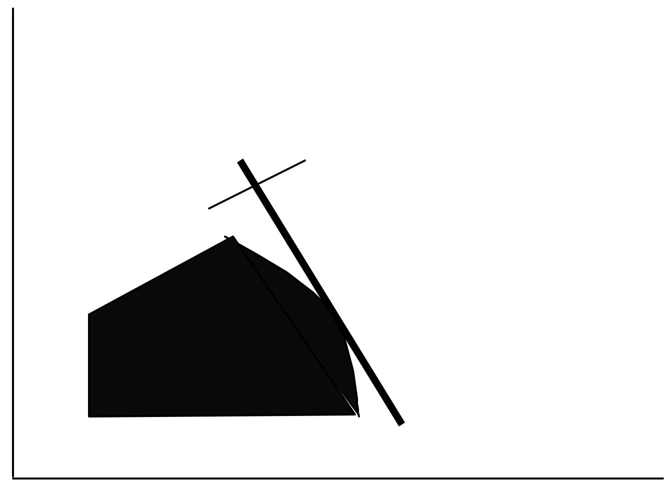
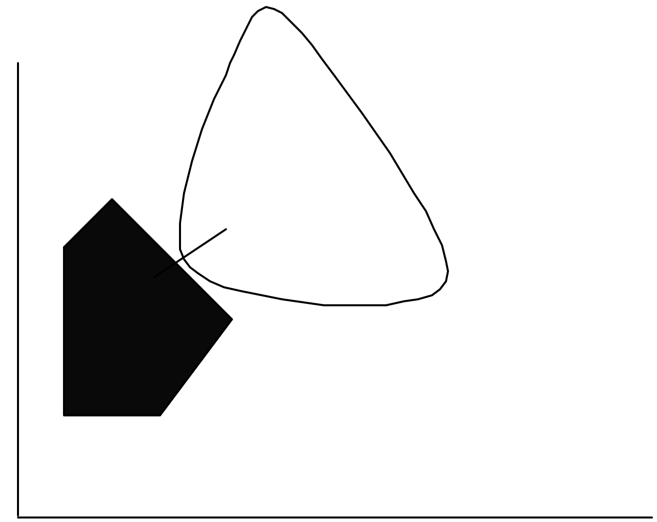
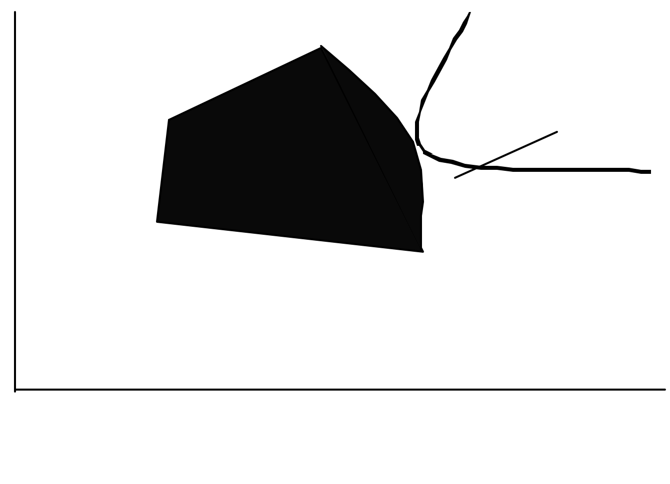






