MODELOS DE DECISIÓN

- <u>NO RESTRINGIDOS</u>

MAX: f(X) o MIN: f(X)

Ejemplo: $$\mathrm{MIN}: 3\cdot x_1 + \frac{2}{x_1} + \ln x_2 + 4\cdot x_1 \cdot x_2$$

- <u>RESTRINGIDOS</u>

  (PROGRAMAS MATEMÁTICOS)

PROGRAMACIÓN MATEMÁTICA

MAX: $Z = f(x)$

Sujeto a:

$$\begin{cases} g_1(x) \le b_1 \\ g_2(x) \le b_2 \\ \cdots\cdots\cdots \\ g_m(x) \le b_m \end{cases}$$

PROGRAMACIÓN LINEAL

Maximizar $\Sigma\, c_j\, x_j$

sujeto a un conjunto de restricciones $\Sigma\, a_{ij}\, x_j \leq b_i$

siendo $x_j \geq 0$

FUNCIÓN OBJETIVO

Maximizar $$\underbrace{Z = \Sigma\, c_j\, x_j}_{\text{FUNCIONAL}}$$

Ejemplo: $Z = 6\; x_1 + 8\; x_2 + 3\; x_3$

Coeficientes del funcional

FUNCIÓN OBJETIVO

Maximizar $\underbrace{Z = \Sigma\, c_j\, x_j}_{\text{FUNCIONAL}}$

Ejemplo: $Z = 6\,x_1 + 8\,x_2 + 3\,x_3$

Variables de decisión

RESTRICCIONES

Conjunto de inecuaciones o ecuaciones

CONDICIONES DE VÍNCULO

$$
\begin{cases}
\Sigma\, a_{ij}\ x_j \leq b_i \\
\Sigma\, a_{ij}\ x_j \geq b_i \\
\Sigma\, a_{ij}\ x_j = b_i
\end{cases}
$$

CONDICIONES DE VÍNCULO

Ejemplo: $\underbrace{12}\ x_1 + \underbrace{9}\ x_2 + \underbrace{4}\ x_3 \leq \underbrace{500}$

$\boxed{\text{COEFICIENTES TECNOLÓGICOS}}$ $\boxed{\text{RHS}}$

CONDICIONES DE LAS VARIABLES X_j

- NO NEGATIVIDAD

- CONTINUIDAD

CONDICIONES DE LOS TÉRMINOS INDEPENDIENTES (“RHS”)

b_i

- NO NEGATIVIDAD

  (en su forma estándar)

INECUACIÓN → ECUACIÓN

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \;\; \pounds \;\; 500$$

VARIABLES “SLACKS”

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \;\; \pounds \;\; 500$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 + x_4 = 500$$

DE HOLGURA

INECUACIÓN $\longrightarrow$ ECUACIÓN

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

VARIABLES “SLACKS”

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 - x_4 = 100$$

SUPERFLUA

FORMA NATURAL

MAX: $6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

Sujeto a:

$$\begin{cases} 12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500 \\ 3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100 \\ 7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200 \end{cases}$$

siendo: $x_j \geq 0$

FORMAS DE FORMULACIÓN DE UN MODELO DE PL

- **FORMA NATURAL**: Restricciones de "$\leq$", "$\geq$" y "="

- **FORMA CANÓNICA**
  - De MAX: Todas las restricciones de "$\leq$"
  - De MIN: Todas las restricciones de "$\geq$"

- **FORMA ESTÁNDAR**
  - Todas las restricciones de "="

FORMA ESTÁNDAR

MAX: $6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

Sujeto a:

$$\left\{\begin{array}{rrrrrrrl}
12 \cdot x_1 + & 9 \cdot x_2 + & 4 \cdot x_3 & + x_4 & & & = & 500 \\
3 \cdot x_1 + & 15 \cdot x_2 + & 6 \cdot x_3 & & + x_5 & & = & 700 \\
2 \cdot x_1 + & 2 \cdot x_2 + & 3 \cdot x_3 & & & - x_6 & = & 100 \\
7 \cdot x_1 + & 4 \cdot x_2 + & 3 \cdot x_3 & & & & = & 200
\end{array}\right.$$

siendo: x_j **³ 0**

FORMA CANÓNICA DE MAX

NATURAL

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \text{ £ } 500$$
$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \text{ £ } 700$$
$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \text{ ³ } 100$$
$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200$$

$$x_j \text{ ³ } 0$$

CANÓNICA

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \text{ £ } 500$$
$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \text{ £ } 700$$
$$- 2 \cdot x_1 - 2 \cdot x_2 - 3 \cdot x_3 \text{ £ } - 100$$
$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 \text{ £ } 200$$
$$- 7 \cdot x_1 - 4 \cdot x_2 - 3 \cdot x_3 \text{ £ } - 200$$

$$x_j \text{ ³ } 0$$

FORMA CANÓNICA DE MIN

NATURAL

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \le 500$$
$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \le 700$$
$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \ge 100$$
$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200$$

$$x_j \ge 0$$

CANÓNICA

$$\text{MIN: } -6 \cdot x_1 - 8 \cdot x_2 - 3 \cdot x_3$$

$$-12 \cdot x_1 - 9 \cdot x_2 - 4 \cdot x_3 \ge -500$$
$$-3 \cdot x_1 - 15 \cdot x_2 - 6 \cdot x_3 \ge -700$$
$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \ge 100$$
$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 \ge 200$$
$$-7 \cdot x_1 - 4 \cdot x_2 - 3 \cdot x_3 \ge -200$$

$$x_j \ge 0$$

FORMA CANÓNICA

MAX: $c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots\ldots + c_k x_k$

Sujeto a:

$$\left\{ \begin{array}{l} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \ldots\ldots + a_{1k} x_k \leq b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \ldots\ldots + a_{2k} x_k \leq b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \ldots\ldots + a_{3k} x_k \leq b_3 \\ \ldots\ldots \\ a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \ldots\ldots + a_{mk} x_k \leq b_m \end{array} \right.$$

siendo: $x_j \geq 0$

FORMA ESTÁNDAR

MAX: $c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots\dots + c_k x_k$

Sujeto a

$$\left\{ \begin{array}{l} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots\dots + a_{1k} x_k + x_{k+1} = b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots\dots + a_{2k} x_k + x_{k+2} = b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \dots\dots + a_{3k} x_k + x_{k+3} = b_3 \\ \dots\dots\dots\dots \\ a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \dots\dots + a_{mk} x_k + x_n = b_m \end{array} \right.$$

siendo x_j ³ 0

FORMA MATRICIAL EXPANDIDA

X1	**X2**	**X3**		**RHS**
6	8	3	$\Rightarrow$	MAX
12	9	4	≤	500
3	15	6	≤	700
2	2	3	≥	100
7	4	3	=	200

En un taller metalúrgico se fabrican dos tipos de piezas A y B, que deben seguir los siguientes procesos:

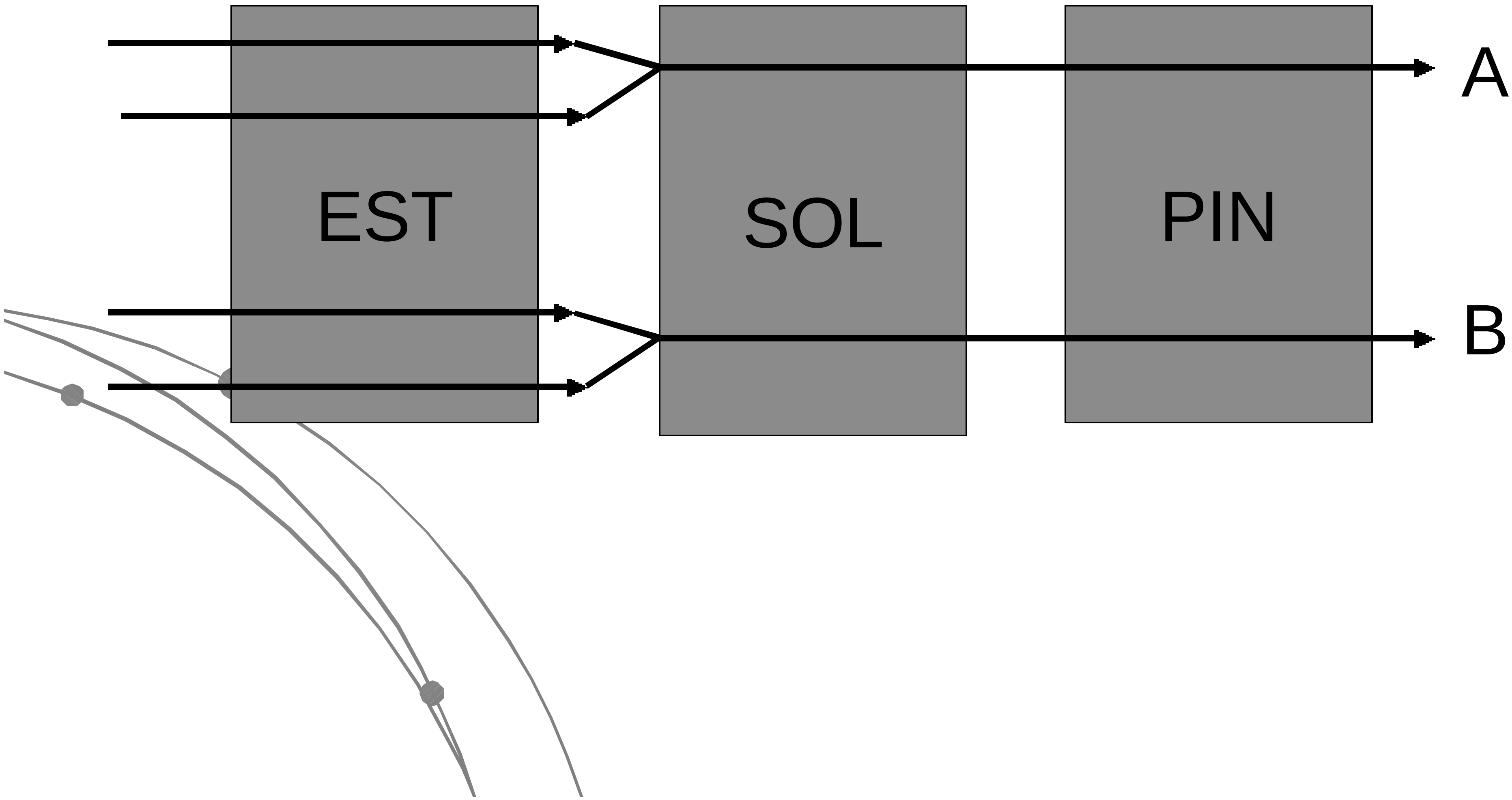
1. Estampado en hojas metálicas
2. Soldado
3. Pintado

La operación de estampado consiste en preparar partes idénticas que luego serán soldadas de a pares, formando la pieza A. El mismo proceso se realiza para la pieza B.

La utilidad unitaria es de $ 4 para la pieza A y $ 3 para la pieza B. Se desea establecer el programa semanal de producción que maximice la utilidad del taller con respecto a las piezas consideradas.

Los insumos de equipos son los siguientes, para la realización de cada una de las operaciones (expresados en segundos por pieza):

Operación	A	B	Tiempo disponible (seg./semana)
Estampado de c/parte	3	8	48.000
Soldado	12	6	42.000
Pintado	9	9	36.000


EST
SOL
PIN
A
B

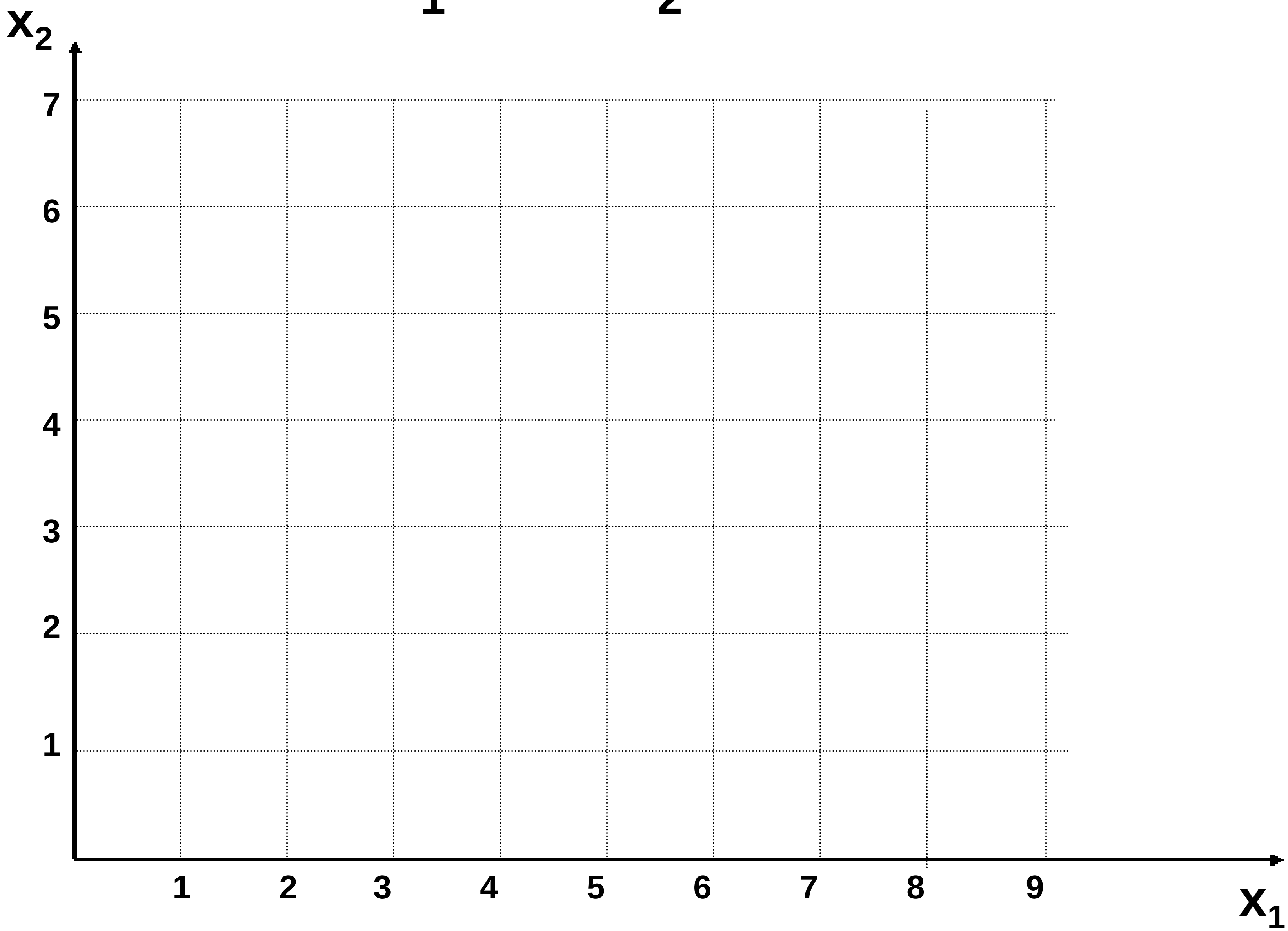
MAX: $Z = 4\,x_1 + 3\,x_2$

Sujeto a:

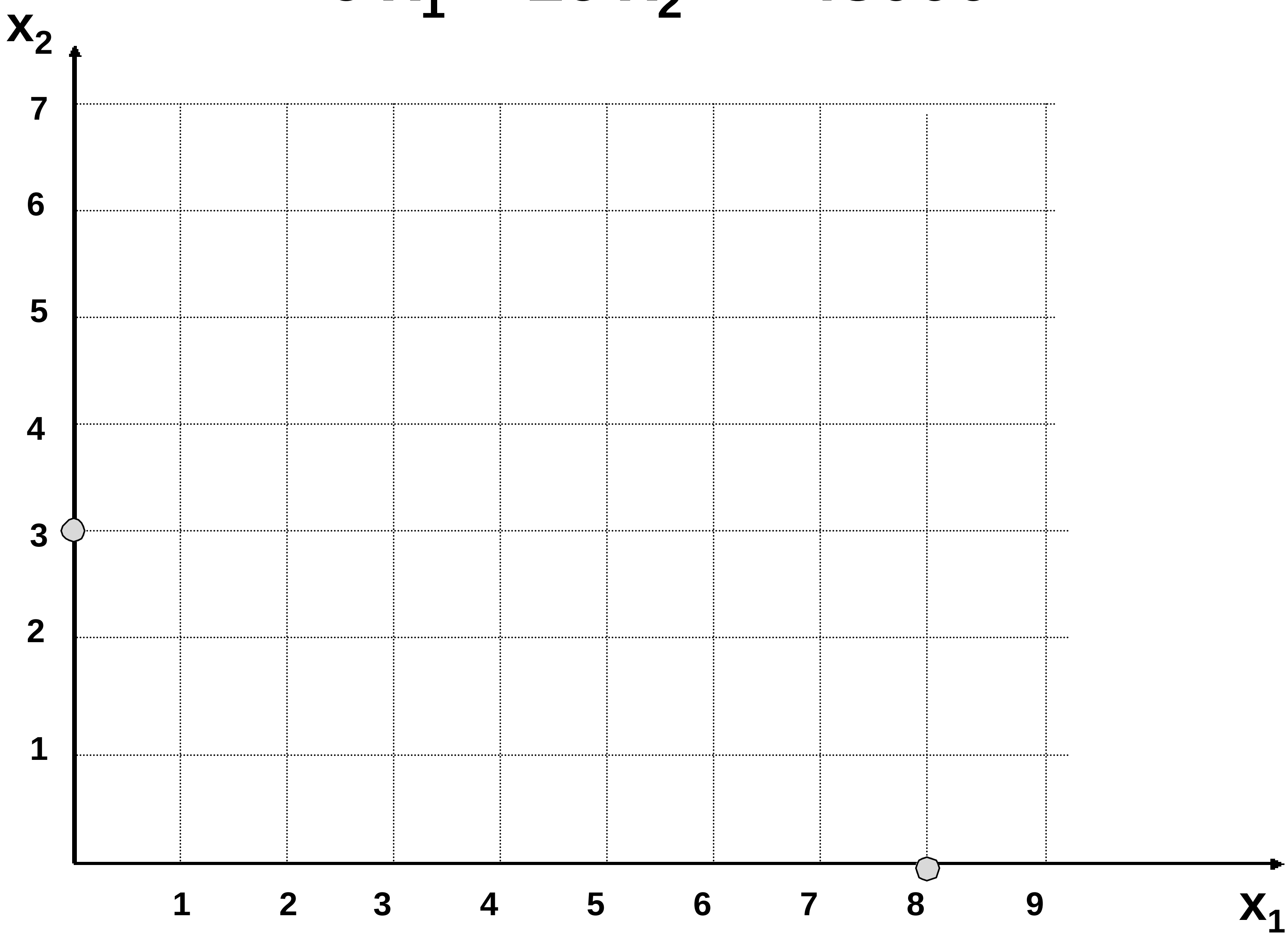
$$\begin{cases} 6\,x_1 + 16\,x_2 \text{ £ } 48000 \\ 12\,x_1 + 6\,x_2 \text{ £ } 42000 \\ 9\,x_1 + 9\,x_2 \text{ £ } 36000 \end{cases}$$

