{\text{h}} = 5\ \frac{\text{cl}}{\text{h}} \\ \mu = \frac{1}{10}\frac{\text{cl}}{\text{min}} \cdot 60\frac{\text{cl}}{\text{h}} = 6\ \frac{\text{cl}}{\text{h}} \\ \rho = \frac{\lambda}{\mu} = \frac{5}{6} = 0{,}8333 \end{cases}$$

$$\text{Ingreso} = u \cdot \bar{\lambda} = u \cdot \lambda = 50 \cdot 5 = 250\frac{\$}{\text{h}}$$

$$W_C = \frac{L_C}{\bar{\lambda}} = \frac{4{,}1667}{5} = 0{,}83333\text{h} = 50\ \text{min}$$

$$W = W_C + T_S = 0{,}8333 + 0{,}1667 = 1\ \text{h} = 60\ \text{min}$$

$$\varpi(0{,}08333) = e^{-(6-5).0{,}0833} = 0{,}9200 = 92\%$$

$$\varpi_C(0{,}05) = 0.1667 \cdot e^{-(6-5).0{,}05} = 0{,}7927 = 79{,}27\%$$

$$\mu_o = \lambda + \sqrt{\lambda \cdot \frac{c_e}{c_s}} = 5 + \sqrt{5 \cdot \frac{20}{15}} = 7{,}5820\ \frac{\text{cl}}{\text{h}}$$