Funciones separables

$$f(x_i) = f_1(x_1) + f_2(x_2) + f_3(x_3) + \ldots\ldots + f_N(x_N)$$

$$x_1^2 + 2 \cdot x_2 + e^{x_3}$$

$$x_1 + x_2^2 + 3 \cdot \log x_3$$

Funciones no separables

$$x_1 - \frac{x_2}{1+x_3} + x_4 + x_4 \cdot e^{-x_3}$$

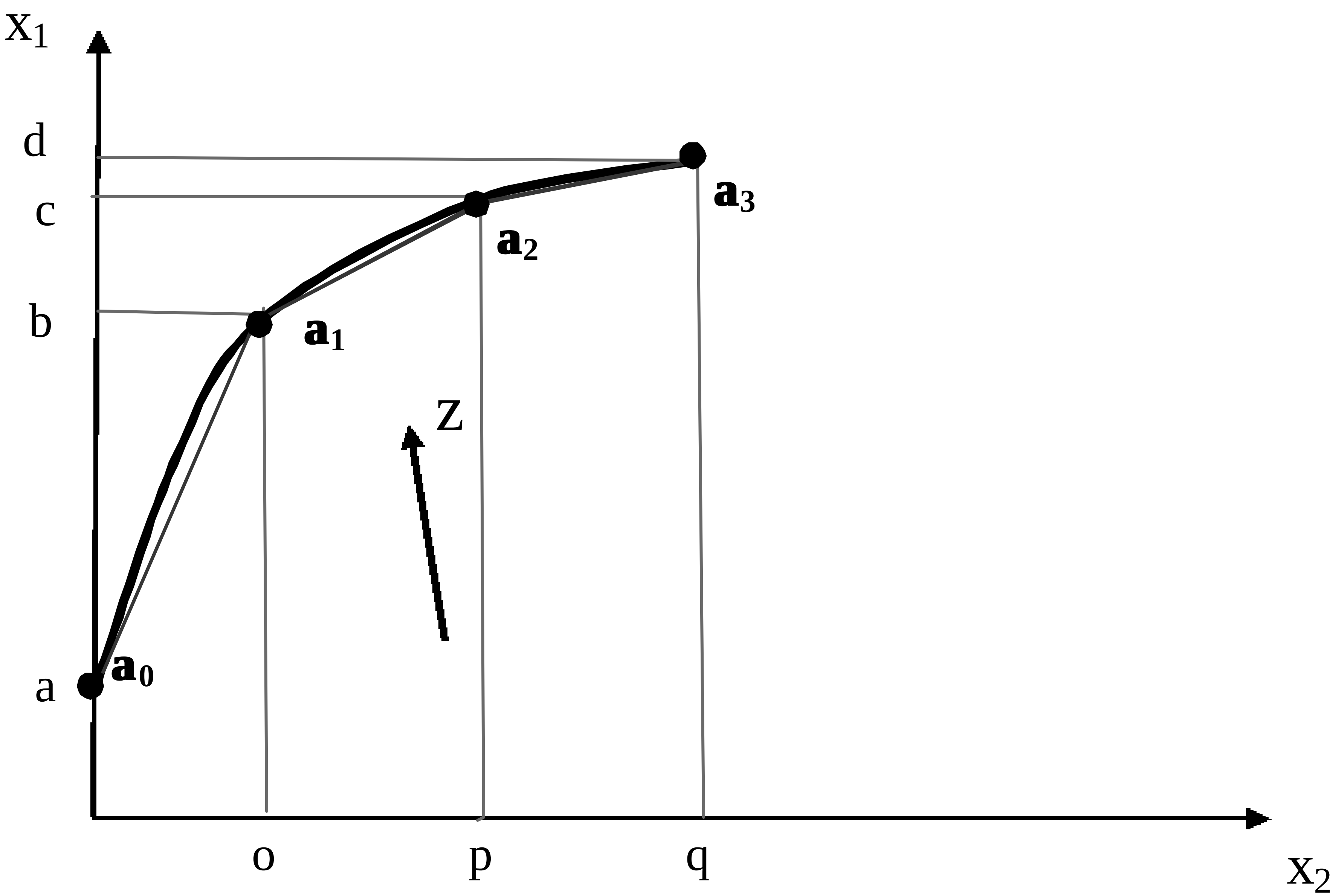
$$x_1 \; x_2 + x_3 \; \ln x_4$$

$$x_1 \; x_2$$

$$x_1 = p + q$$

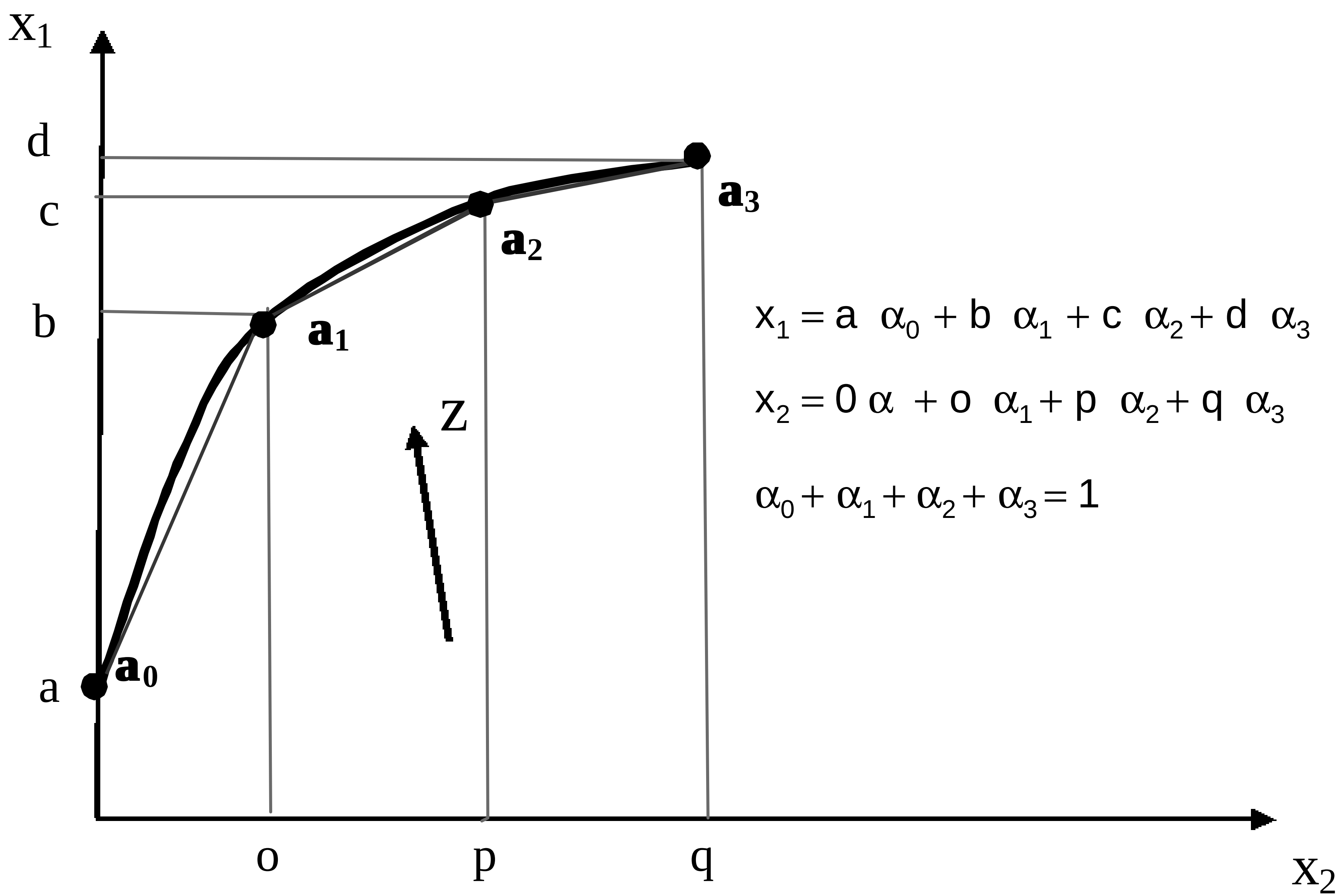
$$x_2 = p - q$$

$$x_1 \; x_2 = (p + q)\,(p - q) = p^2 - q^2$$


x_1
d
c
b
a
$\mathbf{a}_3$
$\mathbf{a}_2$
$\mathbf{a}_1$
$\mathbf{a}_0$
z
o
p
q
x_2

Vectores “mesh”

- La aproximación es tanto más precisa cuanto más vectores α_i se introduzcan
- El procedimiento es aplicable sólo a funciones convexas
- En la solución, debe estar activo una sola variable mesh o a lo sumo dos adyacentes.



$$x_1 = a\ \ \alpha_0 + b\ \ \alpha_1 + c\ \ \alpha_2 + d\ \ \alpha_3$$

$$x_2 = 0\ \alpha\ + o\ \ \alpha_1 + p\ \ \alpha_2 + q\ \ \alpha_3$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$

Ejemplo:

$$\text{MIN} \quad 3x_1^2 + 4x_1 + 15x_2$$

$$\begin{aligned} x_1 + \quad x_2 &\geq 4 \\ 2\,x_1 + 0.5\,x_2 &\leq 5 \\ -\,3\,x_1 + 2.8\,x_2 &\geq 2 \end{aligned}$$

Ejemplo:

$$y = x_1^2$$

$$\text{MIN} \; 3x_1^2 + 4x_1 + 15x_2$$

$$x_1 + x_2 \geq 4$$
$$2x_1 + 0.5x_2 \leq 5$$
$$-3x_1 + 2.8x_2 \geq 2$$

x_1	**y**	**Vector**
0	0	α_0
1	1	α_1
2	4	α_2
2.5	6.25	α_3

$$y = x_1^2$$

x_1	y	Vector
0	0	α_0
1	1	α_1
2	4	α_2
2.5	6.25	α_3

$$\text{MIN } 3y + 4x_1 + 15x_2$$

$$x_1 + x_2 \geq 4$$

$$2x_1 + 0.5x_2 \leq 5$$

$$-3x_1 + 2.8x_2 \geq 2$$

$$-x_1 + \alpha_1 + 2\alpha_2 + 2.5\alpha_3 = 0$$

$$-y + \alpha_1 + 4\alpha_2 + 6.25\alpha_3 = 0$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$

OBJECTIVE FUNCTION VALUE

Z) 50.82759

VARIABLE	VALUE
x_1	1.586207
x_2	2.413793
y	2.758621
α_1	0.413793
α_2	0.586207
α_3	0.000000
α_0	0.000000