siendo: x_1, x_2 ³ 0 **y continuas**

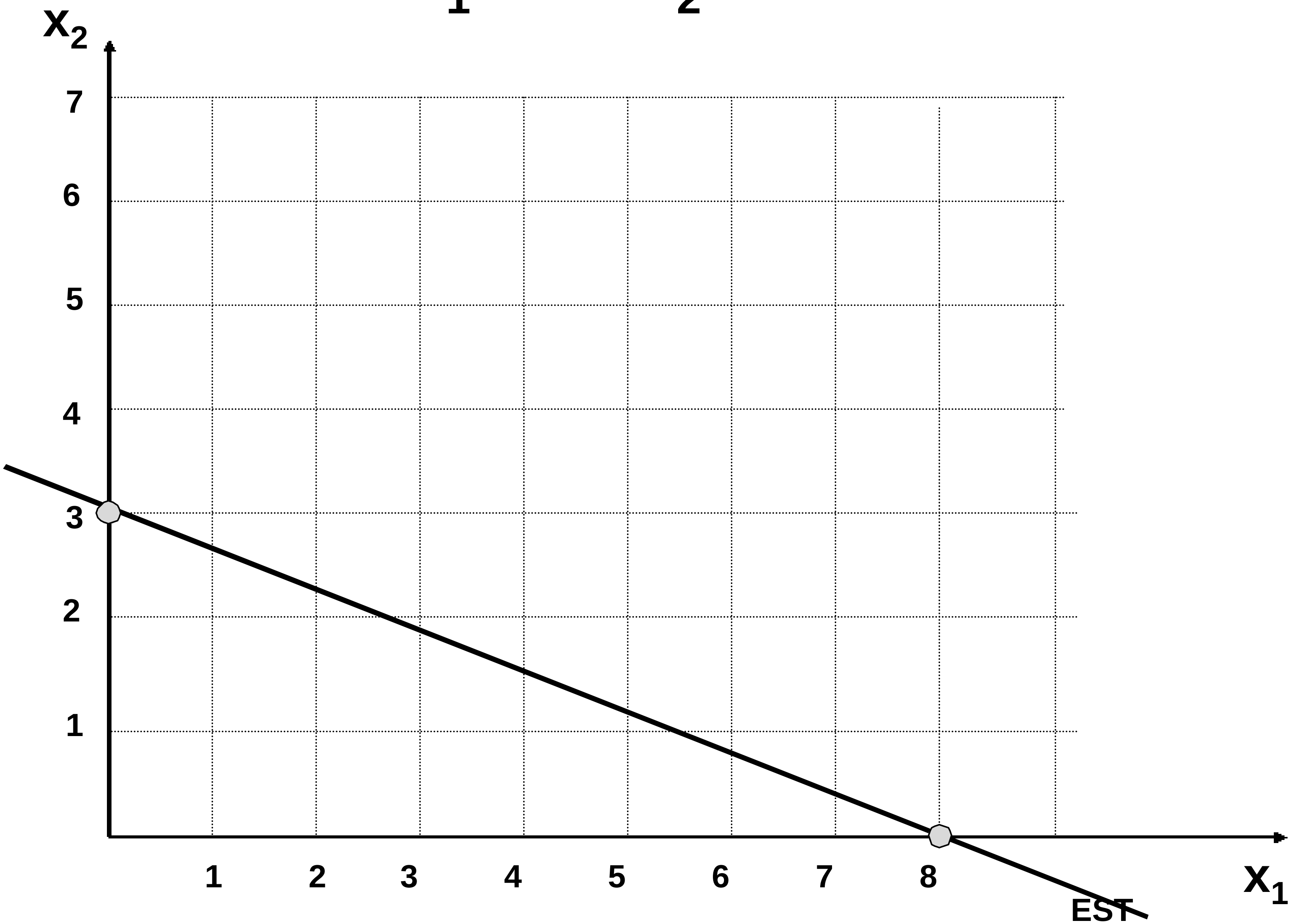
$$6\,x_1 + 16\,x_2 \text{ £ } 48000$$



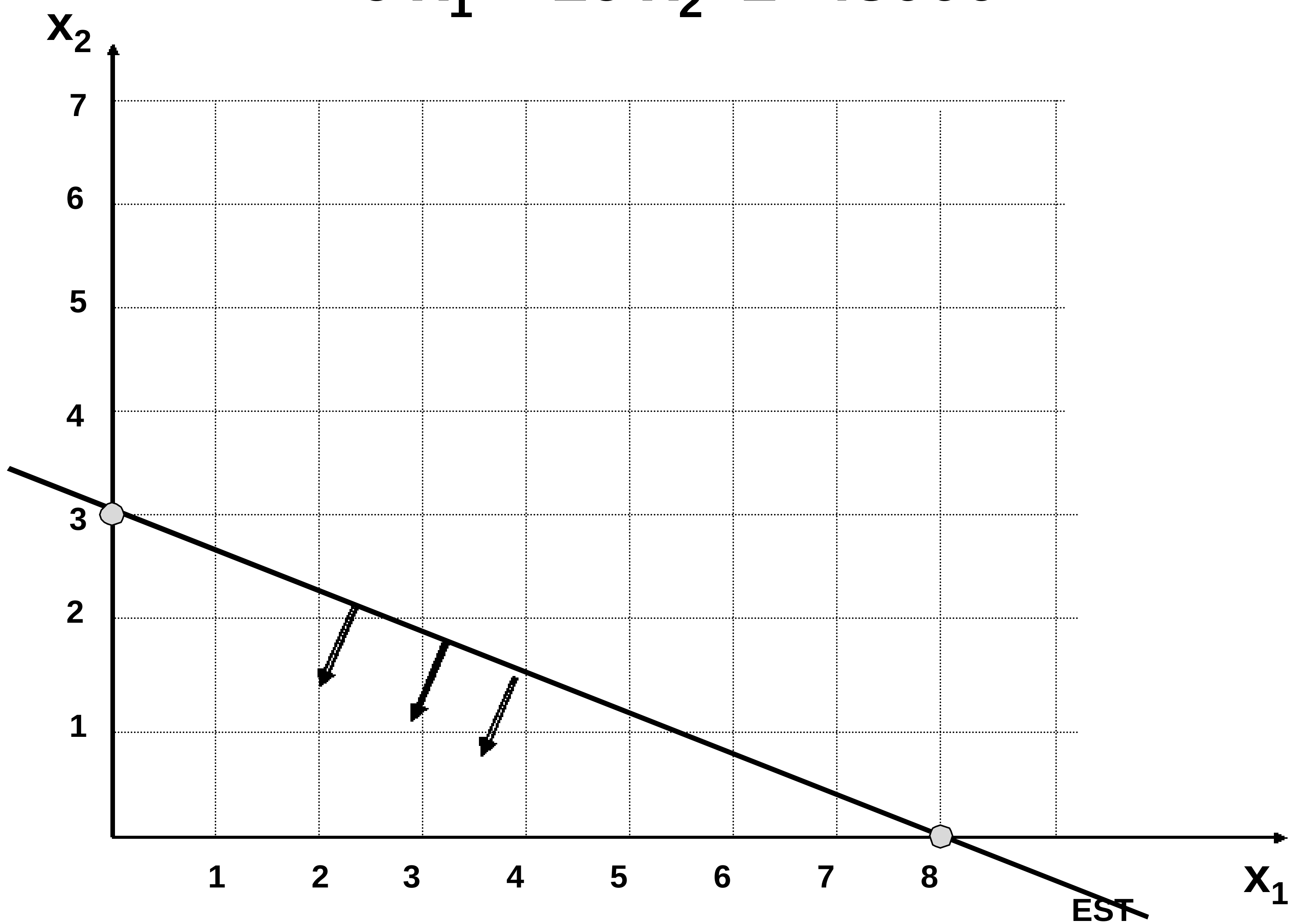
$$6\,x_1 + 16\,x_2 = 48000$$



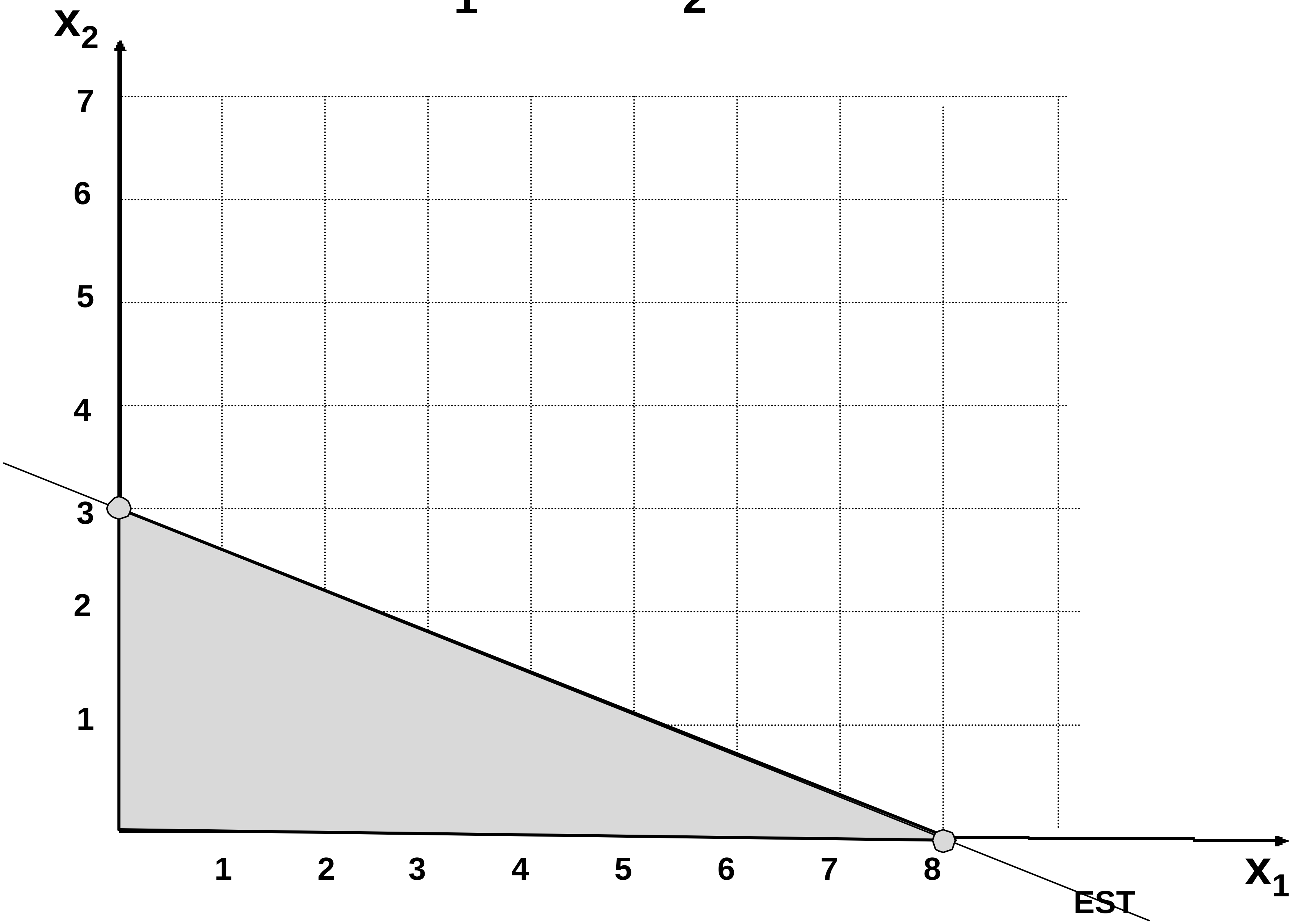
$$6\,x_1 + 16\,x_2 = 48000$$



$6 x_1 + 16 x_2$ £ 48000

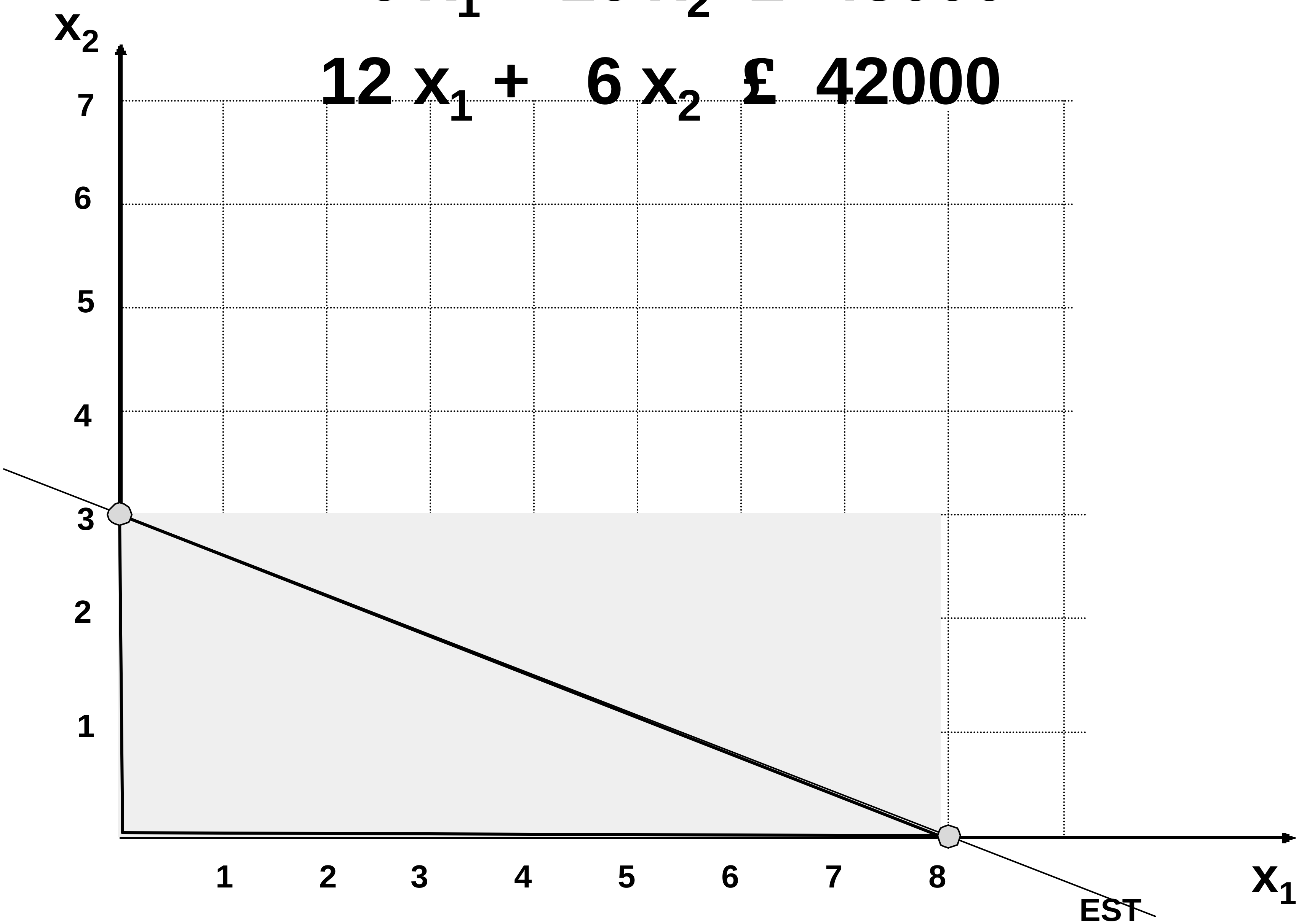


$$6\,x_1 + 16\,x_2 \;\text{£}\; 48000$$



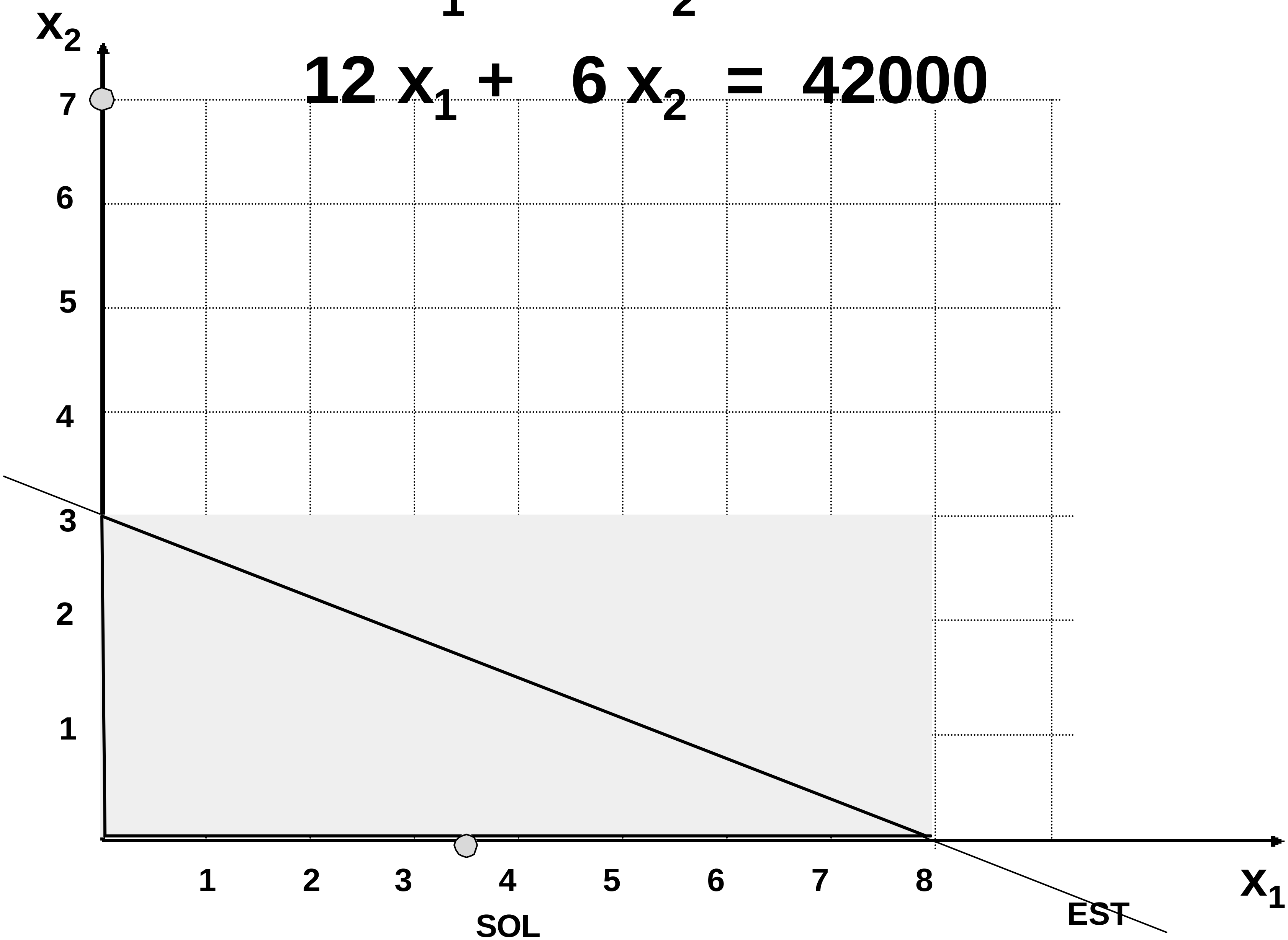
$6\ x_1 + 16\ x_2$ £ 48000

$12\ x_1 + 6\ x_2$ £ 42000



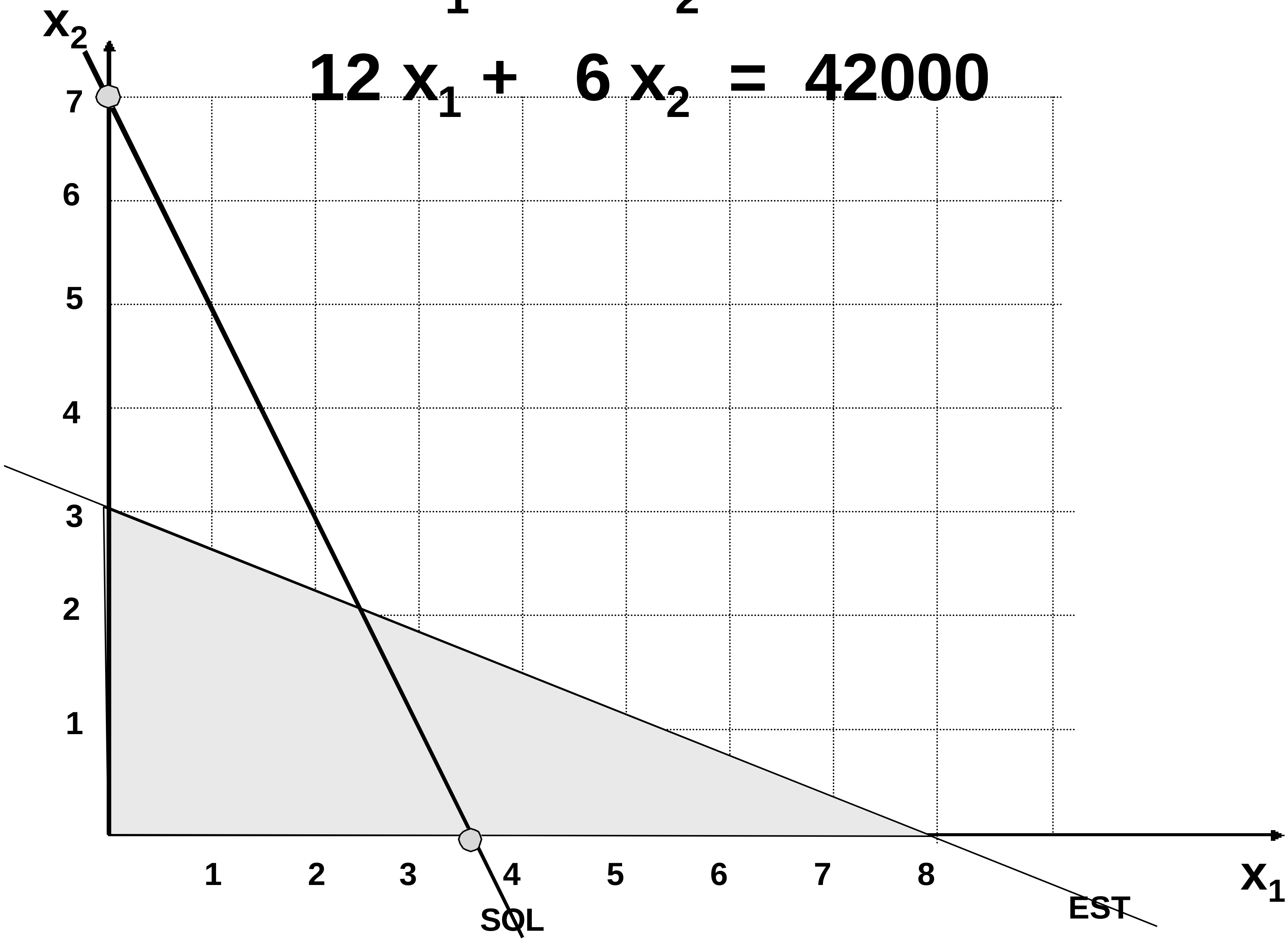
$$6\,x_1 + 16\,x_2 \text{ £ } 48000$$

$$12\,x_1 + 6\,x_2 = 42000$$



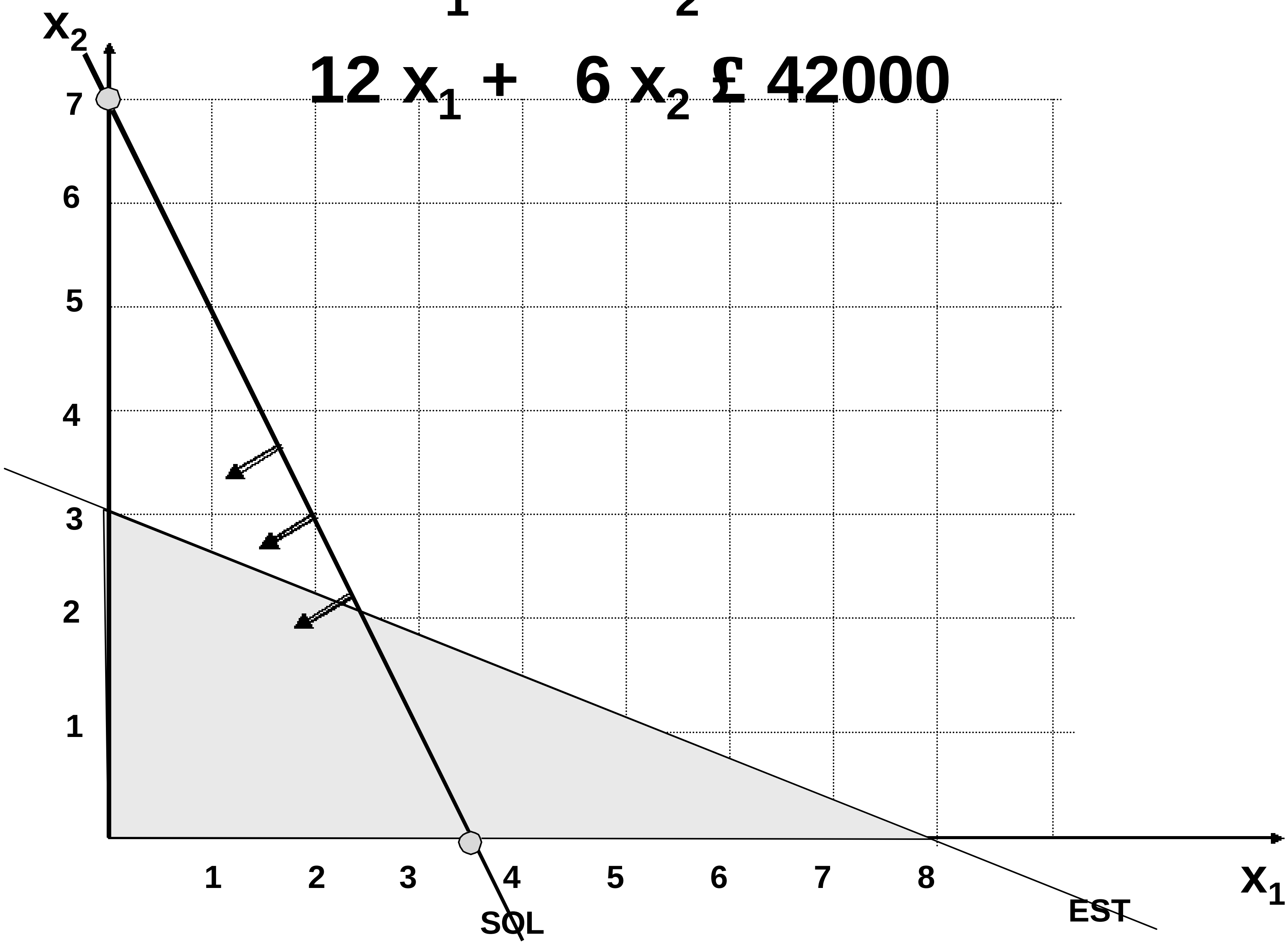
$$6\,x_1 + 16\,x_2 \;\text{£}\; 48000$$

$$12\,x_1 + \;\;6\,x_2 \;=\; 42000$$



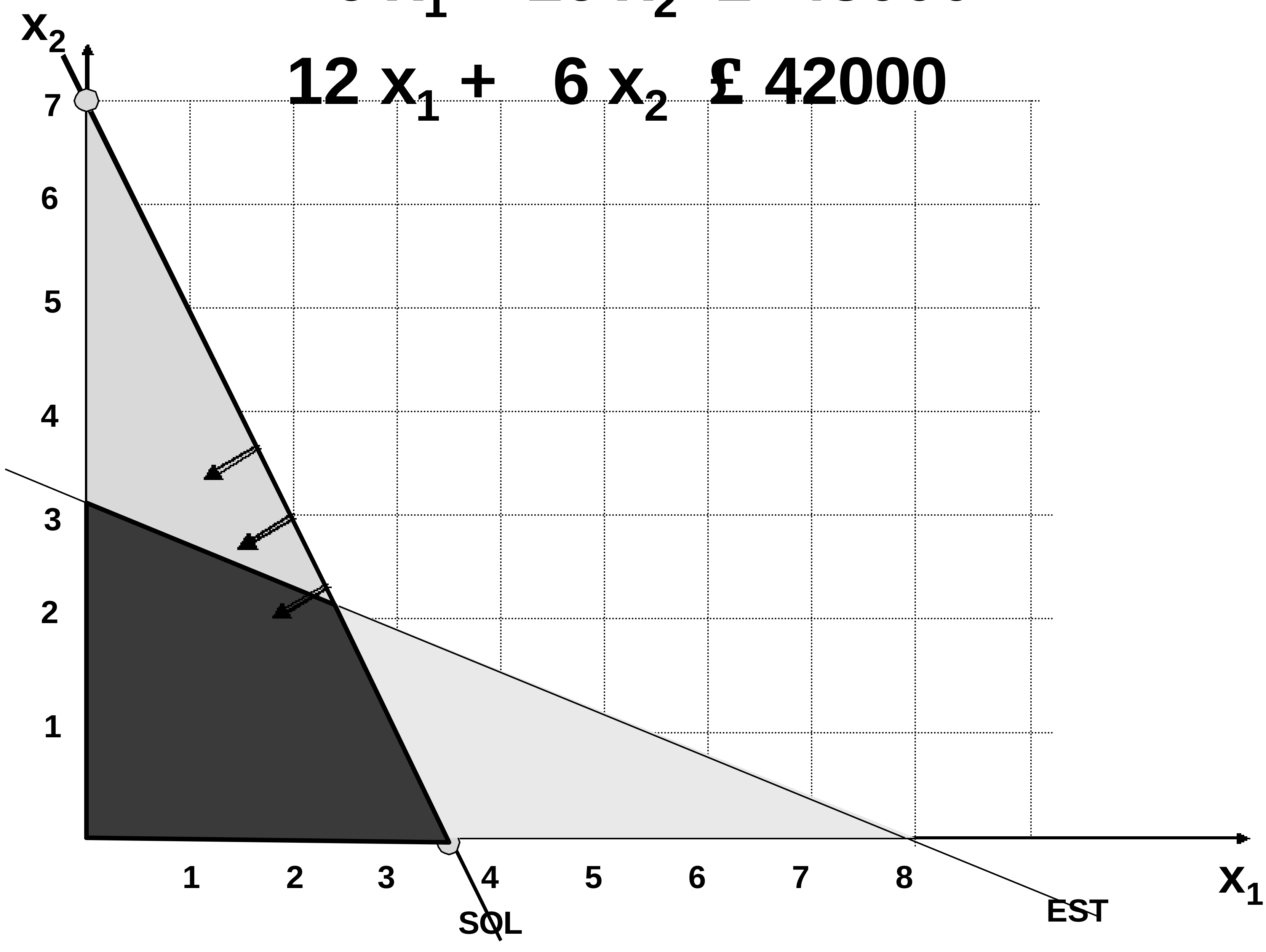