Otro ejemplo

$$\text{Max} \quad x_1 - x_2$$

$$\text{st} \quad -x_1^2 + x_2 \geq 1$$

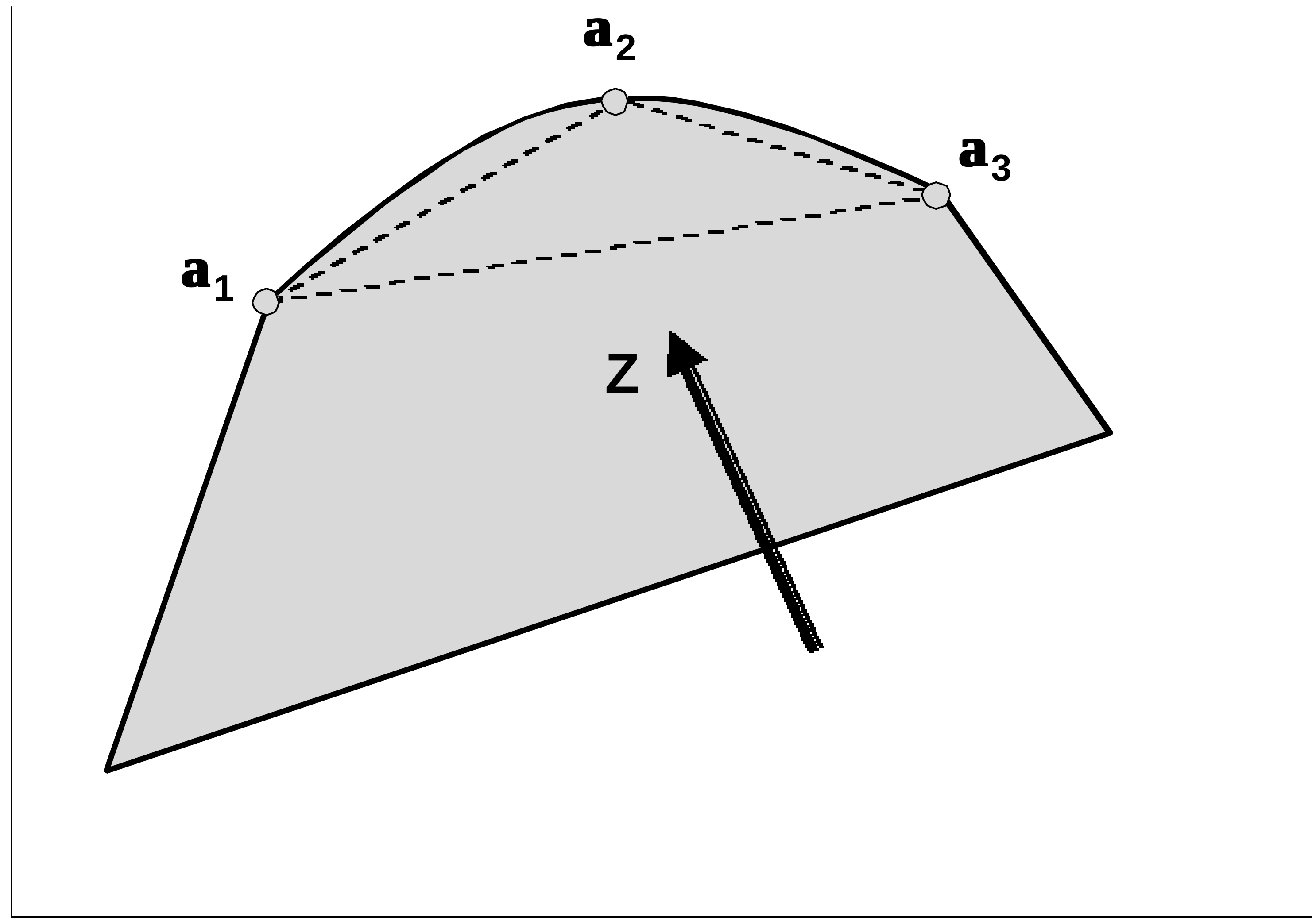
$$x_1 + x_2 \leq 3$$

$$-x_1 + x_2 \leq 2$$

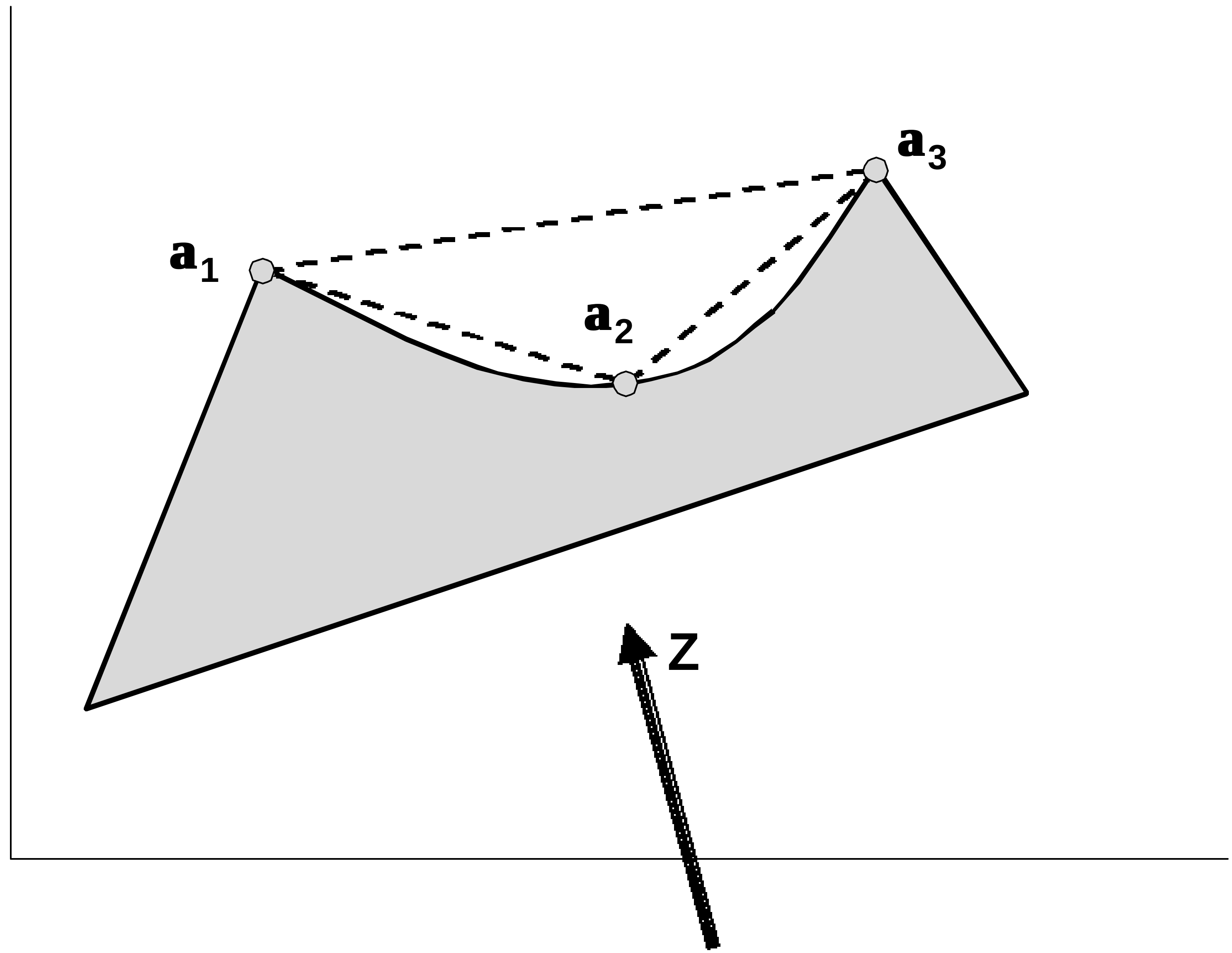
$$x_1, x_2 \geq 0$$

x_1	A	
0.0	0.00	A_0
0.2	0. 04	A_1
0.4	0. 16	A_2
0.6	0.36	A_3
0.8	0.64	A_4
1.0	1.00	A_5

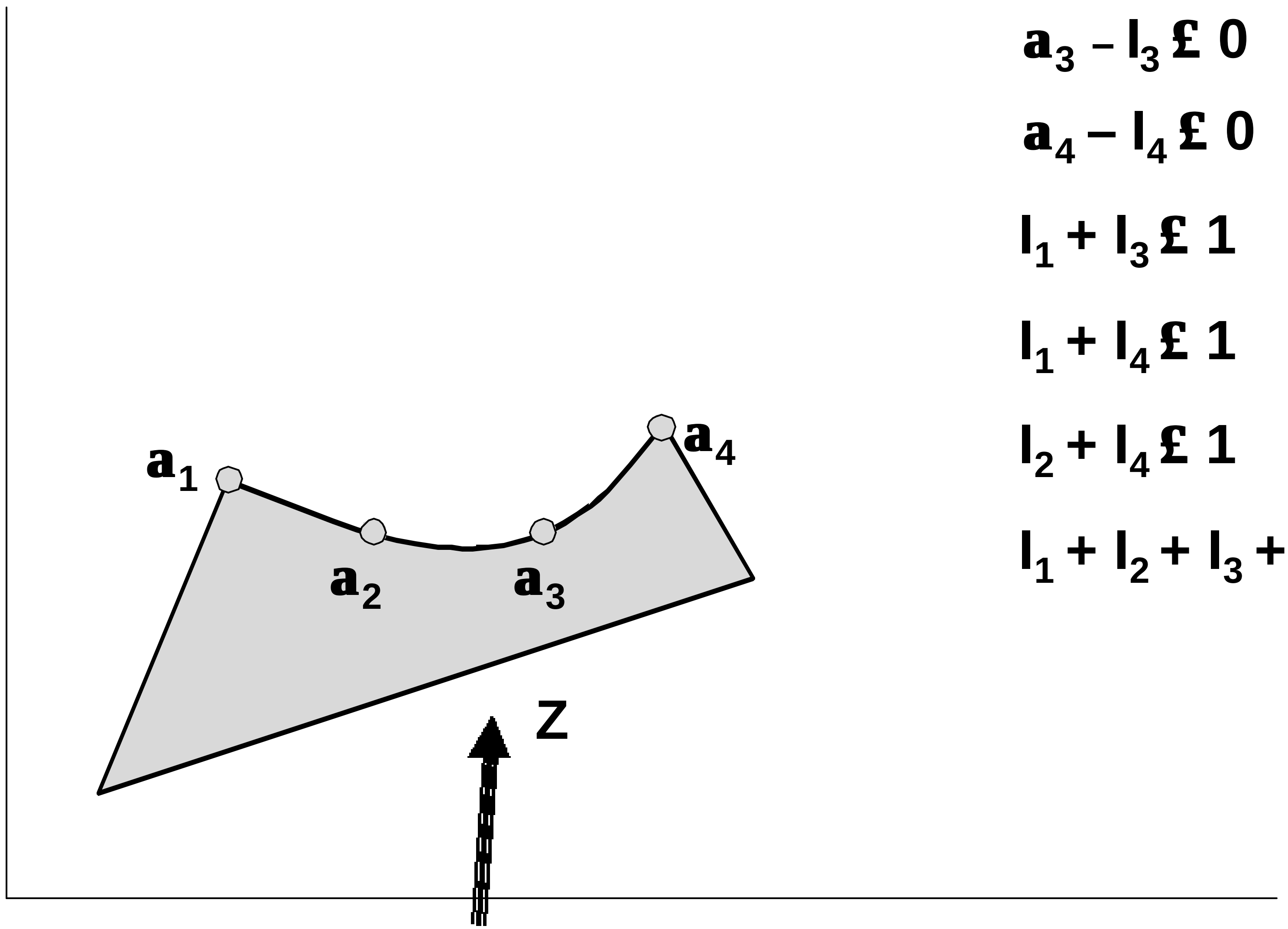
$$\begin{aligned}
&\text{Max} \quad x_1 - x_2 \\
&\text{st} \quad -A + x_2 \geq 1 \\
&\qquad x_1 + x_2 \leq 3 \\
&\quad -x_1 + x_2 \leq 2 \\
&-x_1 + 0.2A_1 + 0.4A_2 + 0.6A_3 + 0.8A_4 + A_5 = 0 \\
&-A + 0.04A_1 + 0.16A_2 + 0.36A_3 + 0.64A_4 + A_5 = 0 \\
&A_0 + A_1 + A_2 + A_3 + A_4 + A_5 = 1
\end{aligned}$$



a2
a3
a1
Z



a_1
a_2
a_3
Z



$a_1 - l_1$ £ 0

$a_2 - l_2$ £ 0

$a_3 - l_3$ £ 0

$a_4 - l_4$ £ 0

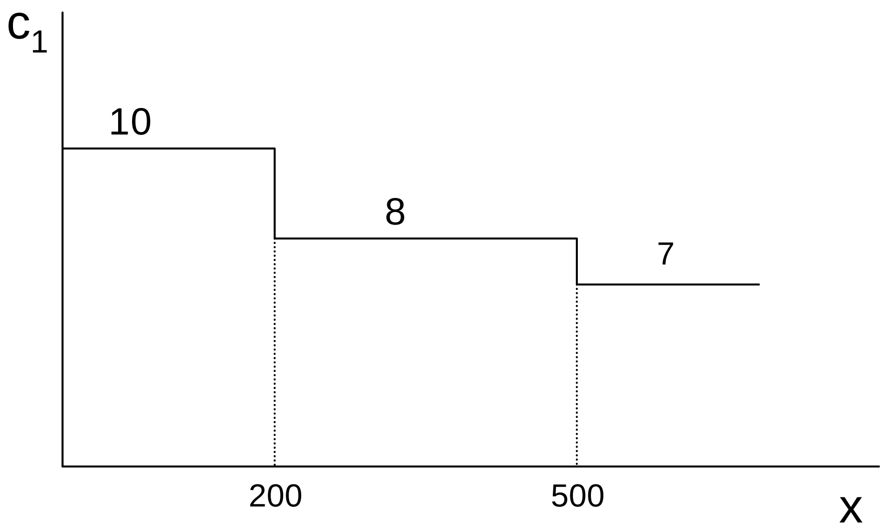
$l_1 + l_3$ £ 1

$l_1 + l_4$ £ 1

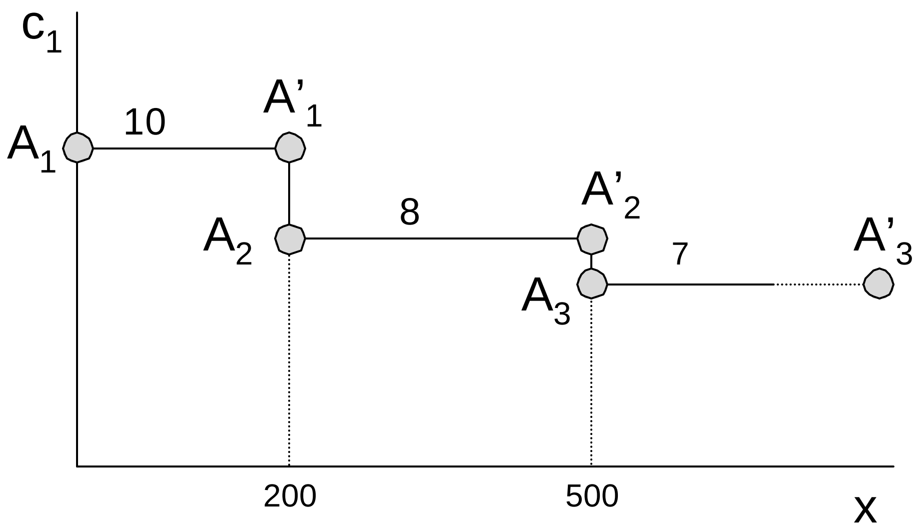
$l_2 + l_4$ £ 1

$l_1 + l_2 + l_3 + l_4$ £ 2

Descuentos por cantidad



Descuentos por cantidad

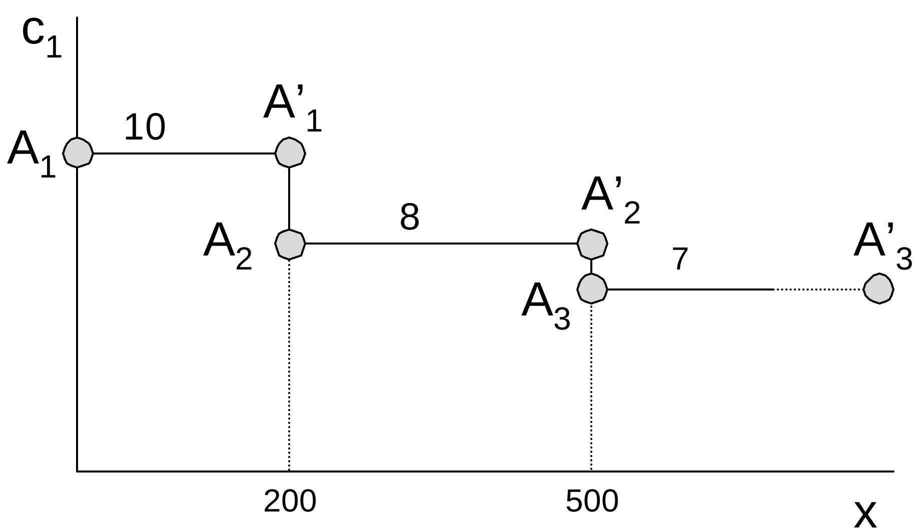


Descuentos por cantidad

$$- X + 0\ A_1 + 200\ A'_1 + 200\ A_2 + 500\ A'_2 + 500\ A_3 + M\ A'_3 = 0$$

$$A_1 + A'_1 + A_2 + A'_2 + A_3 + A'_3 = 1$$

$$Z = \ldots + 0\ A_1 + 2000\ A'_1 + 1600\ A_2 + 4000\ A'_2 + 3500\ A_3 + 7M\ A'_3 \ldots \qquad \text{MIN}$$



$$\textbf{MAX:} \quad Z = 4\,x_1 + c_2\,x_2$$

Sujeto a:

$$\begin{cases} 6\,x_1 + 16\,x_2 \leq 48000 \\ 12\,x_1 + 6\,x_2 \leq 42000 \\ 9\,x_1 + 9\,x_2 \leq 36000 \end{cases}$$

siendo: $x_1, x_2 \geq 0$ **y continuas**

$c_2 = 3$ **para** $x_2 \leq 500$

$c_2 = 5$ **para** $x_2 > 500$

$$6\,x_1 + 16\,x_2 \;\pounds\; 48000$$

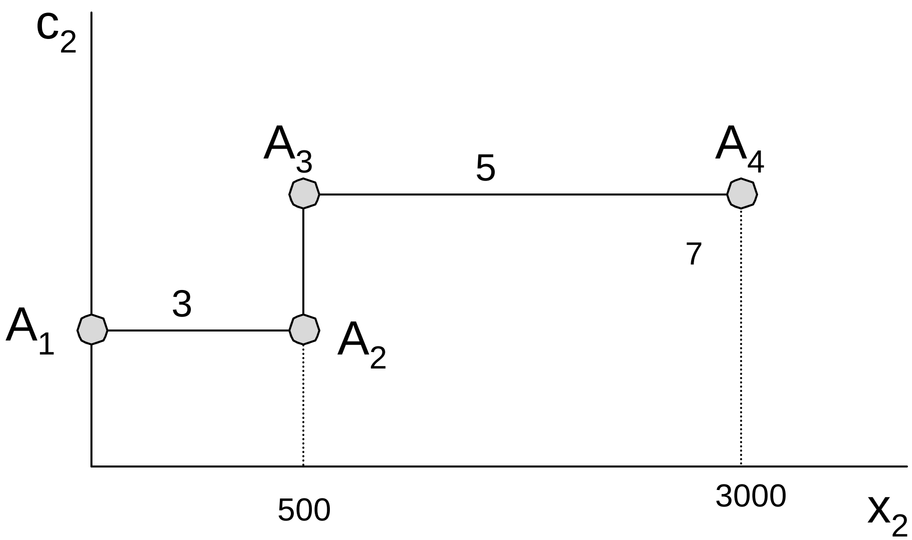
$$12\,x_1 + 6\,x_2 \;\pounds\; 42000$$

$$9\,x_1 + 9\,x_2 \;\pounds\; 36000$$

$$-x_2 + 0\,A_1 + 500\,A_2 + 500\,A_3 + 3000\,A_4 = 0$$

$$A_1 + A_2 + A_3 + A_4 = 1$$

$$\text{MAX:} \quad Z = 4\,x_1 + 1500\,A_2 + 2500\,A_3 + 15000\,A_4$$



OBJECTIVE FUNCTION VALUE

1)	18400.00

VARIABLE	VALUE	REDUCED COST
X1	1600.000000	0.000000
A2	0.000000	1000.000000
A3	0.240000	0.000000
A4	0.760000	0.000000
X2	2400.000000	0.000000
A1	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.100000
3)	8400.000000	0.000000
4)	0.000000	0.377778
5)	0.000000	-5.000000
6)	0.000000	0.000000

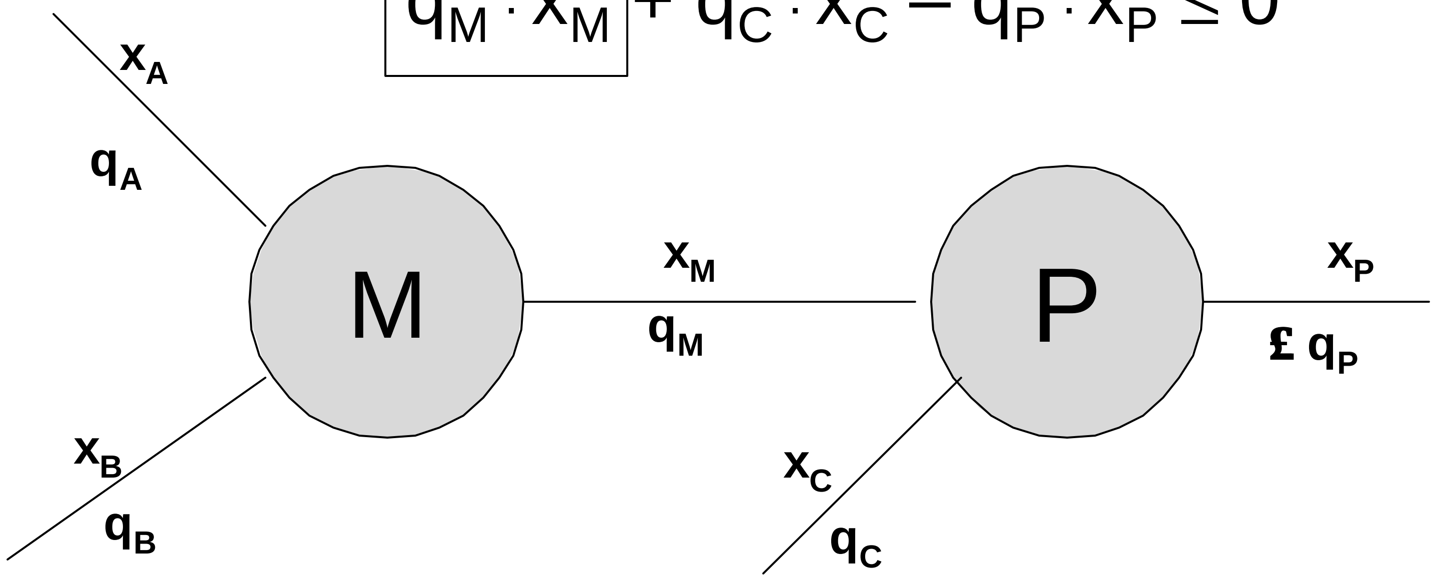
RECURRENCIA

$$x_A + x_B - x_M = 0$$

$$x_M + x_C - x_P = 0$$

$$q_A \cdot x_A + q_B \cdot x_B - \boxed{q_M \cdot x_M} = 0$$

$$\boxed{q_M \cdot x_M} + q_C \cdot x_C - q_P \cdot x_P \leq 0$$



$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

$$x_A + x_B - x_M = 0$$

$$x_M + x_C - x_P = 0$$

$$q_M \cdot x_M + q_C \cdot x_C - q_P \cdot x_P \leq 0$$

$$q_A \cdot x_A + q_B \cdot x_B - q_M \cdot x_M = 0 \Rightarrow \boxed{q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B}}$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\ X_A + 12\ x_B + 15\ x_C \Rightarrow \text{Min}$$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}25 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \mathrm{Min}$$

q_A = 3,5
q_B = 3
q_C = 2
q_P = 2,8

X_A	**40,00**
X_B	**24,00**
X_C	**36,00**
X_M	**64,00**
X_P	**100,00**
Z	**1228,00**

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3125$$

$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}3125 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\ X_A + 12\ x_B + 15\ x_C \Rightarrow \text{Min}$$

X_A	**40,00**
X_B	**20,95**
X_C	**39,04**
X_M	**60,95**
X_P	**100,00**
Z	**1237,14**

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3281$$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}3281 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

q_A = 3,5
q_B = 3
q_C = 2
q_P = 2,8

X_A	**40,00**
X_B	**20,24**
X_C	**39,76**
X_M	**60,23**
X_P	**100,00**
Z	**1239,29**

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3320$$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}3320 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

q_A = 3,5
q_B = 3
q_C = 2
q_P = 2,8

X_A	**40,00**
X_B	**20,06**
X_C	**39,94**
X_M	**60,06**
X_P	**100,00**
Z	**1239,82**

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3330$$

$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}3330 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