$6\ x_1 + 16\ x_2$ £ 48000

$12\ x_1 + \ \ 6\ x_2$ £ 42000



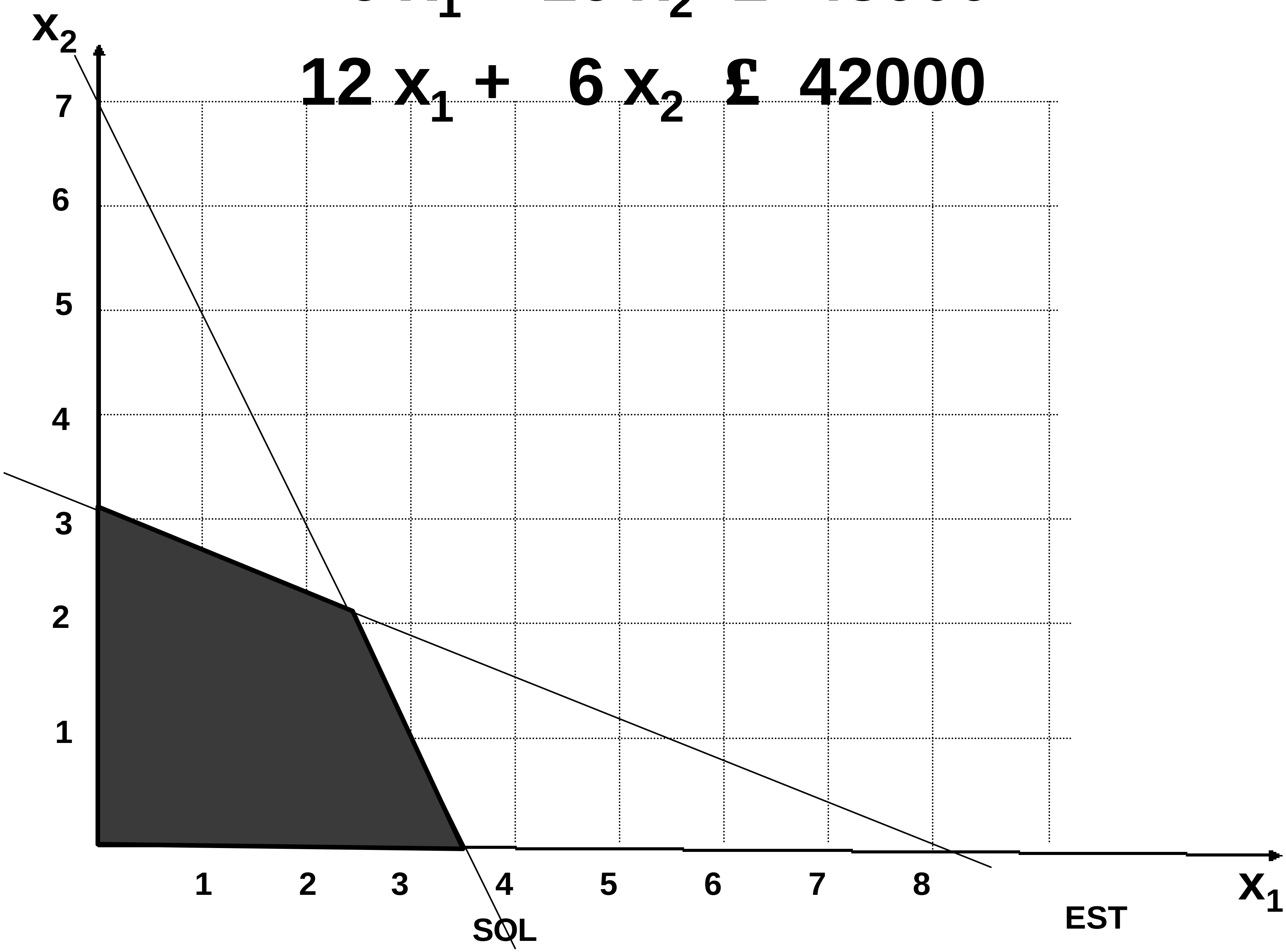
$6\ x_1 + 16\ x_2$ £ 48000

$12\ x_1 + 6\ x_2$ £ 42000



$$6\,x_1 + 16\,x_2 \text{ £ } 48000$$

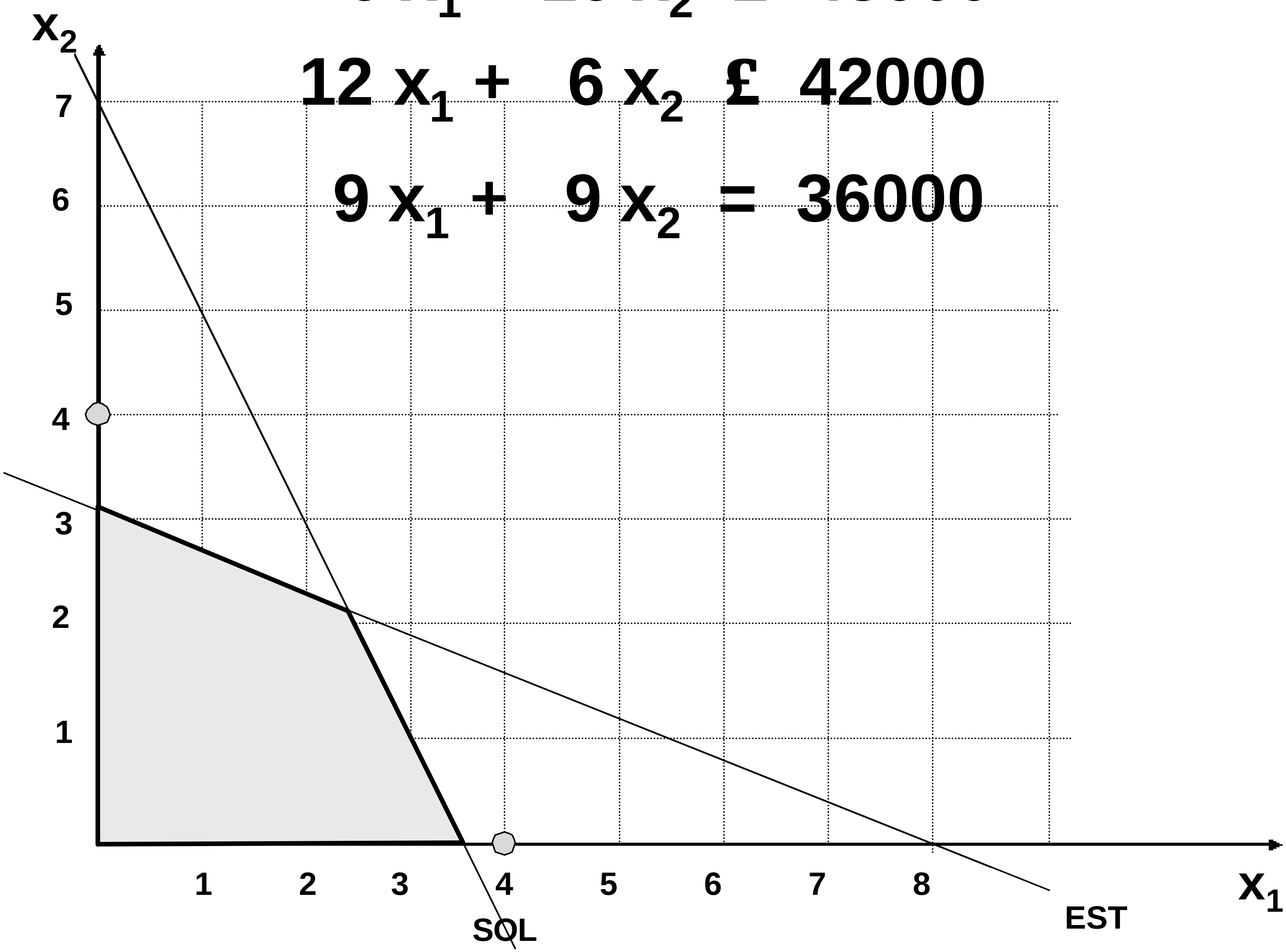
$$12\,x_1 + 6\,x_2 \text{ £ } 42000$$



$$6\ x_1 + 16\ x_2\ \ £\ \ 48000$$

$$12\ x_1 + 6\ x_2\ \ £\ \ 42000$$

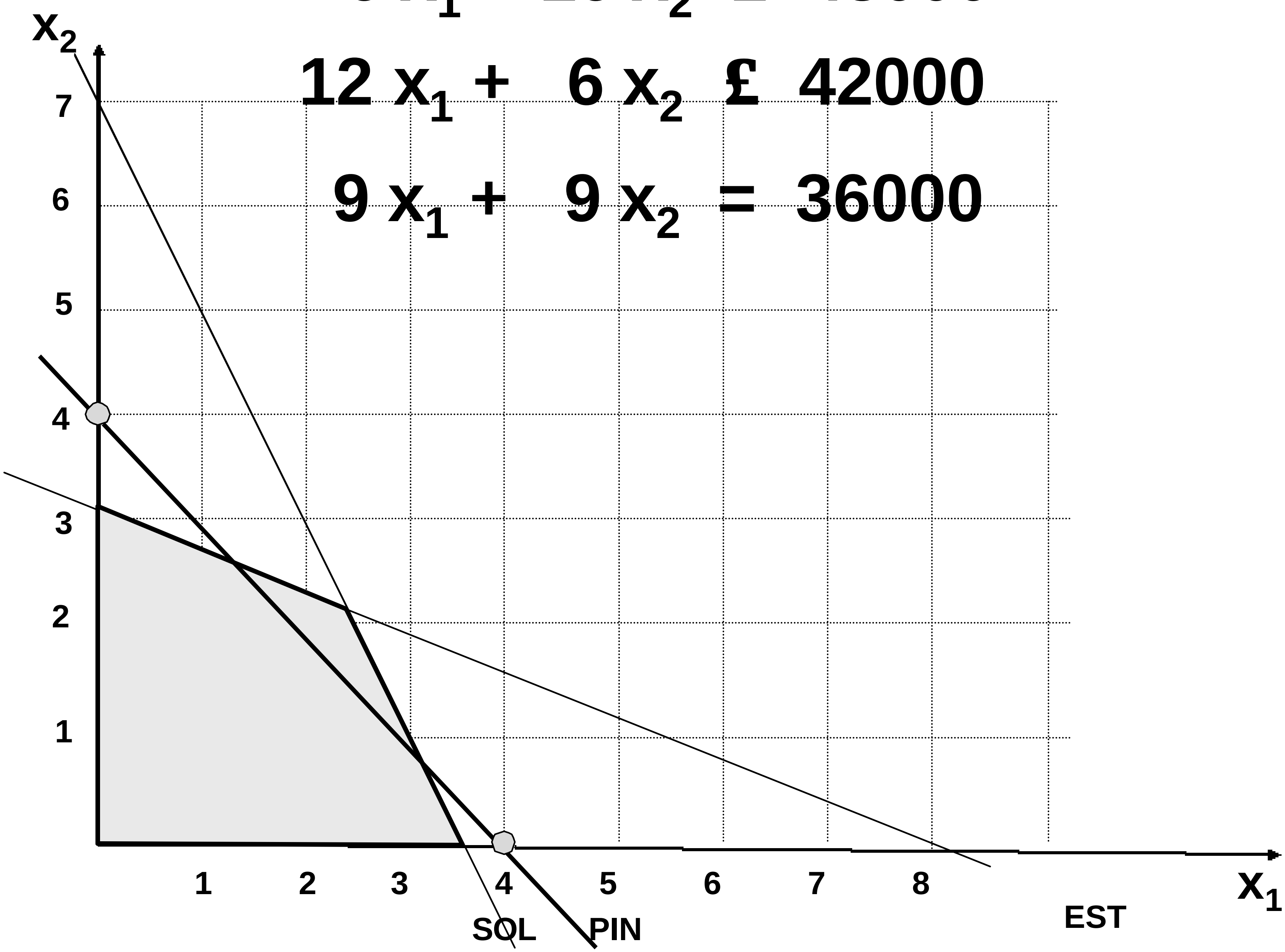
$$9\ x_1 + 9\ x_2 = 36000$$

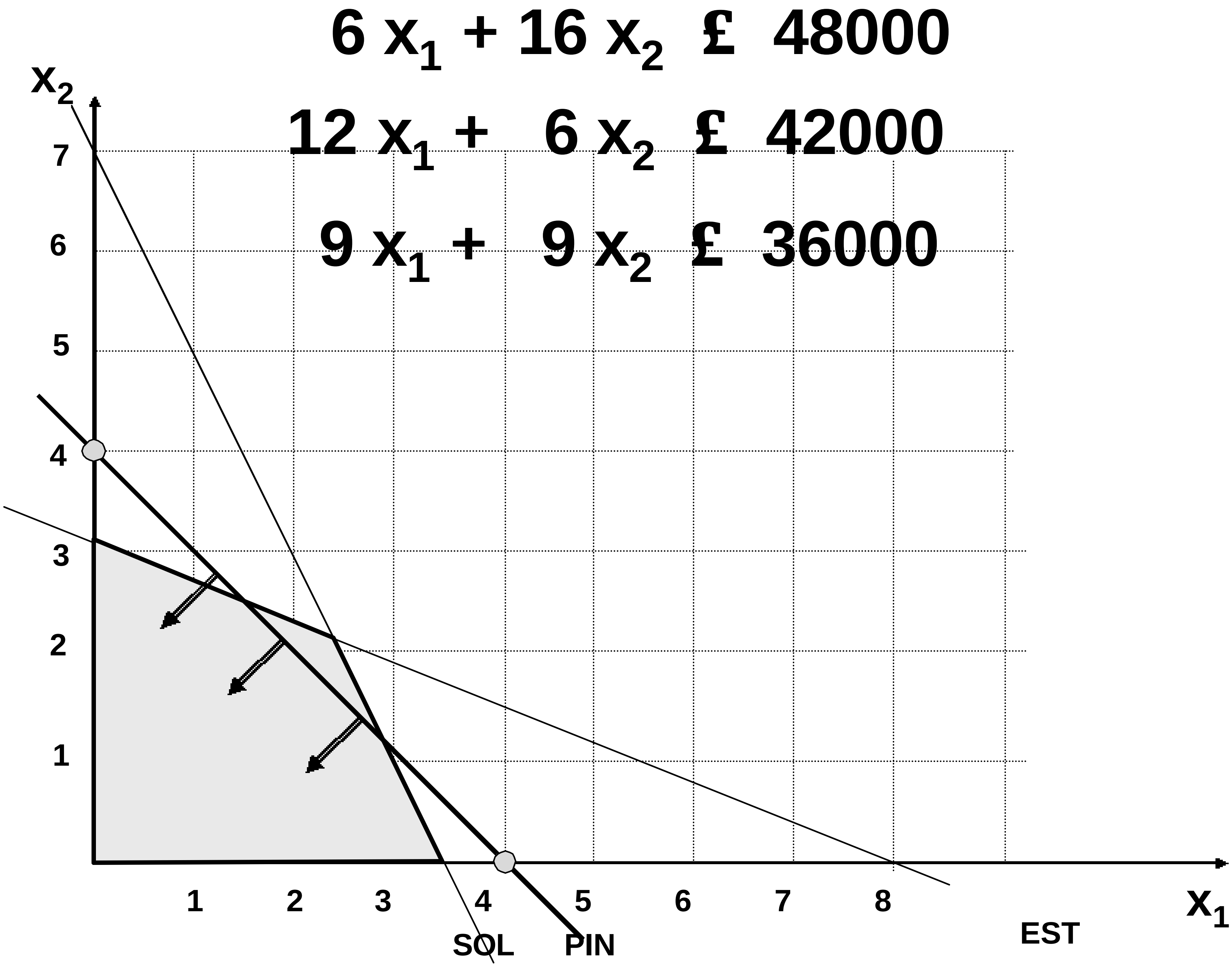


$6 x_1 + 16 x_2$ £ 48000

$12 x_1 + 6 x_2$ £ 42000

$9 x_1 + 9 x_2 = 36000$

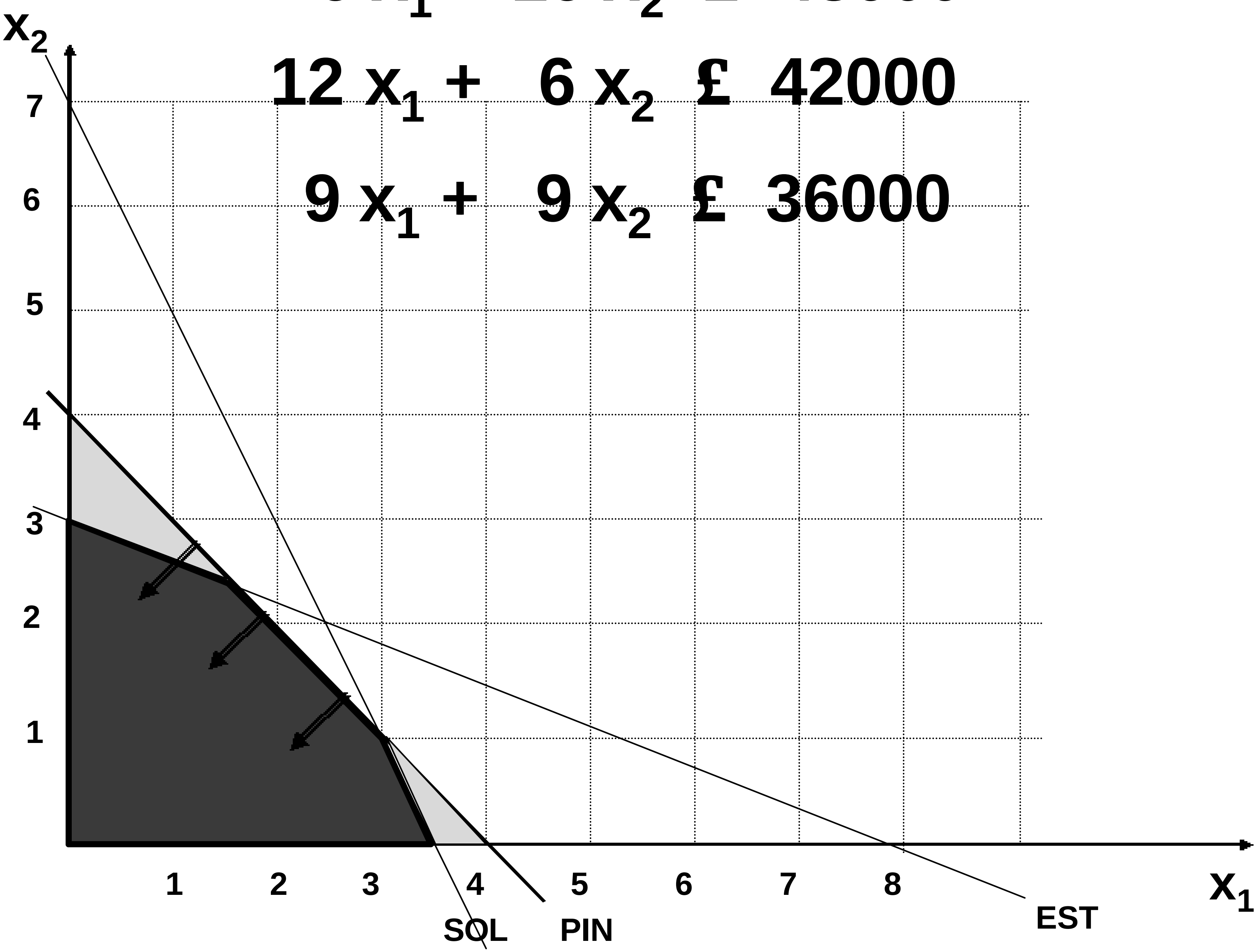



6 x1 + 16 x2 £ 48000
12 x1 + 6 x2 £ 42000
9 x1 + 9 x2 £ 36000
x2
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
x1
SOL
PIN
EST