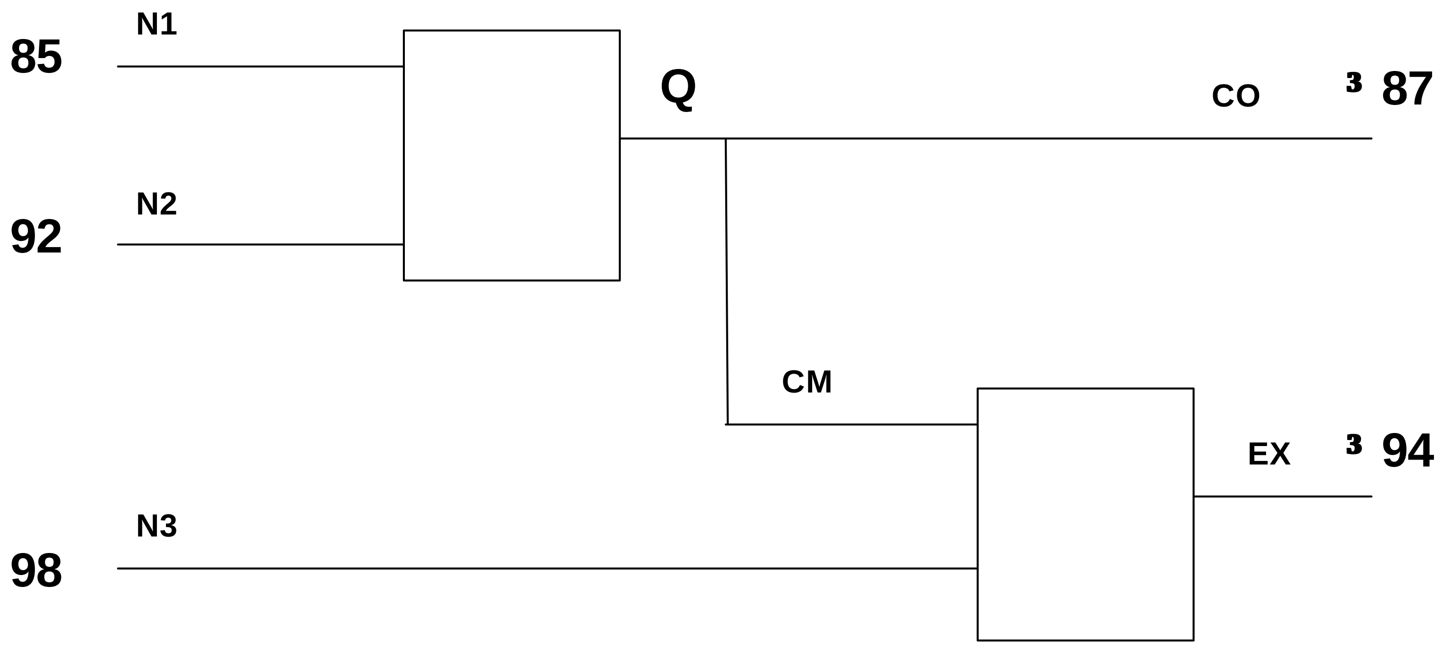
$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \mathrm{Min}$$

X_A	40,00
X_B	20,06
X_C	39,94
X_M	60,06
X_P	100,00
Z	1239,82

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3330$$



85
N1
92
N2
Q
CO
³ 87
CM
EX
³ 94
98
N3

APROXIMACIÓN SUCESIVA

MAX $1.2\ EX + 1\ CO - 0.8\ N1 - 0.85\ N2 - 1.25\ N3$

2) $N1 \leq 5000$

3) $N2 \leq 3000$

4) $N3 \leq 2000$

5) $CO \geq 2500$

6) $CO \leq 3500$

7) $EX \leq 4000$

8) $N1 + N2 - CM - CO = 0$

9) $CM + N3 - EX = 0$

10) $85\ N1 + 92\ N2 - \boxed{Q \cdot CM} - \boxed{Q \cdot CO} = 0$

11) $\boxed{Q\ \ CM} + 98\ N3 - 94\ EX \geq 0$

Iteración	Q	Z
1	87	990,82
2	88	1029,76
3	89	1094,29
4	90	945,00
5	88.5	1057,96
6	89.5	1038,89
7	88.75	1074,96
8	89,25	1075,08
9	88,875	1084,71
10	89,125	1.096,39
11	89,05	1098,46
12	89,09	1101,88

$$\text{MAX:} \quad Z = 7\,x_1 + 3\,x_2$$

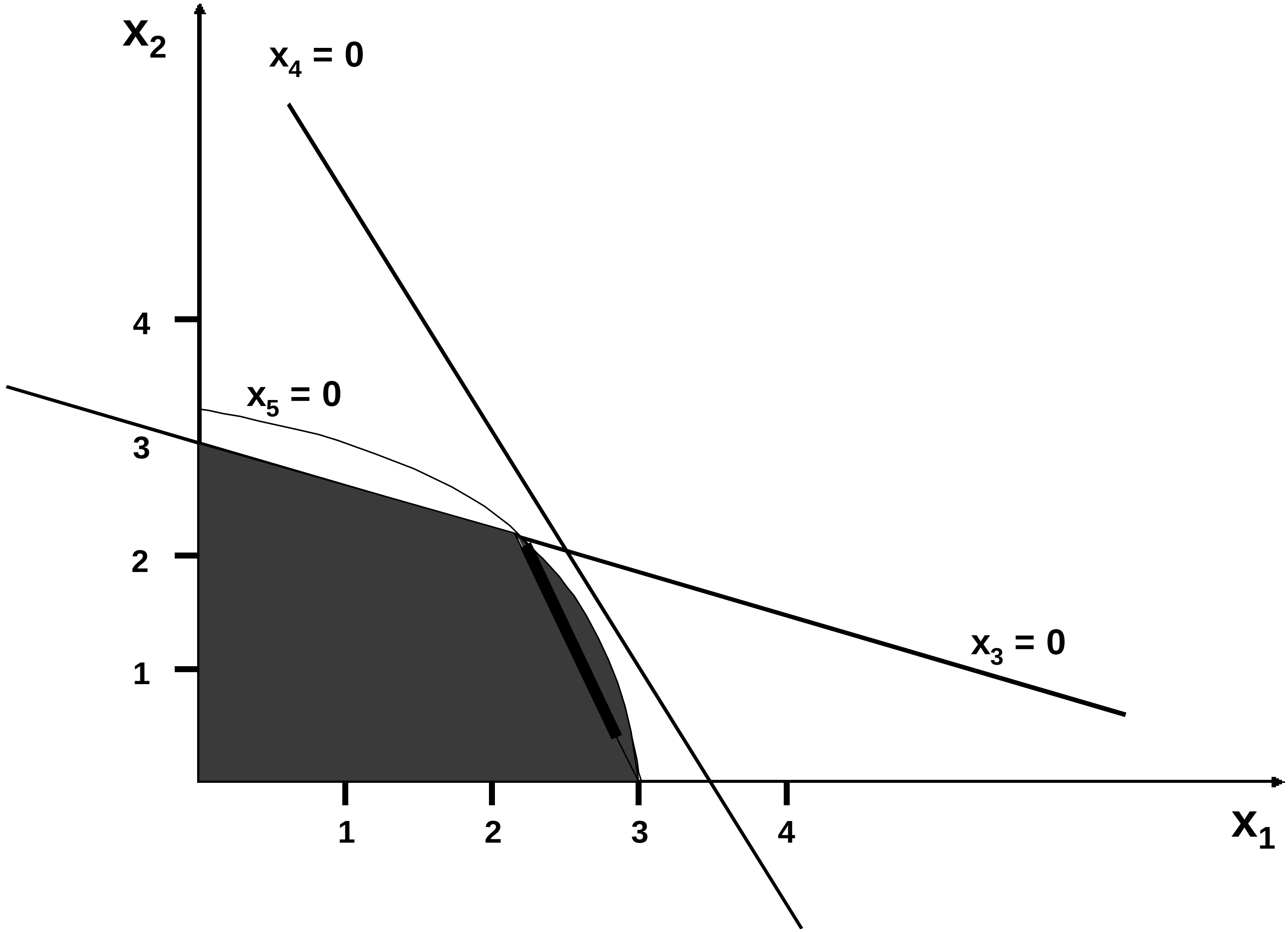
Sujeto a:

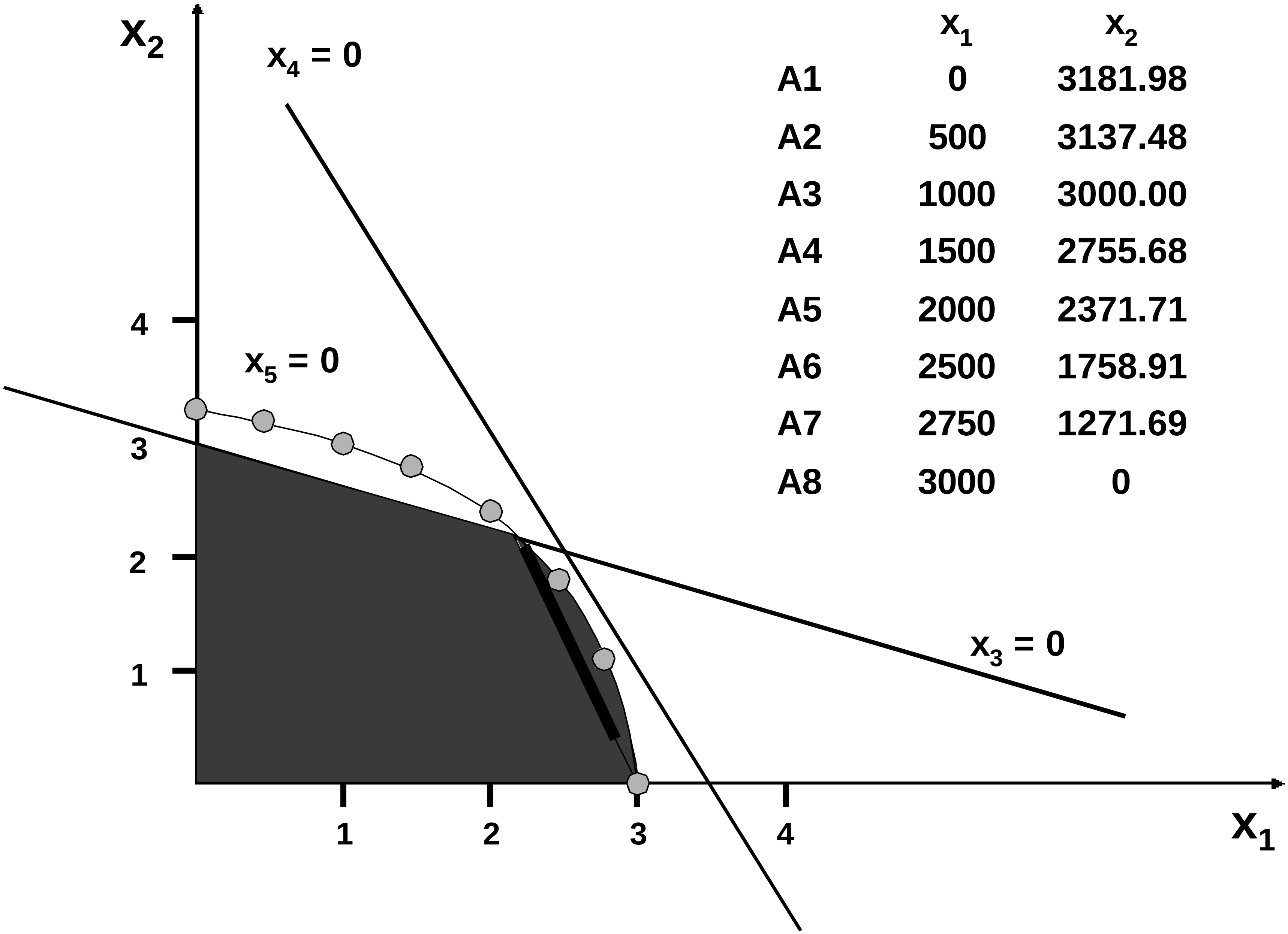
$$6\,x_1 + 16\,x_2 \quad \text{£} \quad 48000$$