$$6\ x_1 + 16\ x_2\ \text{£}\ 48000$$

$$12\ x_1 + 6\ x_2\ \text{£}\ 42000$$

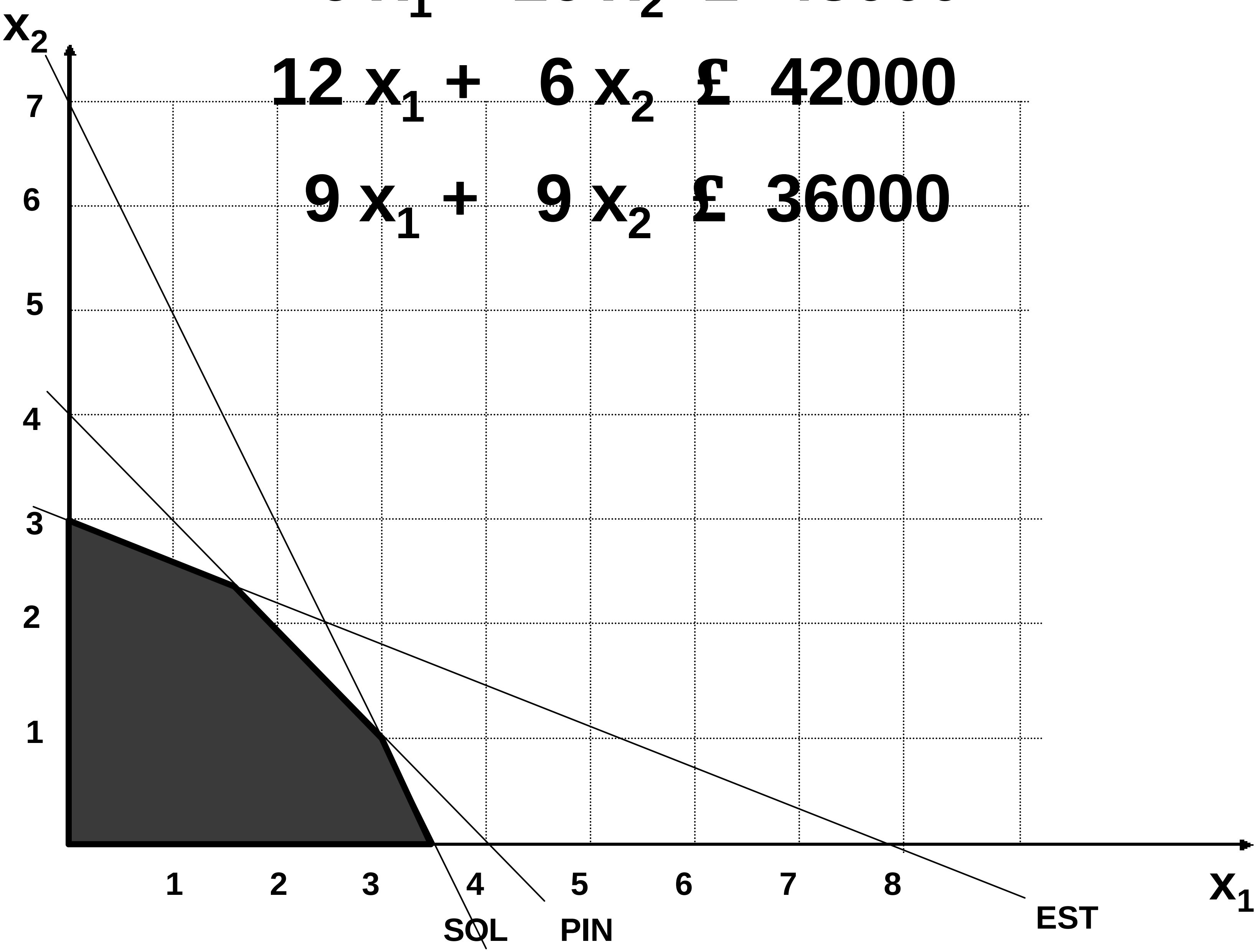
$$9\ x_1 + 9\ x_2\ \text{£}\ 36000$$

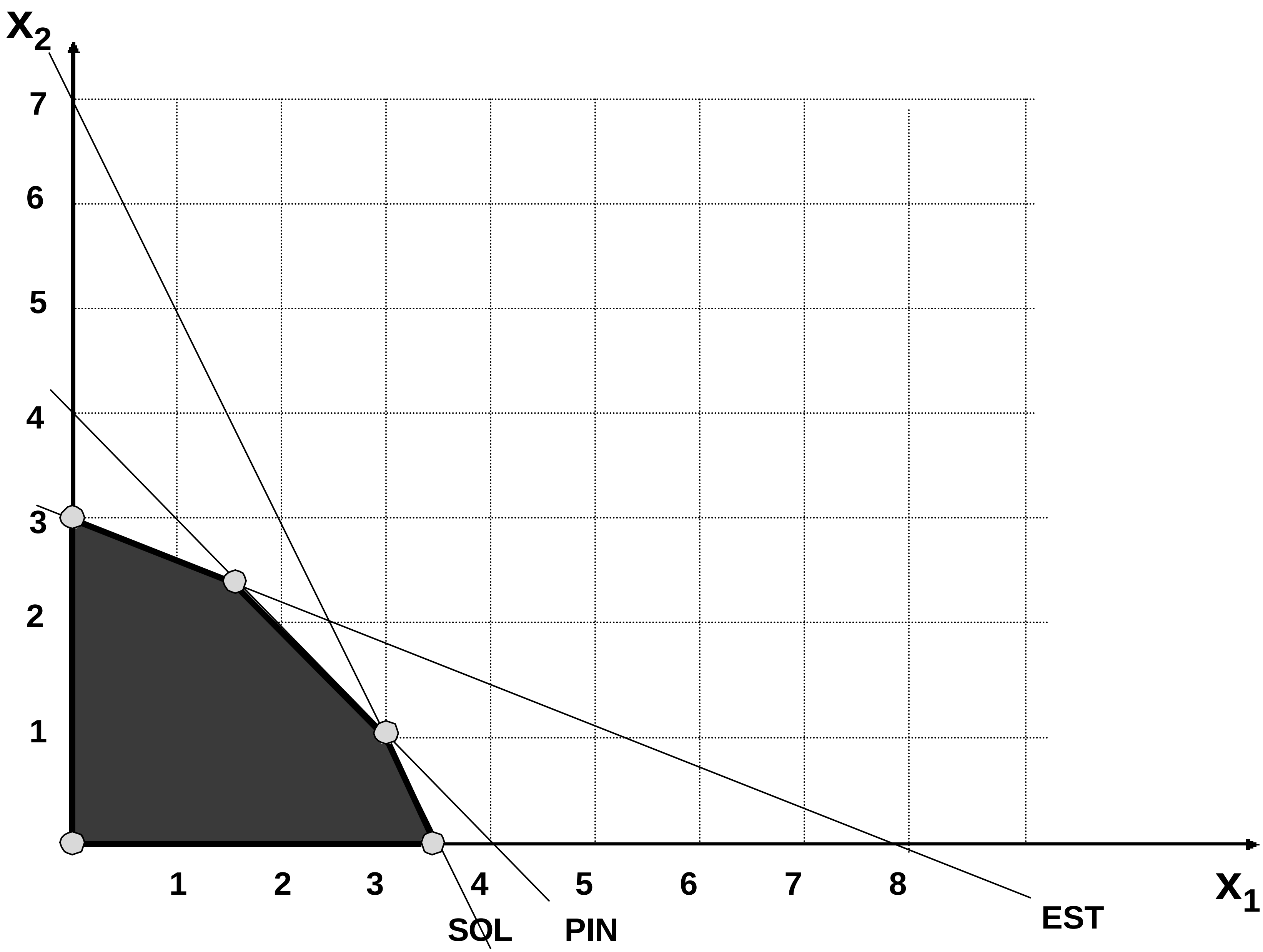


$$6\,x_1 + 16\,x_2 \;\; \pounds \;\; 48000$$

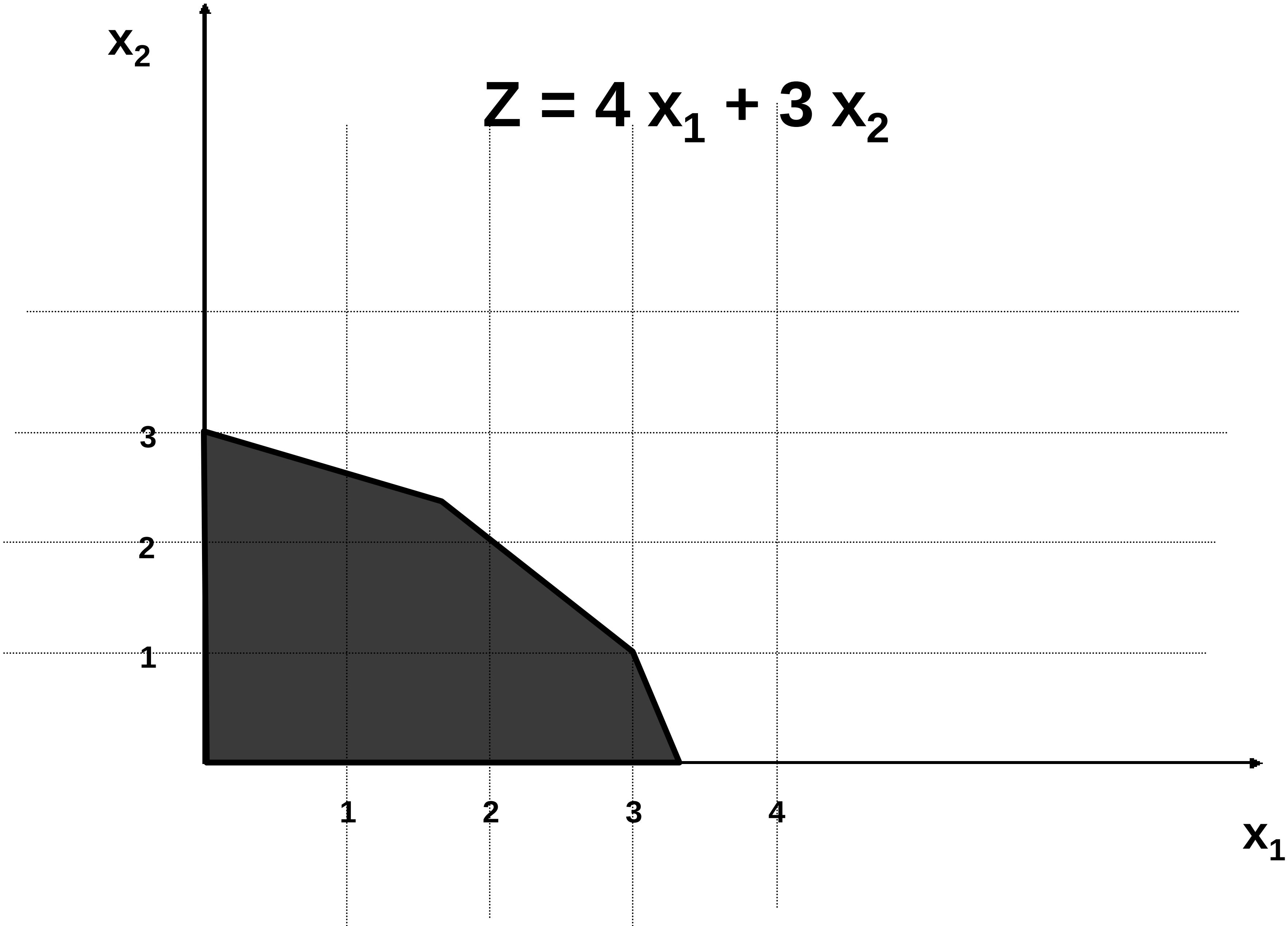
$$12\,x_1 + \;\; 6\,x_2 \;\; \pounds \;\; 42000$$

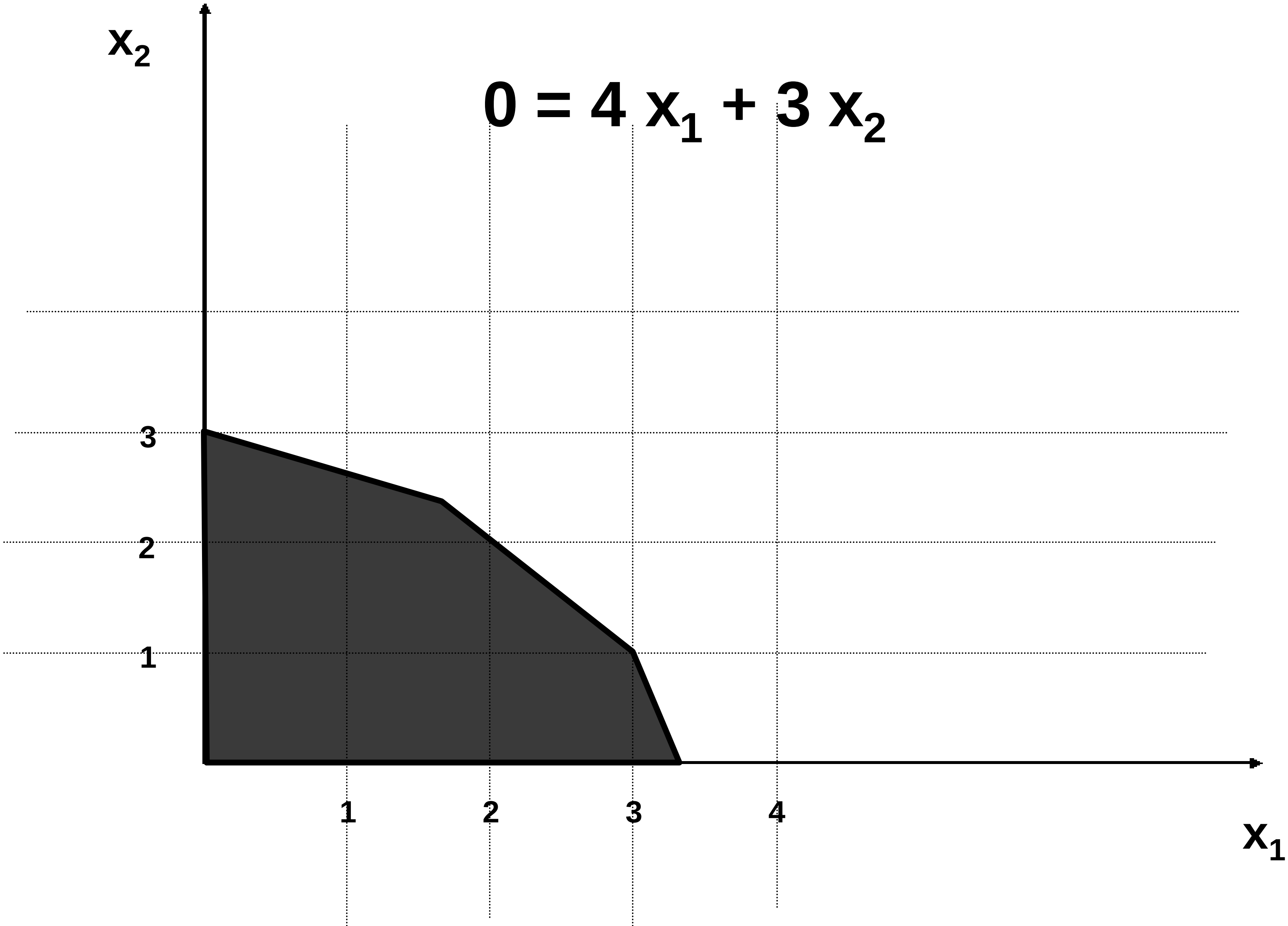
$$9\,x_1 + \;\; 9\,x_2 \;\; \pounds \;\; 36000$$





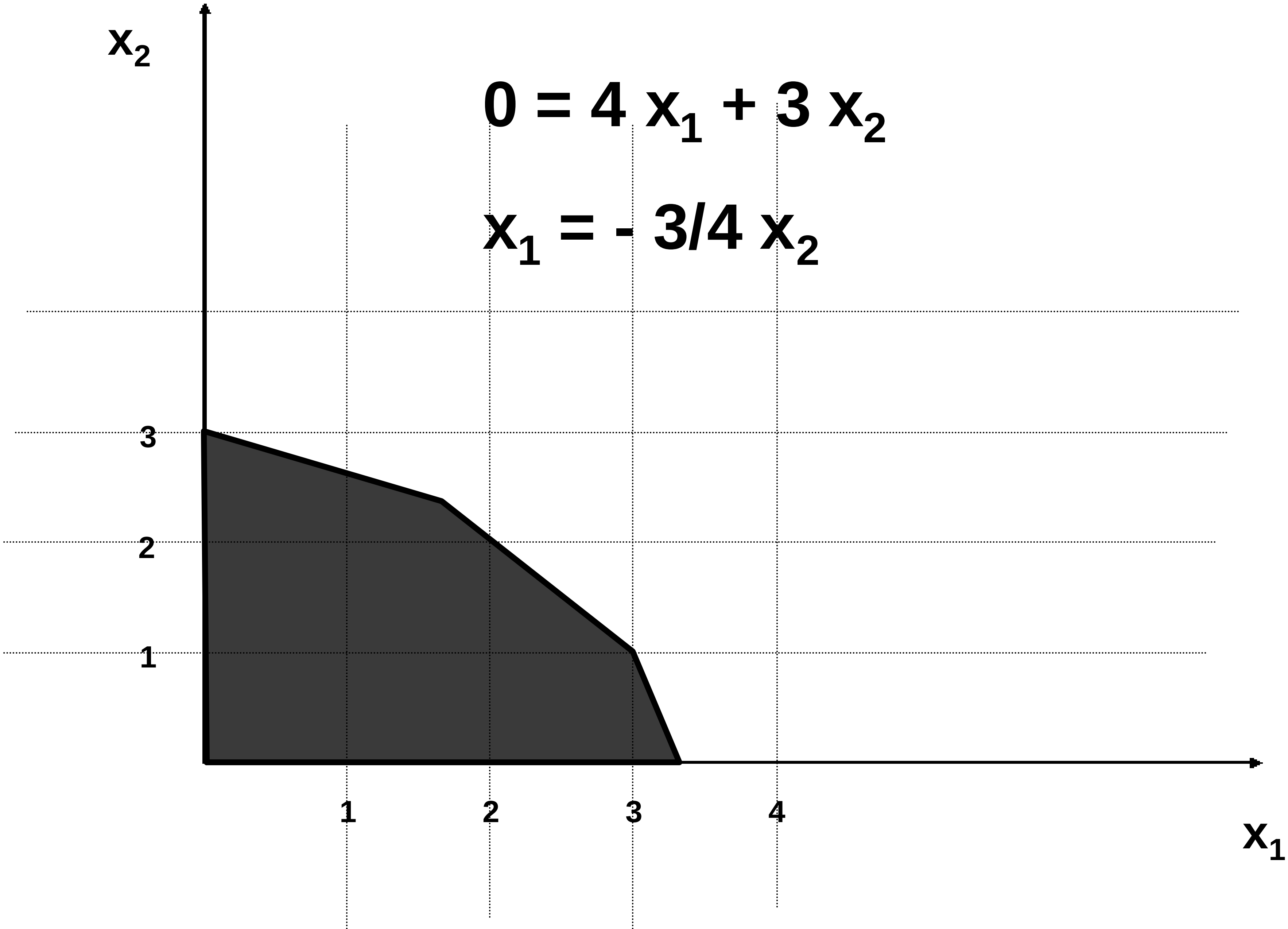
x2
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
x1
SOL
PIN
EST