$$12\,x_1 + 6\,x_2 \quad \text{£} \quad 42000$$

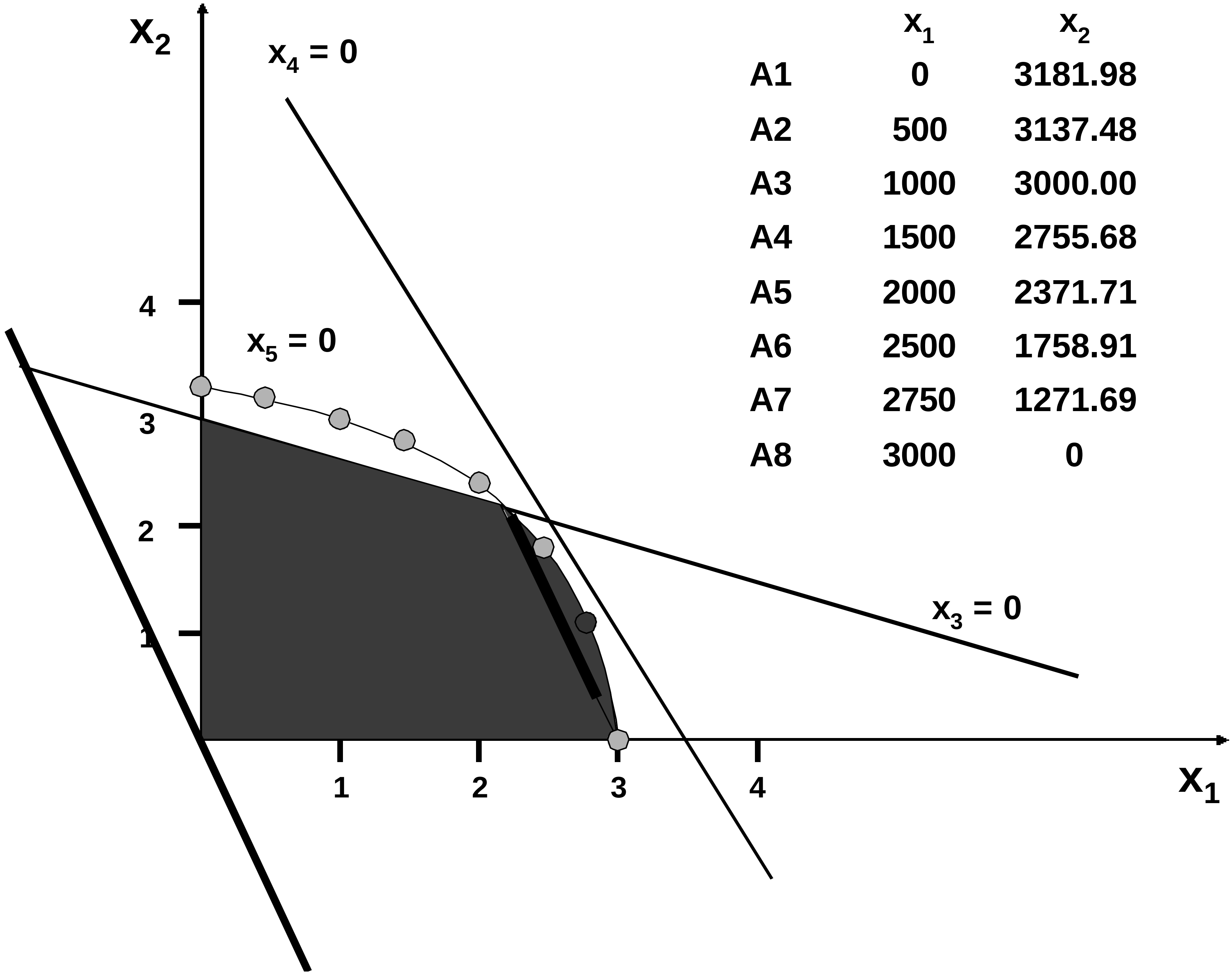
$$0{,}00225\,x_1^2 + 0{,}002\,x_2^2 \quad \text{£} \quad 20250$$

siendo: x_1, x_2 ³ 0 **y continuas**


x2
x4 = 0
4
x5 = 0
3
2
1
x3 = 0
1
2
3
4
x1



	x_1	x_2
A1	0	3181.98
A2	500	3137.48
A3	1000	3000.00
A4	1500	2755.68
A5	2000	2371.71
A6	2500	1758.91
A7	2750	1271.69
A8	3000	0



	x_1	x_2
A1	0	3181.98
A2	500	3137.48
A3	1000	3000.00
A4	1500	2755.68
A5	2000	2371.71
A6	2500	1758.91
A7	2750	1271.69
A8	3000	0

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 23065.07

VARIABLE	VALUE	REDUCED COST
X1	2750.000000	0.000000
X2	1271.689941	0.000000
A1	0.000000	13519.129883
A2	0.000000	10152.629883
A3	0.000000	7065.069824
A4	0.000000	4298.030273
A5	0.000000	1949.940063
A6	0.000000	288.339905
A7	1.000000	0.000000
A8	0.000000	2065.070068

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	11152.959961	0.000000
3)	1369.859985	0.000000
4)	0.000000	-7.000000
5)	0.000000	-3.000000
6)	0.000000	23065.070312

FORMULACIÓN CON “LINGO”

```
max= 7 * x1 + 3 * x2;

 6 * x1 + 16 * x2 < 48000;
12 * x1 + 6 * x2 < 42000;
0.00225 * x1^2 + 0.002 * X2^2 < 20250;
```

SOLUCIÓN REAL CON “LINGO”

```
Local optimal solution found at step:              8
Objective value:                            23067.83

Variable           Value        Reduced Cost
      X1        2731.076           0.0000000
      X2        1316.769           0.0000000

     Row    Slack or Surplus      Dual Price
       1        23067.83           1.000000
       2        10545.25           0.6511758E-08
       3        1326.482           0.0000000
       4        0.0000000          0.5695761
```

MÉTODOS NO BASADOS EN PL

MULTIPLICADORES DE LAGRANGE

MAX = $f(x_1, x_2, \ldots\ldots, x_n)$

ST

$g_1 (x_1, x_2,\ldots, x_n) = b_1$

$g_2 (x_1, x_2,\ldots, x_n) = b_2$

....

$g_m (x_1, x_2,\ldots, x_n) = b_m$

$$L = f(x_1, x_2, \ldots, x_N) + \sum_{1}^{m} \lambda_i \cdot \left[b_i - g_i(x_1, x_2, \ldots, x_n) \right]$$

Ejemplo

MAX: $Z = 0{,}003\, x_1^2 + 0{,}005\, x_2^2$

Sujeto a:

$$6\, x_1 + 16\, x_2 = 48000$$

siendo: $x_1, x_2 \geq 0$ **y continuas**

$$L = 0{,}003 \cdot x_1^2 + 0{,}005 \cdot x_2^2 + \lambda \cdot [6 \cdot x_1 + 16 \cdot x_2 - 48000]$$

$$\frac{\partial L}{\partial x_1} = 0{,}006 \cdot x_1 + 6 \cdot \lambda = 0 \quad \Rightarrow \quad x_1 = -\frac{6 \cdot \lambda}{0{,}006}$$

$$\frac{\partial L}{\partial x_2} = 0{,}01 \cdot x_1 + 16 \cdot \lambda = 0 \quad \Rightarrow \quad x_2 = -\frac{16 \cdot \lambda}{0{,}01}$$

$$\frac{\partial L}{\partial \lambda} = 6 \cdot x_1 + 16 \cdot x_2 = 48000$$

$x_1 = 1.265,8$

$x_2 = 2.025,28$

$ë = -1,2658$

$Z = 25.315,54$

CONDICIONES DE KARUSH-KUHN-TUCKER

MIN (o MAX) = $f(x_1, x_2, \ldots\ldots, x_n)$
$g_1 (x_1, x_2,\ldots, x_n) \leq b_1$
$g_2 (x_1, x_2,\ldots, x_n) \leq b_2$
....
$g_m (x_1, x_2,\ldots, x_n) \leq b_m$

x_i no negativas

$$L = f(x_1, x_2, \ldots, x_N) \pm \sum_{1}^{m} \lambda_i \cdot \left[b_i - g_i(x_1, x_2, \ldots, x_n)\right]$$

$$\frac{\partial L}{\partial x_i} = 0$$

$$\lambda_i \cdot \left[b_i - g_i(x_1, x_2, \ldots, x_n)\right] = 0$$

$$\lambda_i \geq 0$$

EJEMPLO:

$$\text{MIN}: \quad 5 \cdot x_1 + \frac{125}{x_1} + 10 \cdot x_2 + \frac{160}{x_2}$$

ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \leq 3 \\ \dfrac{4}{x_1} + \dfrac{6}{x_2} \leq 5 \end{cases}$$

$$x_i \geq 0$$

$$L = 5 \cdot x_1 + \frac{125}{x_1} + 10 \cdot x_2 + \frac{160}{x_2} + \lambda_1 \cdot \left[0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 - 3\right]$$

$$+ \lambda_2 \cdot \left[\frac{4}{x_1} + \frac{6}{x_2} - 5\right] \to \text{Mín}$$

$$\frac{\partial L}{\partial x_1} = 5 - \frac{125}{x_1^2} + \lambda_1 \cdot 0{,}6 - \lambda_2 \cdot \frac{4}{x_1^2} = 0$$

$$x_1 = \sqrt{\frac{125 + 4 \cdot \lambda_2}{5 + 0{,}6 \cdot \lambda_1}}$$

$$L = 5 \cdot x_1 + \frac{125}{x_1} + 10 \cdot x_2 + \frac{160}{x_2} + \lambda_1 \cdot [0,6 \cdot x_1 + 0,5 \cdot x_2 - 3]$$

$$+ \lambda_2 \cdot \left[\frac{4}{x_1} + \frac{6}{x_2} - 5\right] \to \text{Mín}$$

$$\frac{\partial L}{\partial x_2} = 10 - \frac{160}{x_2^2} + \lambda_1 \cdot 0,5 - \lambda_2 \cdot \frac{6}{x_1^2} = 0$$

$$x_2 = \sqrt{\frac{160 + 6 \cdot \lambda_2}{10 + 0,5 \cdot \lambda_1}}$$

CONDICIONES DE KARUSH-KUHN-TUCKER

(1) $$x_1 = \sqrt{\frac{125 + 4 \cdot \lambda_2}{5 + 0{,}6 \cdot \lambda_1}}$$

(2) $$x_2 = \sqrt{\frac{160 + 6 \cdot \lambda_2}{10 + 0{,}5 \cdot \lambda_1}}$$

(3) $$\lambda_1 \cdot \left[0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 - 3\right] = 0$$

(4) $$\lambda_2 \cdot \left[\frac{4}{x_1} + \frac{6}{x_2} - 5\right] = 0$$

(5) $$\lambda_1 \geq 0$$

(6) $$\lambda_2 \geq 0$$

$ë_1$	$ë_2$	x_1	x_2	
0	0	4	5	VULNERA (3)
>0	**0**	**2,6693**	**2,7969**	**$ë_1$=20,9069**
0	>0			INCOMPATIBLE
>0	>0			INCOMPATIBLE

USANDO EL SISTEMA "LINGO"

```
MIN = 5 * x1 + 125 / x1 + 10 * x2 + 160 / x2;
0.6 * X1 + 0.5 * X2 < 3;
4 / X1 + 6 / X2 < 5;
```

```
 Local optimal solution found at step:              10
 Objective value:                             145.3511

                      Variable           Value        Reduced Cost
                            X1        2.669250           0.0000000
                            X2        2.796900           0.3422487E-06

                           Row    Slack or Surplus      Dual Price
                             1        145.3511            1.000000
                             2       0.0000000            20.90686
                             3        1.356219           0.0000000
```

EJEMPLO:

$$\text{MIN}: \quad 5\cdot x_1 + \frac{2000}{x_1} + 10\cdot x_2 + \frac{5000}{x_2}$$

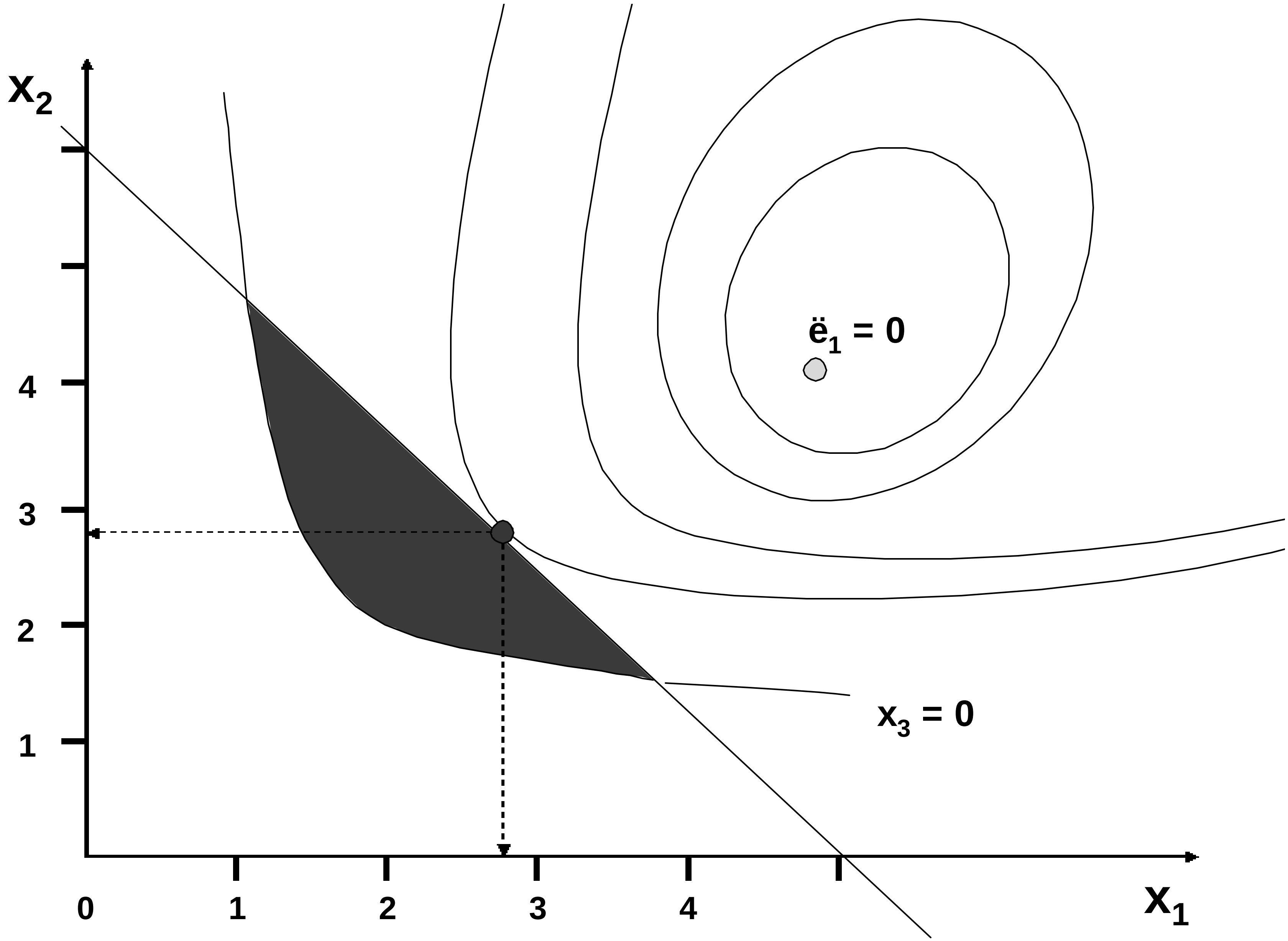
ST

$$0{,}6\cdot x_1 + 0{,}5\cdot x_2 \leq 300$$

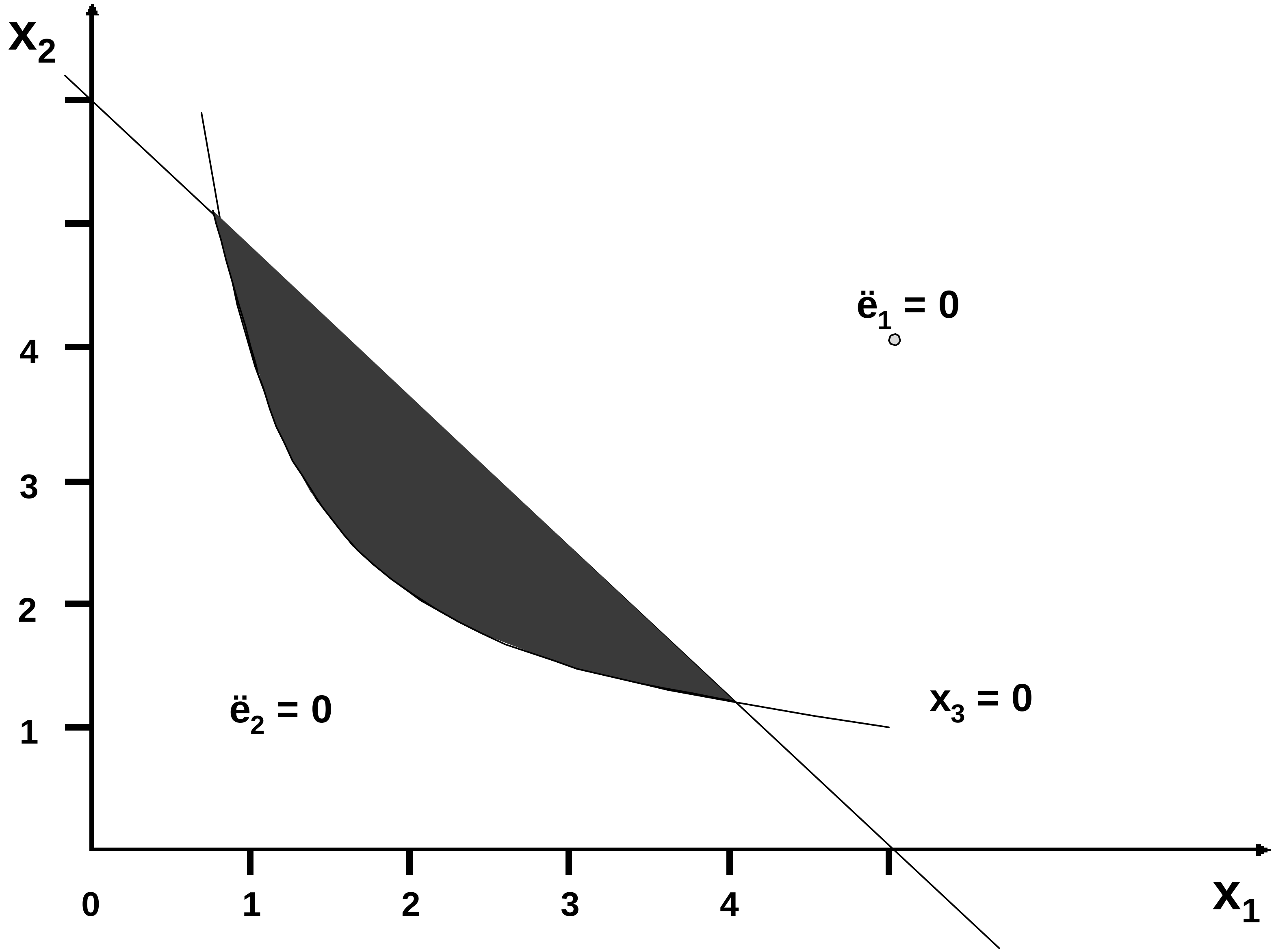
$$\frac{400}{x_1} + \frac{600}{x_2} \leq 0{,}5$$