$Z = 4\,x_1 + 3\,x_2$
x_2
x_1
3
2
1
1
2
3
4


x_2
$0 = 4\ x_1 + 3\ x_2$
3
2
1
1
2
3
4
x_1

$$0 = 4\,x_1 + 3\,x_2$$

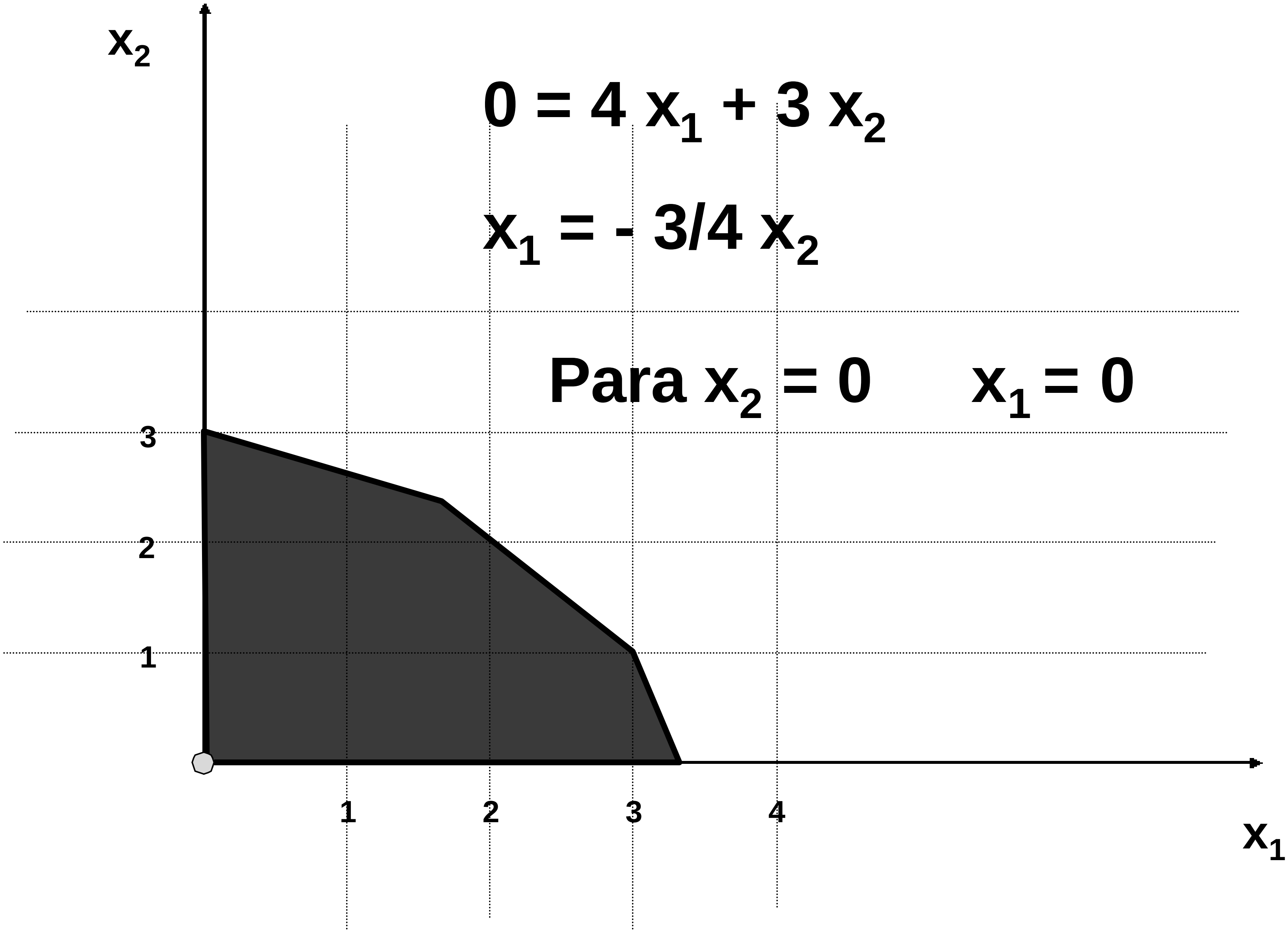
$$x_1 = -\,3/4\,x_2$$



$$0 = 4\,x_1 + 3\,x_2$$

$$x_1 = -\,3/4\,x_2$$

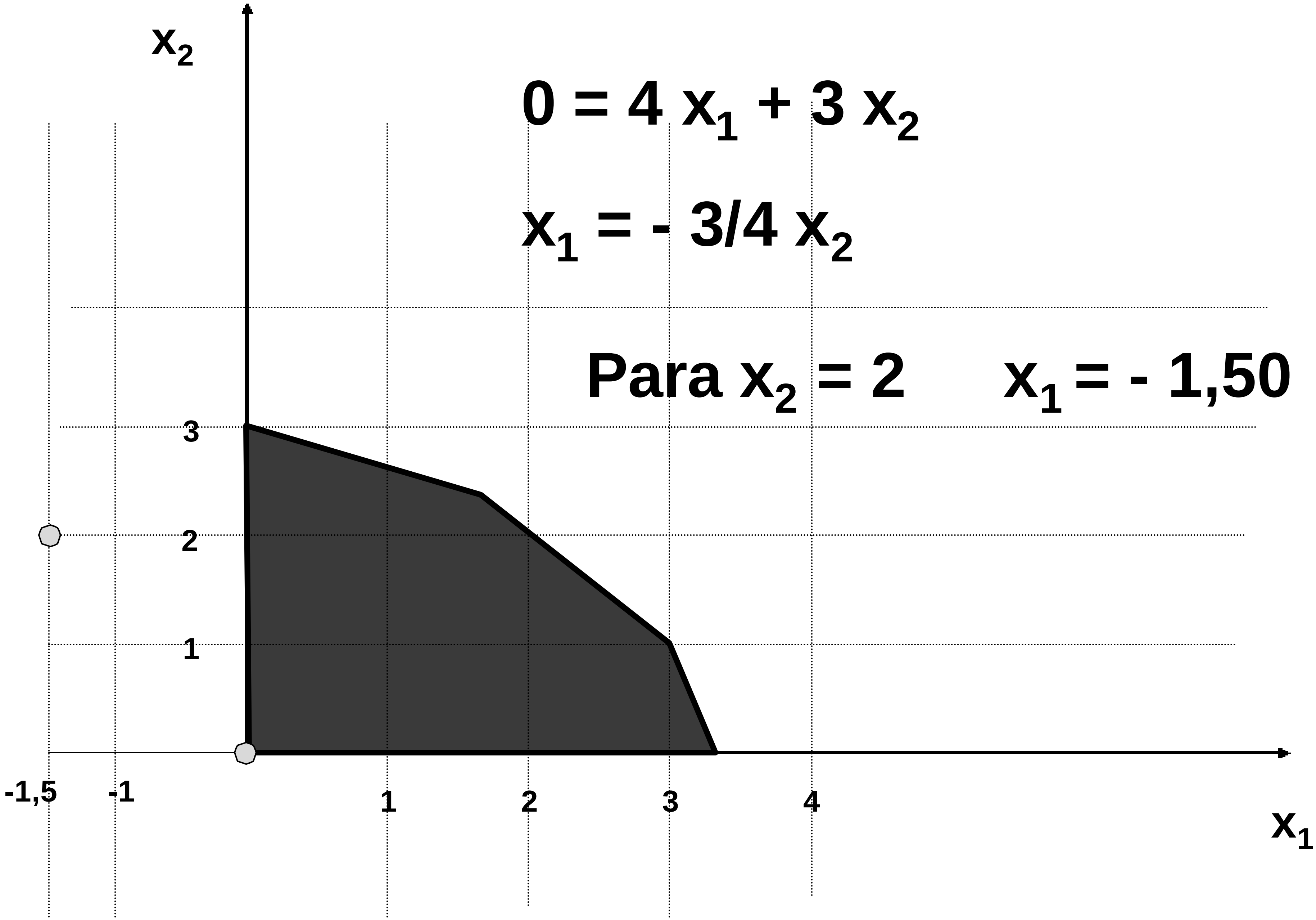
Para $x_2 = 0$ $\quad x_1 = 0$

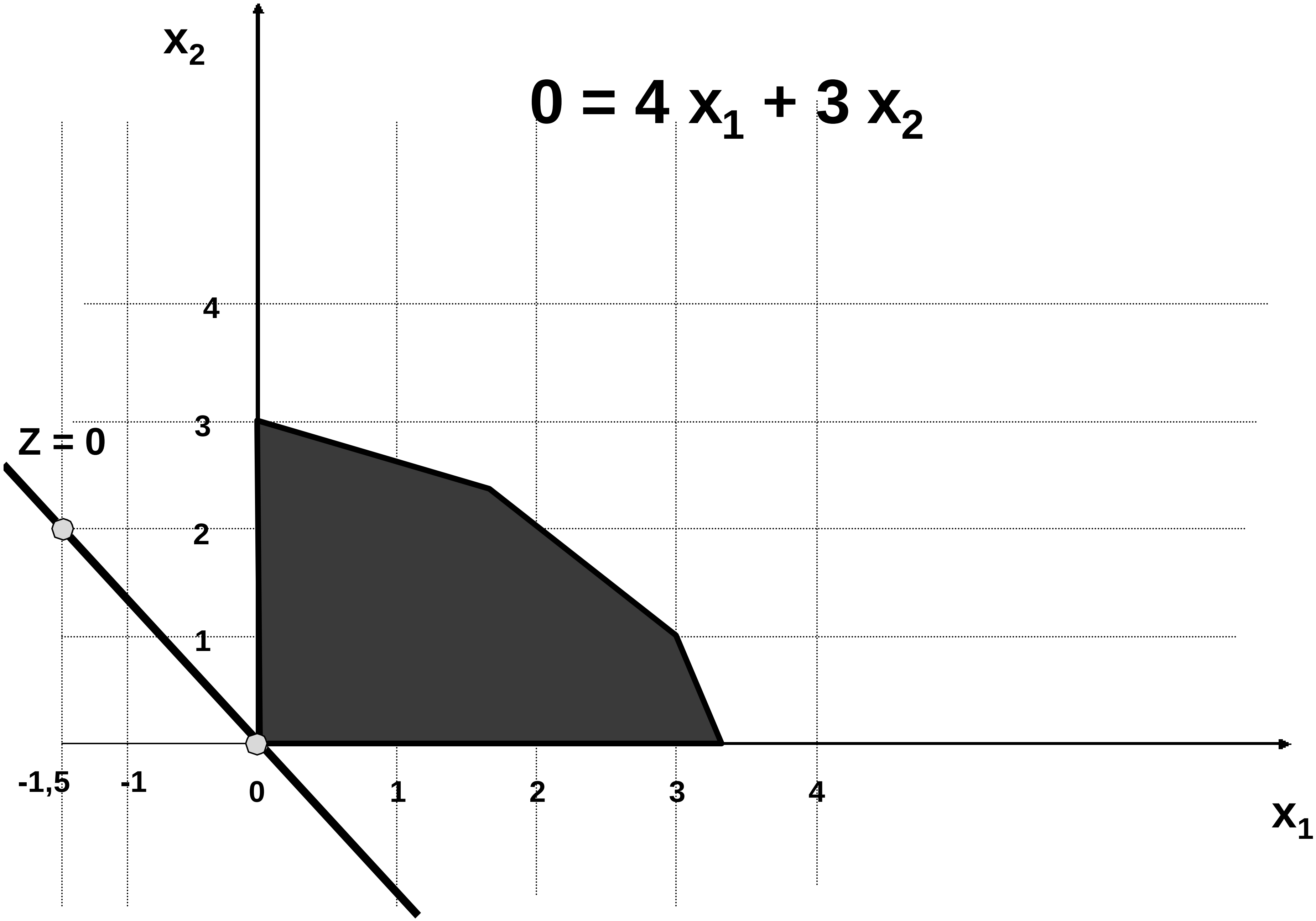


$$0 = 4\,x_1 + 3\,x_2$$

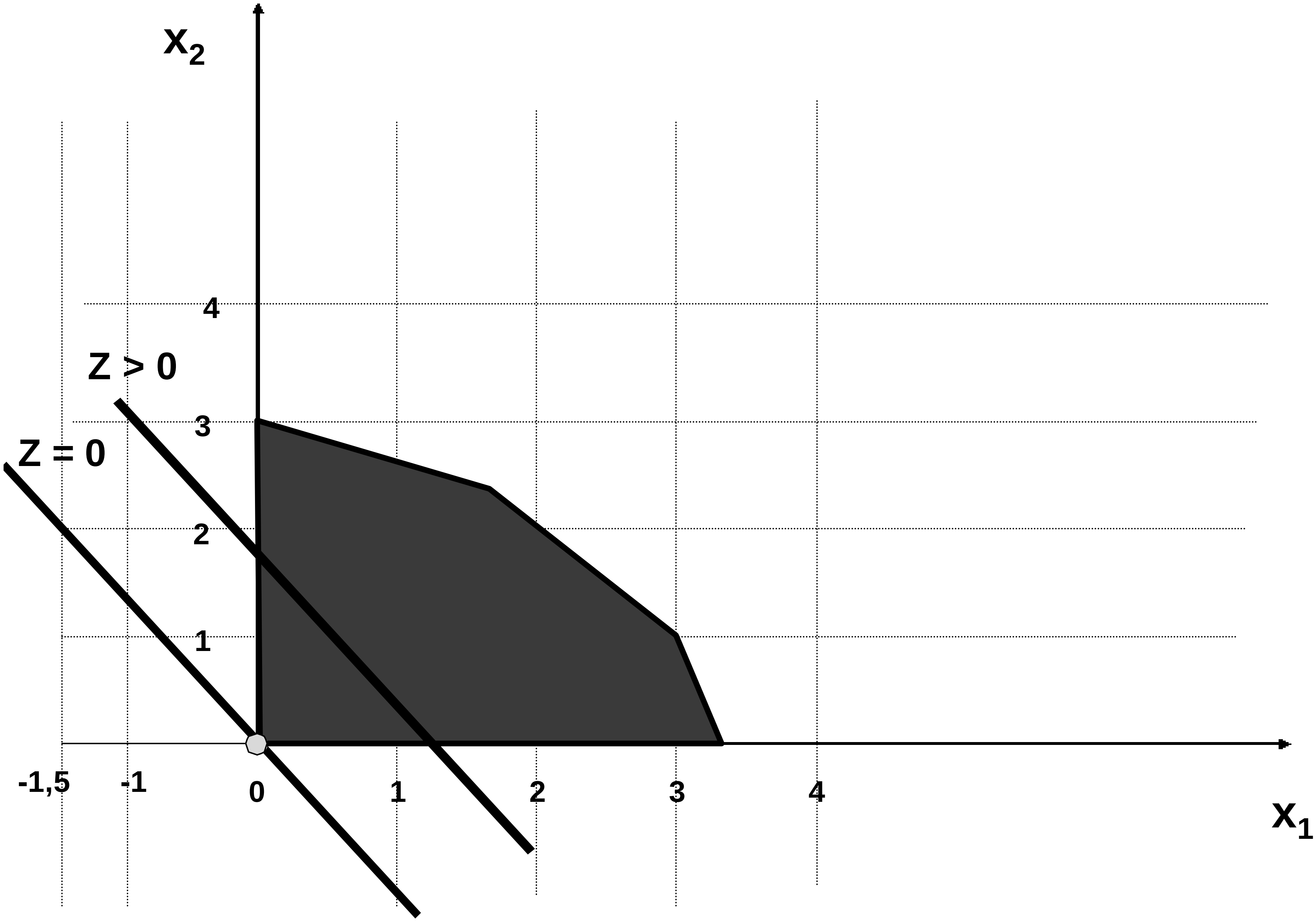
$$x_1 = -\,3/4\,x_2$$

Para $x_2 = 2$ $\quad x_1 = -\,1{,}50$

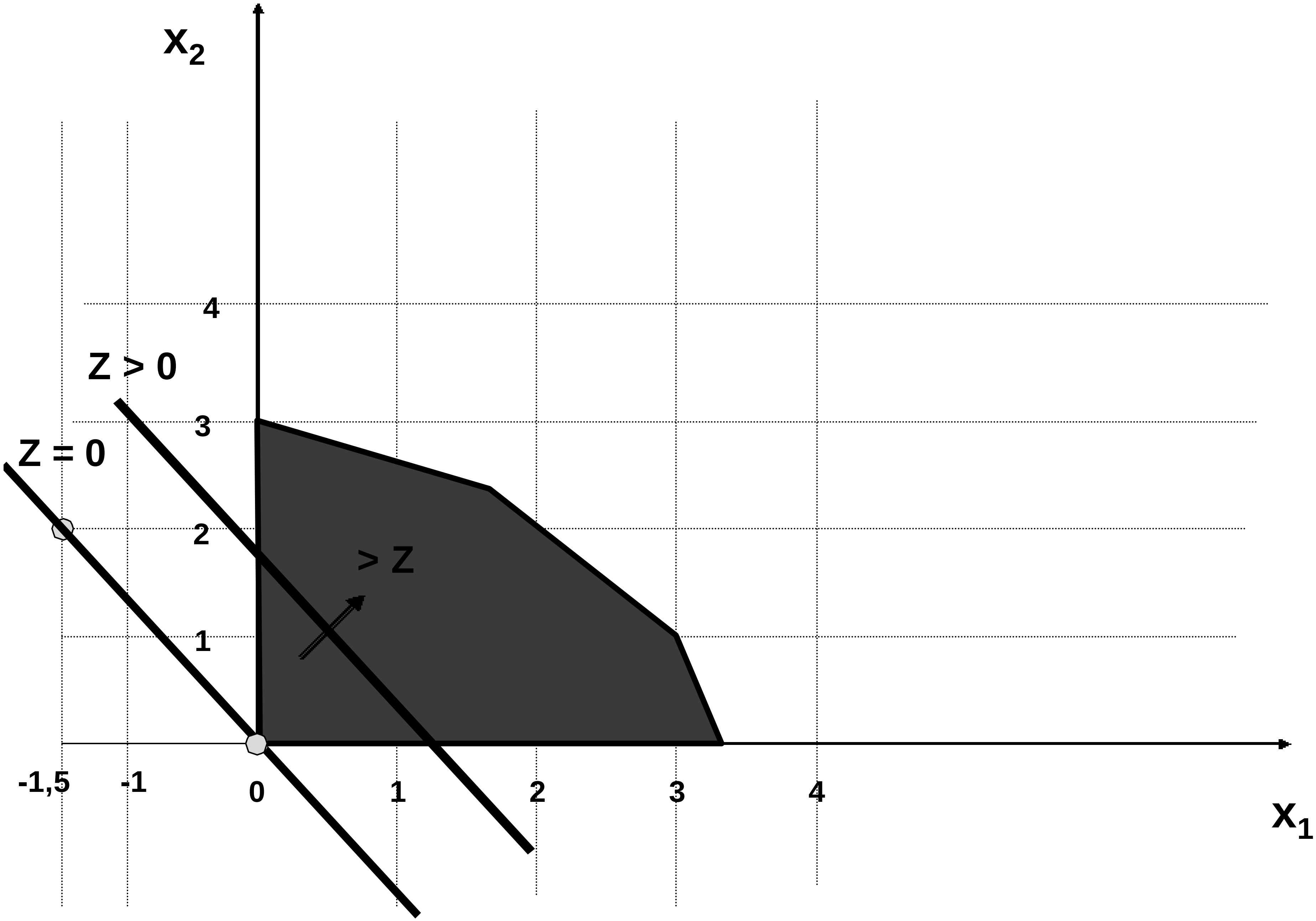




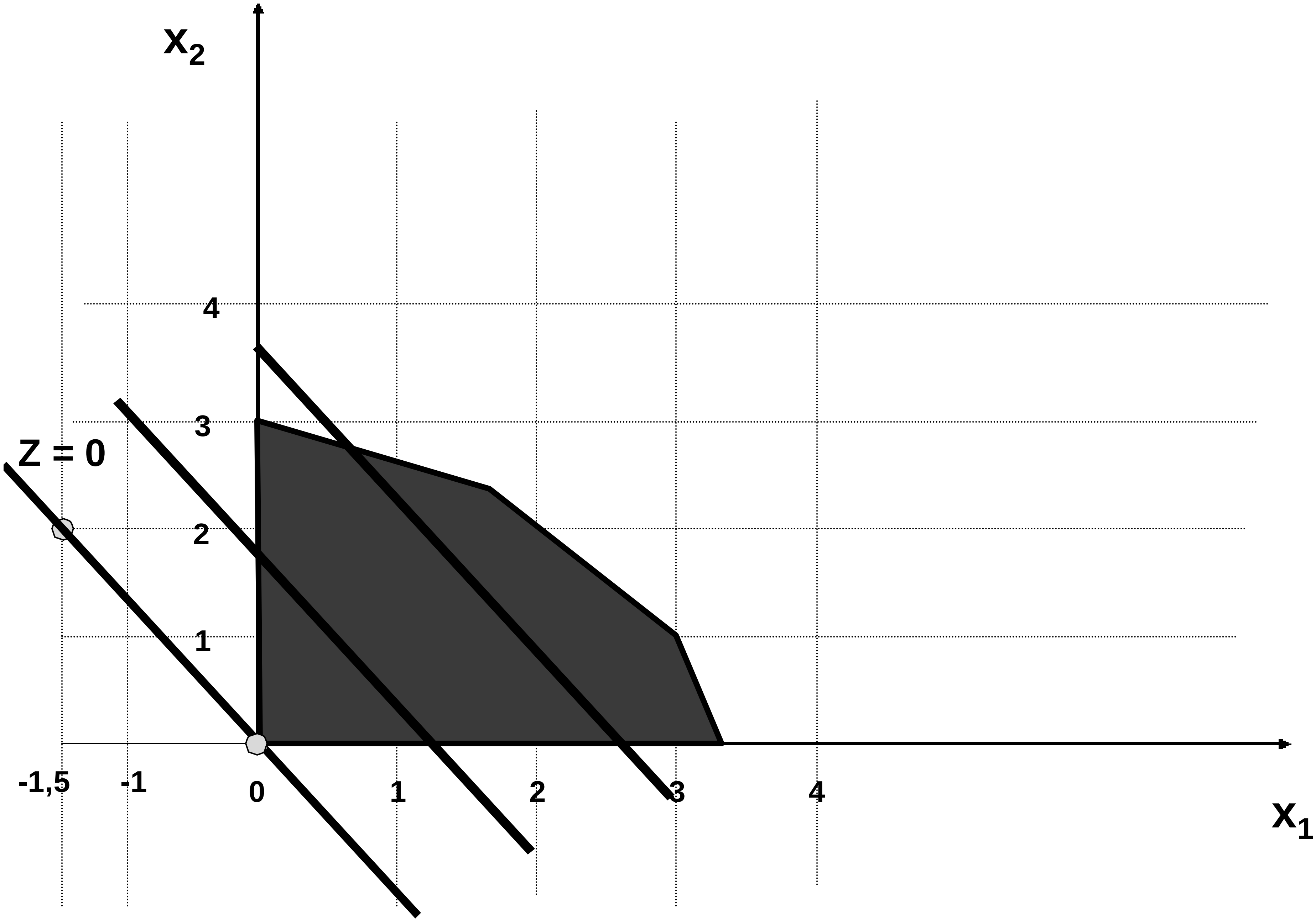
x_2
$0 = 4\,x_1 + 3\,x_2$
4
3
Z = 0
2
1
-1,5
-1
0
1
2
3
4
x_1



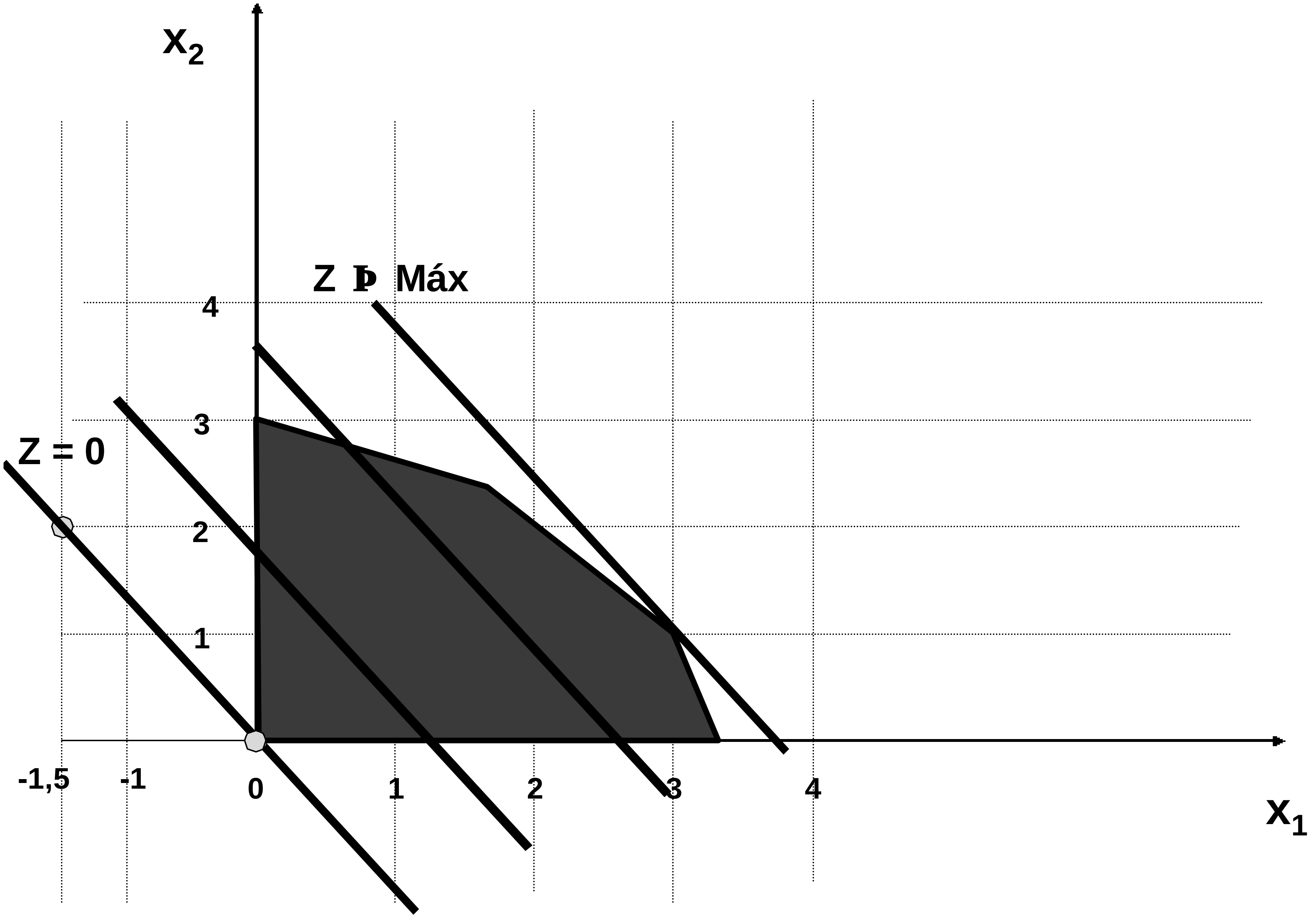
x2
4
Z > 0
3
Z = 0
2
1
-1,5
-1
0
1
2
3
4
x1



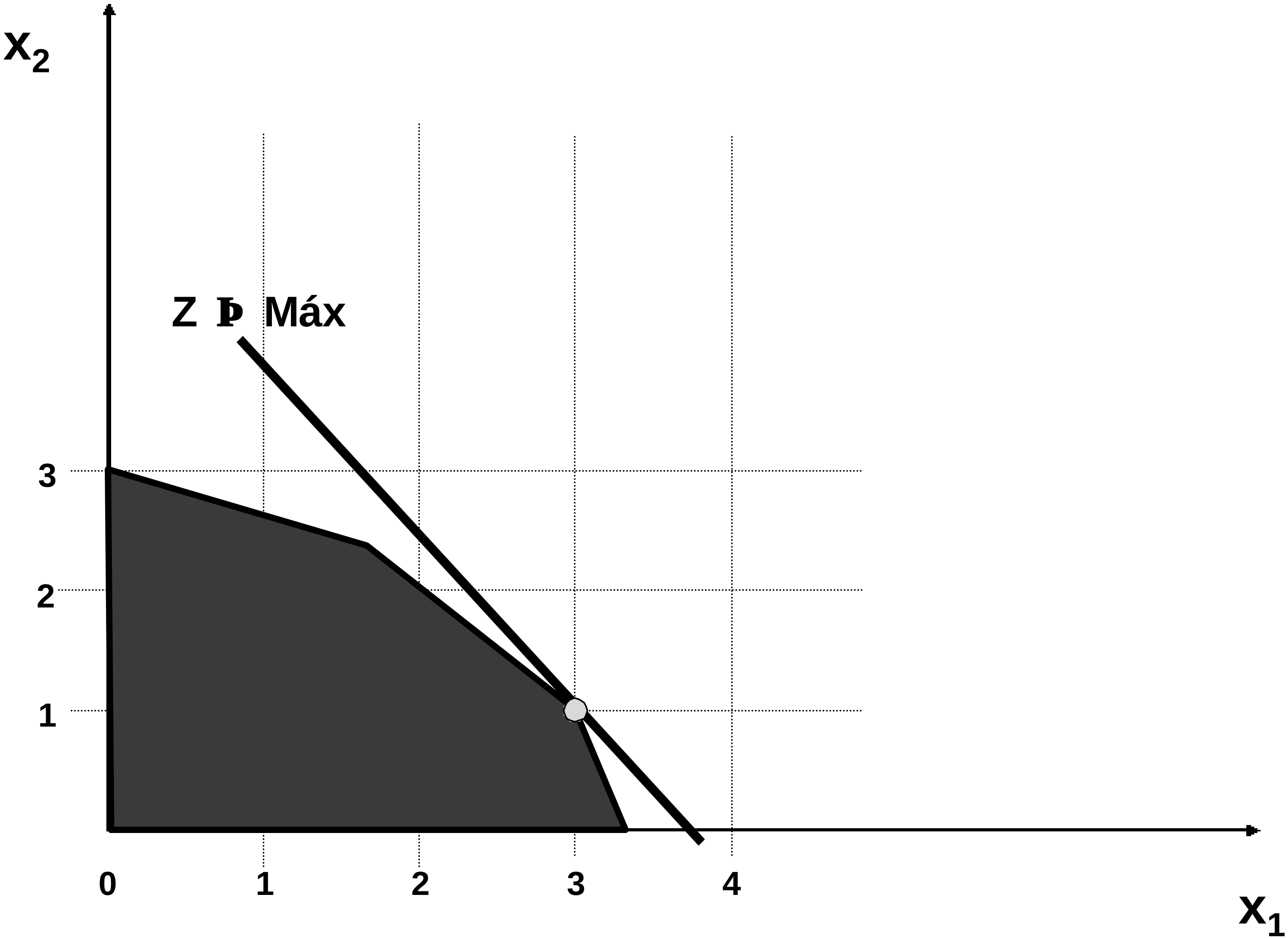
x_2
4
Z > 0
3
Z = 0
2
> Z
1
-1,5
-1
0
1
2
3
4
x_1