$$L = f(x_1, x_2, \ldots, x_N) \pm \sum_{1}^{m} \lambda_i \cdot \left[b_i - g_i(x_1, x_2, \ldots, x_n)\right]$$

$$\lambda_i \cdot \left[b_i - g_i(x_1, x_2, \ldots, x_n)\right] = 0 \qquad \lambda_i \geq 0$$



x_2
4
3
2
1
0
1
2
3
4
x_1
ë$_1$ = 0
x_3 = 0



x2
4
3
2
1
0
1
2
3
4
x1
ë1 = 0
ë2 = 0
x3 = 0

MAX: $Z = 0{,}004\, x_1^2 - 0{,}006\, x_1 + 0{,}003\, x_2^2$

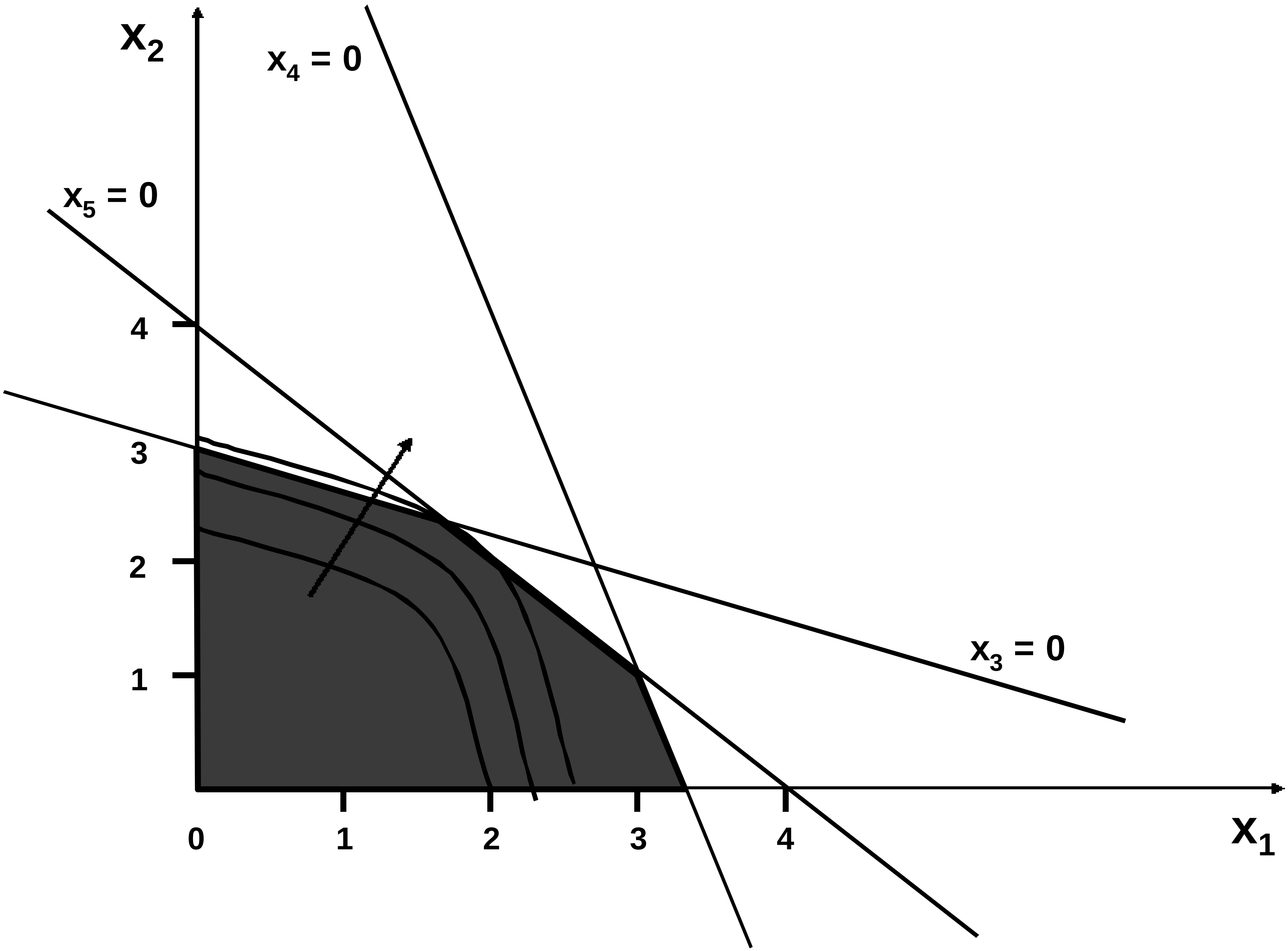
Sujeto a:

$6\, x_1 + 16\, x_2$ £ 48000

$12\, x_1 + 6\, x_2$ £ 42000

$9\, x_1 + 9\, x_2$ £ 36000

siendo: x_1, x_2 ³ 0 y continuas


x2
x4 = 0
x5 = 0
4
3
2
1
x3 = 0
0
1
2
3
4
x1

$$\mathbf{L = 0{,}004\ x_1^2 - 0{,}006\ x_1 + 0{,}003\ x_2^2}$$

$$\mathbf{-\ \ddot{e}_1\ (6\ x_1\ + 16\ x_2 - 48000)}$$

$$\mathbf{-\ \ddot{e}_2\ (12\ x_1\ + 6\ x_2 - 42000)}$$

$$\mathbf{-\ \ddot{e}_3\ (9\ x_1\ + 9\ x_2 - 36000)}$$

$$\frac{\partial L}{\partial x_1} = 0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 - 12 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$L = 0{,}004\ x_1^2 - 0{,}006\ x_1 + 0{,}003\ x_2^2$$
$$- \ddot{e}_1\ (6\ x_1 + 16\ x_2 - 48000)$$
$$- \ddot{e}_2\ (12\ x_1 + 6\ x_2 - 42000)$$
$$- \ddot{e}_3\ (9\ x_1 + 9\ x_2 - 36000)$$

$$\frac{\partial L}{\partial x_2} = 0{,}006 \cdot x_2 - 16 \cdot \lambda_1 - 6 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

CONDICIONES DE KKT

$$0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 - 12 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 16 \cdot \lambda_1 - 6 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$\text{ë}_1\ (6\ x_1 + 16\ x_2 - 48000) = 0$$

$$\text{ë}_2\ (12\ x_1 + 6\ x_2 - 42000) = 0$$

$$\text{ë}_3\ (9\ x_1 + 9\ x_2 - 36000) = 0$$

$\ddot{e}_1 = \ddot{e}_2 = \ddot{e}_3 = 0$ Þ INCOMPATIBLE

$$0{,}008 \cdot x_1 - 0{,}006 = 0$$

$$0{,}006 \cdot x_2 = 0$$

$$\mathbf{6\,x_1 + 16\,x_2 - 48000 = 0}$$

$$\mathbf{12\,x_1 + 6\,x_2 - 42000 = 0}$$

$$\mathbf{9\,x_1 + 9\,x_2 - 36000 = 0}$$

$\ddot{e}_1 = \ddot{e}_2 = 0$ Þ INCOMPATIBLE

$$0{,}008 \cdot x_1 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 9 \cdot \lambda_3 = 0$$

$6\ x_1 + 16\ x_2 - 48000 = 0$

$12\ x_1 + 6\ x_2 - 42000 = 0$

ë$_1$ = ë$_3$ = 0 Þ INCOMPATIBLE

$$0{,}008 \cdot x_1 - 0{,}006 - 12 \cdot \lambda_2 = 0$$

$$0{,}006 \cdot x_2 - 6 \cdot \lambda_2 = 0$$

$$\mathbf{6\ x_1 + 16\ x_2 - 48000 = 0}$$

$$\mathbf{9\ x_1 + 9\ x_2 - 36000 = 0}$$

$\ddot{e}_2 = \ddot{e}_3 = 0$ Þ INCOMPATIBLE

$$0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 = 0$$

$$0{,}006 \cdot x_2 - 16 \cdot \lambda_1 = 0$$

$$\mathbf{12\ x_1 + 6\ x_2 - 42000 = 0}$$

$$\mathbf{9\ x_1 + 9\ x_2 - 36000 = 0}$$

ë$_1$ **= 0 Þ INCOMPATIBLE**

$$0{,}008 \cdot x_1 - 0{,}006 - 12 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 6 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$\mathbf{6\ x_1 + 16\ x_2 - 48000 = 0}$$

$\ddot{e}_2 = 0$

$$0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 16 \cdot \lambda_1 - 9 \cdot \lambda_3 = 0$$

$$6\ x_1 + 16\ x_2 - 48000 = 0$$

$$9\ x_1 + 9\ x_2 - 36000 = 0$$

$X_1 = 1600$

$X_2 = 2400$

$\ddot{e}_1 = 0{,}1606$

$\ddot{e}_2 = 1{,}3145$

FORMULACIÓN CON SISTEMA “LINGO”

```
MAX = 0.004 * x1^2 - 0.006 * X1 + 0.003*x2^2;

 6 * x1 + 16 * x2 < 48000;
12 * x1 +  6 * x2 < 42000;
 9 * x1 +  9 * X2 < 36000;
```

RESOLUCIÓN CON SISTEMA “LINGO”

Local optimal solution found at step: 5
Objective value: 27510.40

Variable	Value	Reduced Cost
X1	1600.000	0.0000000
X2	2400.000	0.0000000

Row	Slack or Surplus	Dual Price
1	27510.40	1.000000
2	0.0000000	0.1606000
3	8400.000	0.0000000
4	0.0000000	1.314489