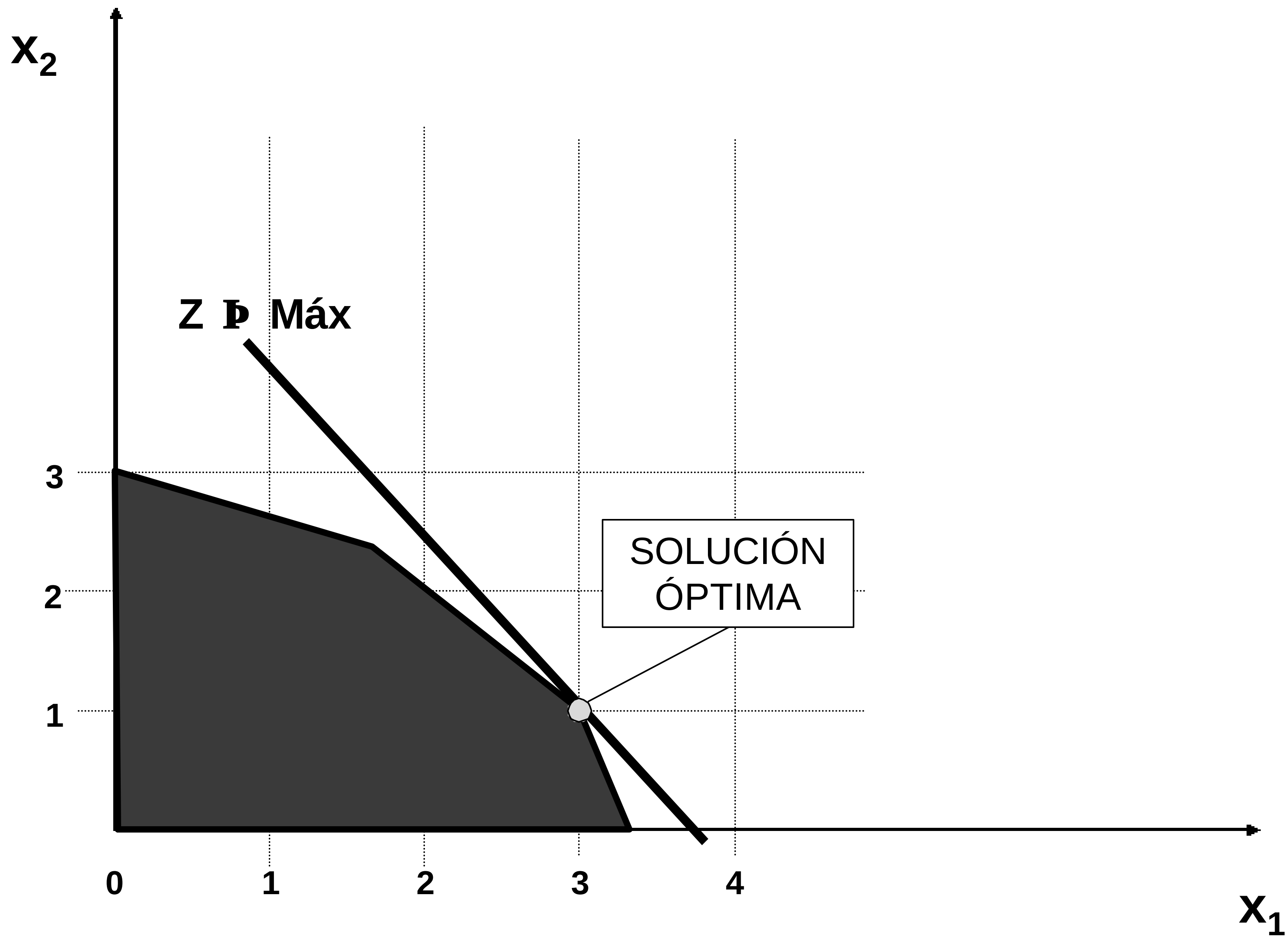


x2
4
3
2
1
Z = 0
-1,5
-1
0
1
2
3
4
x1

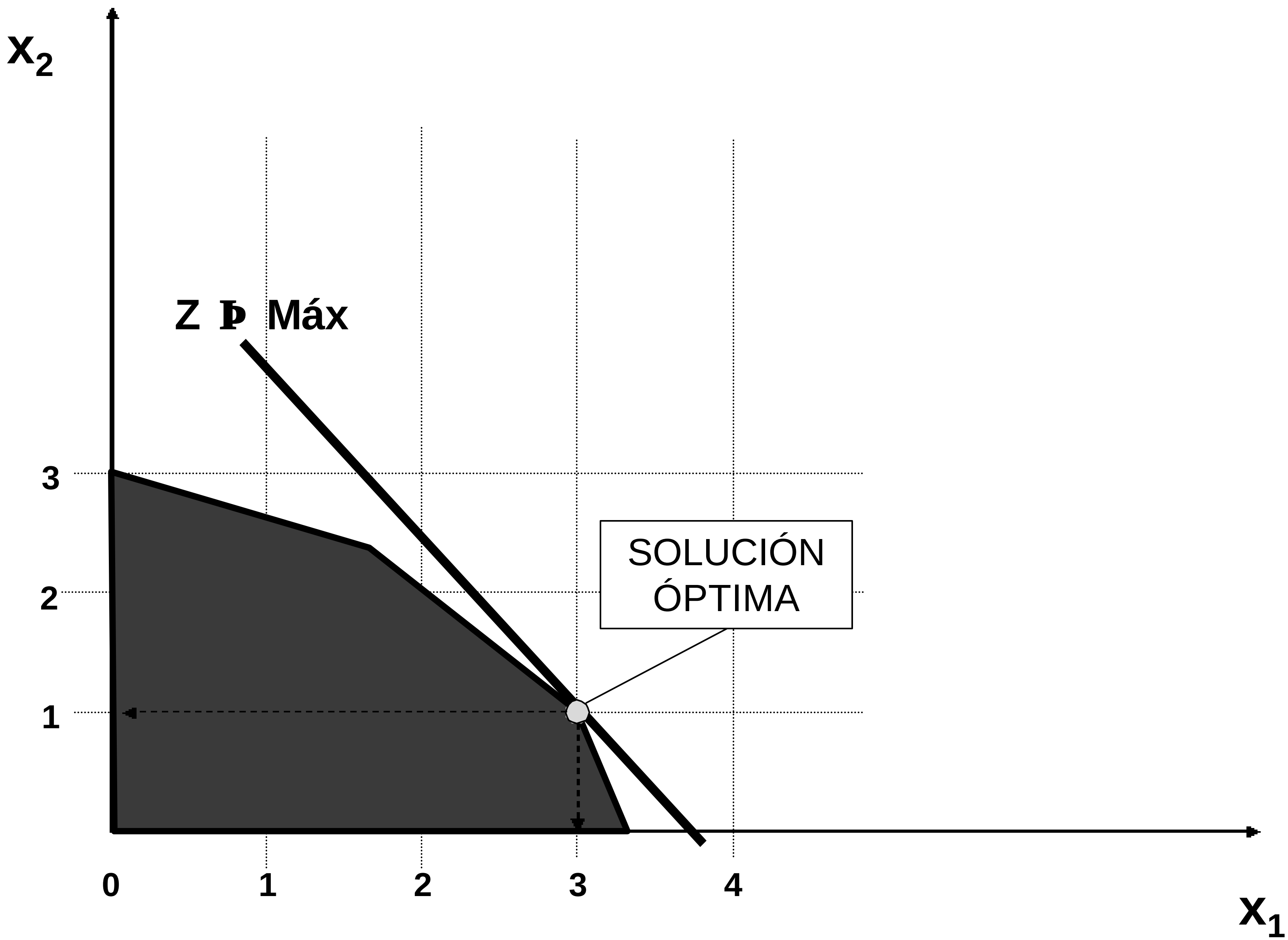


x2
Z Þ Máx
4
3
Z = 0
2
1
-1,5
-1
0
1
2
3
4
x1

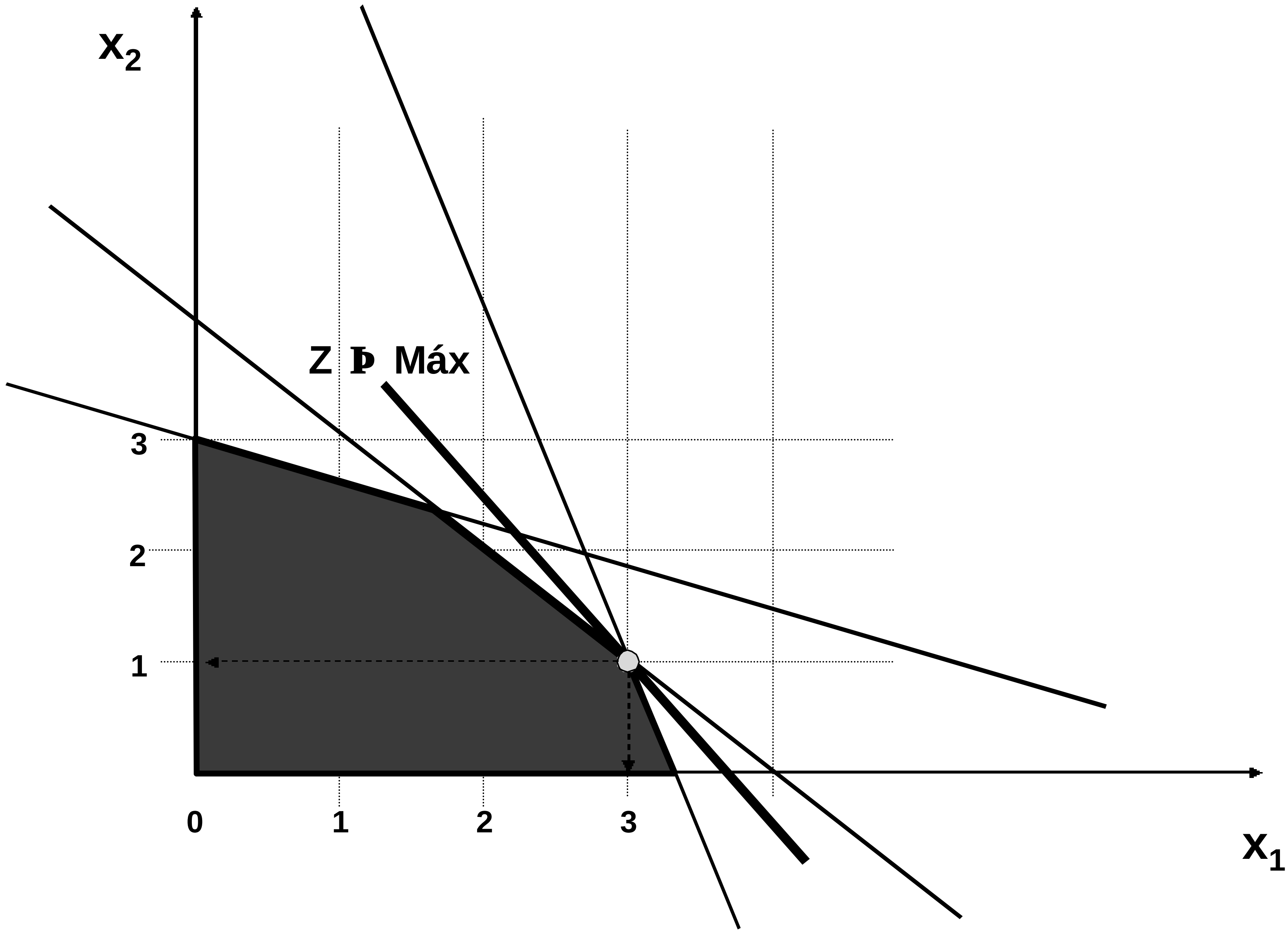

x2
Z Þ Máx
3
2
1
0
1
2
3
4
x1

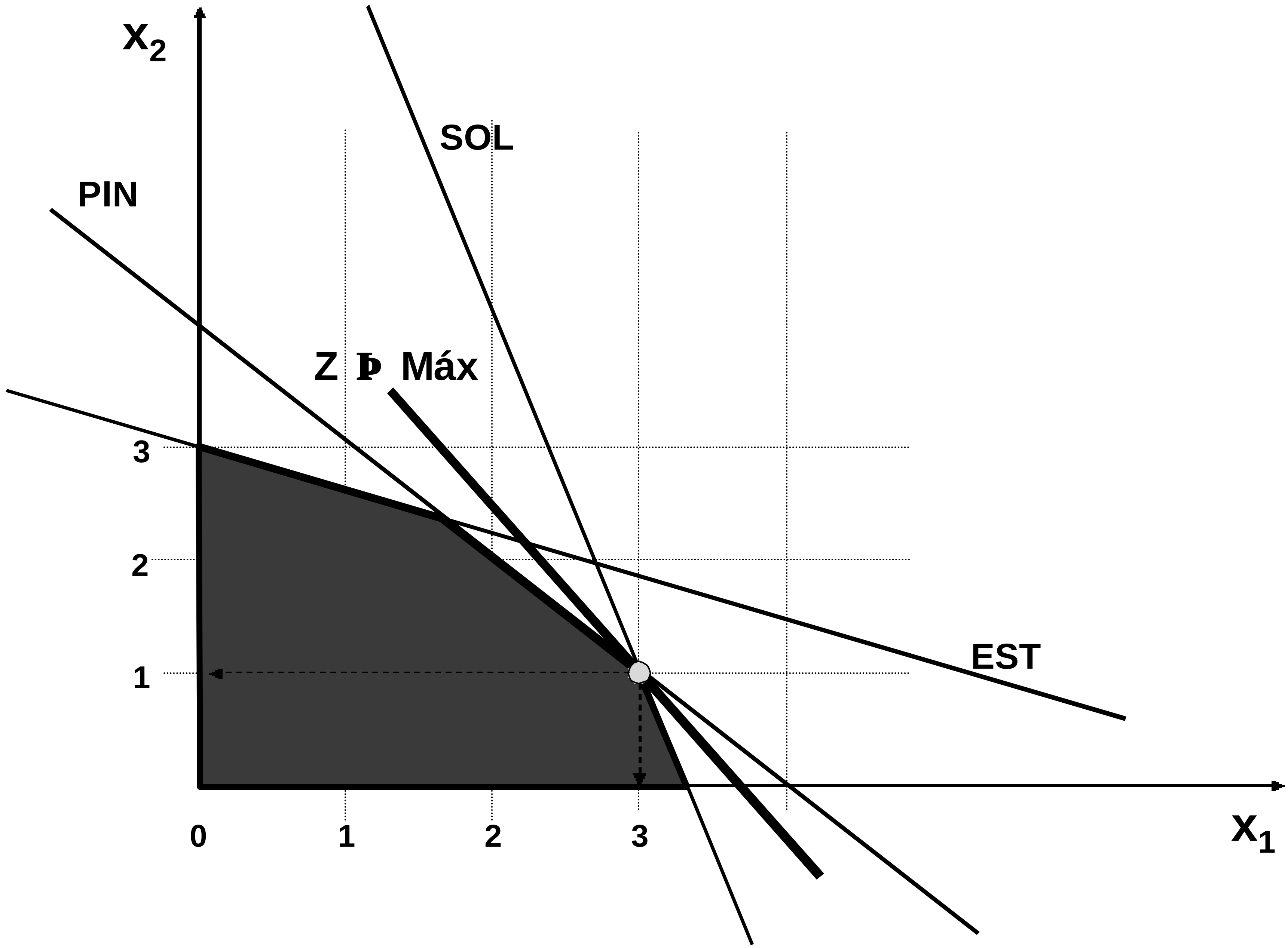


x2
Z Þ Máx
3
2
1
SOLUCIÓN ÓPTIMA
0
1
2
3
4
x1

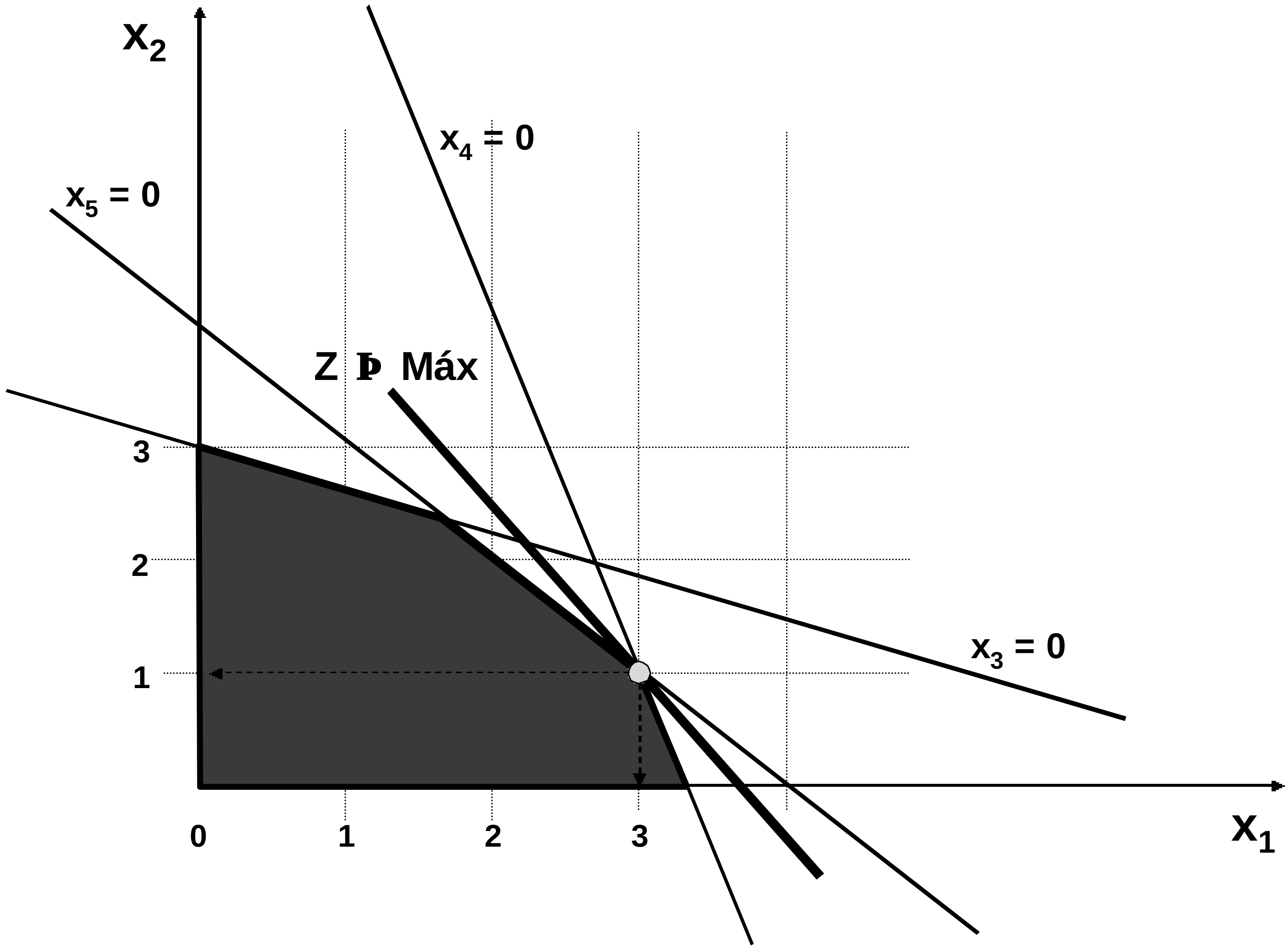


x_2
Z Þ Máx
3
2
1
SOLUCIÓN
ÓPTIMA
0
1
2
3
4
x_1

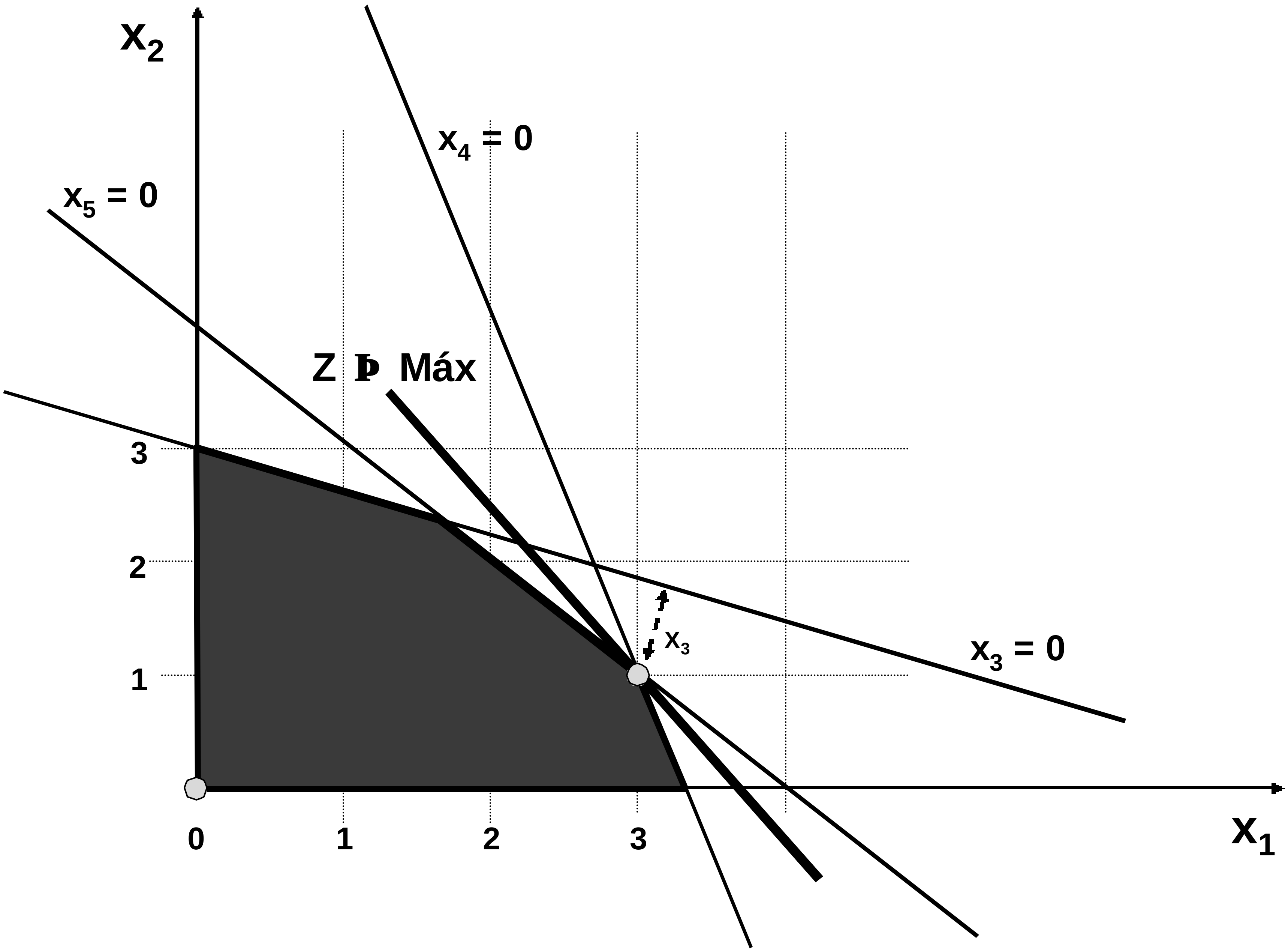

x_2
Z Þ Máx
3
2
1
0
1
2
3
x_1



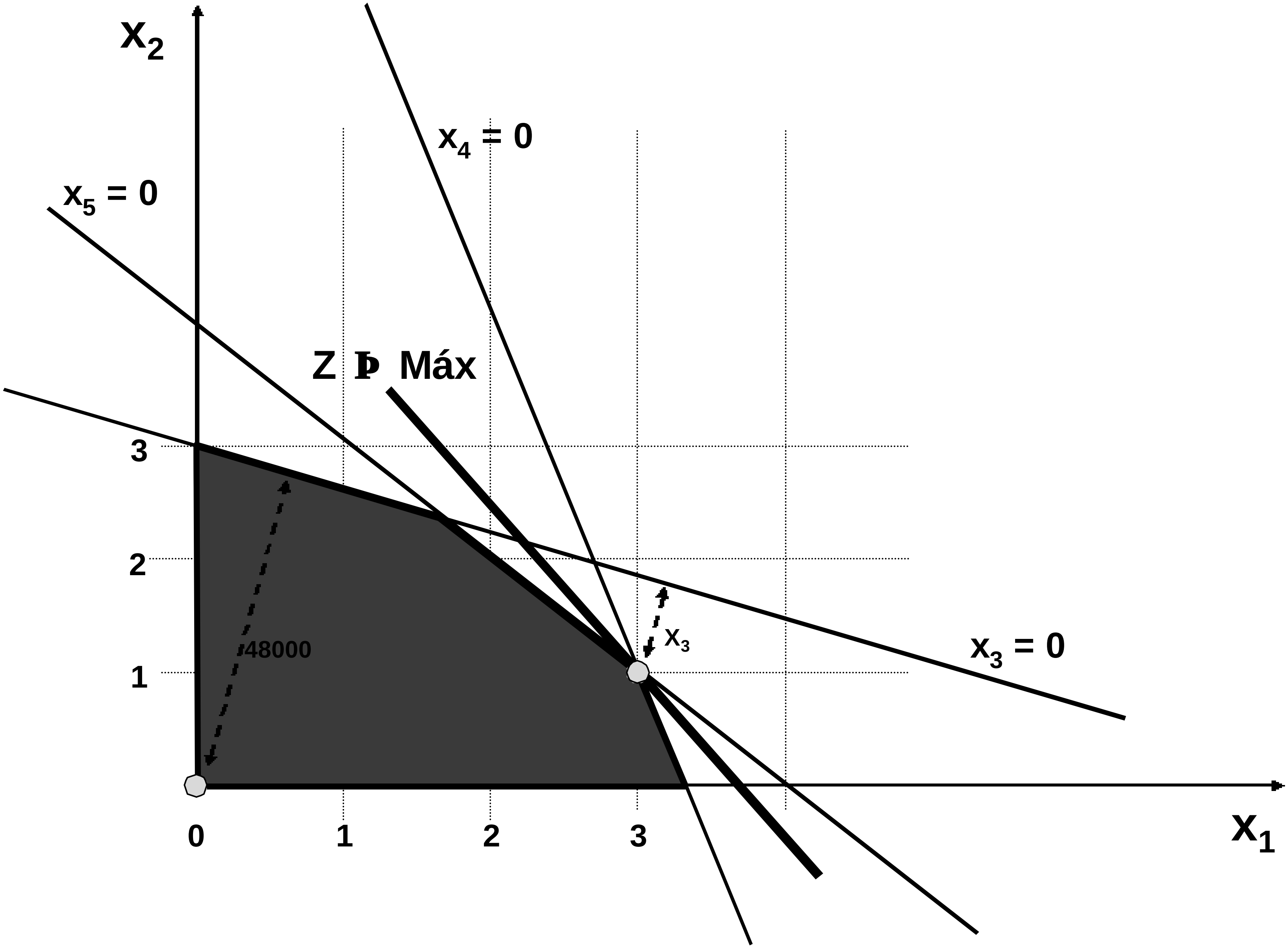
x_2
SOL
PIN
Z Þ Máx
3
2
1
EST
0
1
2
3
x_1



x2
x4 = 0
x5 = 0
Z Þ Máx
3
2
1
x3 = 0
0
1
2
3
x1



x2
x4 = 0
x5 = 0
Z Þ Máx
3
2
1
x3
x3 = 0
0
1
2
3
x1



x2
x4 = 0
x5 = 0
Z Þ Máx
3
2
1
48000
x3
x3 = 0
0
1
2
3
x1

$$\text{MAX:} \quad Z = 4\,x_1 + 3\,x_2$$

$$\left\{ \begin{array}{l} 6\,x_1 + 16\,x_2 \;\text{£}\; 48000 \\ 12\,x_1 + 6\,x_2 \;\text{£}\; 42000 \\ 9\,x_1 + 9\,x_2 \;\text{£}\; 36000 \end{array} \right.$$

$$x_1, x_2 \;\text{³}\; 0$$

FORMA MATRICIAL EXTENDIDA

	X_1	**X_2**	**SIGNO**	**RHS**
Z)	4	3	➡	MAX
EST)	6	16	$\leq$	48000
SOL)	12	6	$\leq$	42000
PIN)	9	9	$\leq$	36000
Var.	NN	NN		

$$\text{MAX:} \quad Z = 4\,x_1 + 3\,x_2$$

$$\begin{cases} 6\,x_1 + 16\,x_2 + x_3 = 48000 \\ 12\,x_1 + 6\,x_2 + x_4 = 42000 \\ 9\,x_1 + 9\,x_2 + x_5 = 36000 \end{cases}$$

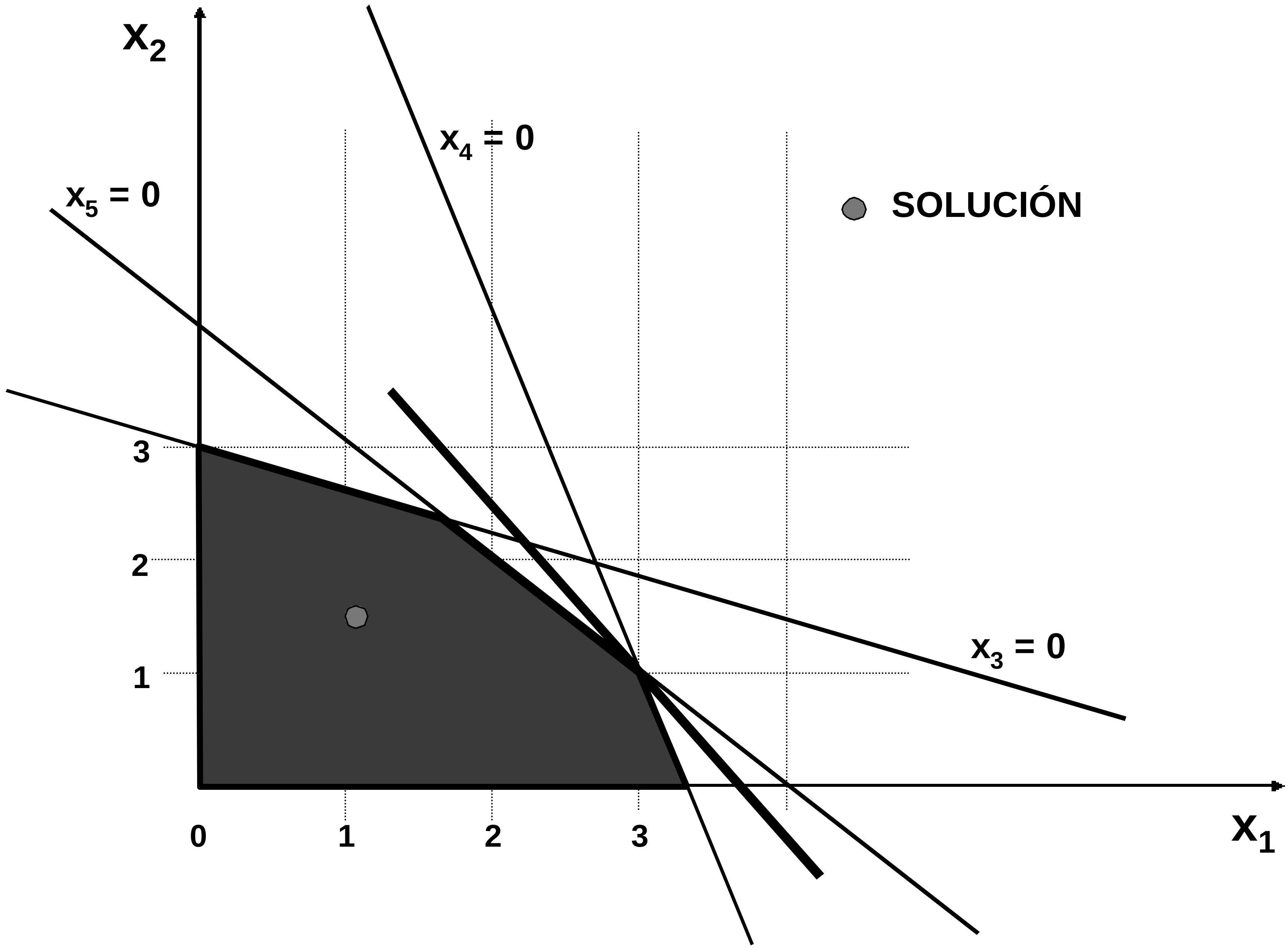
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

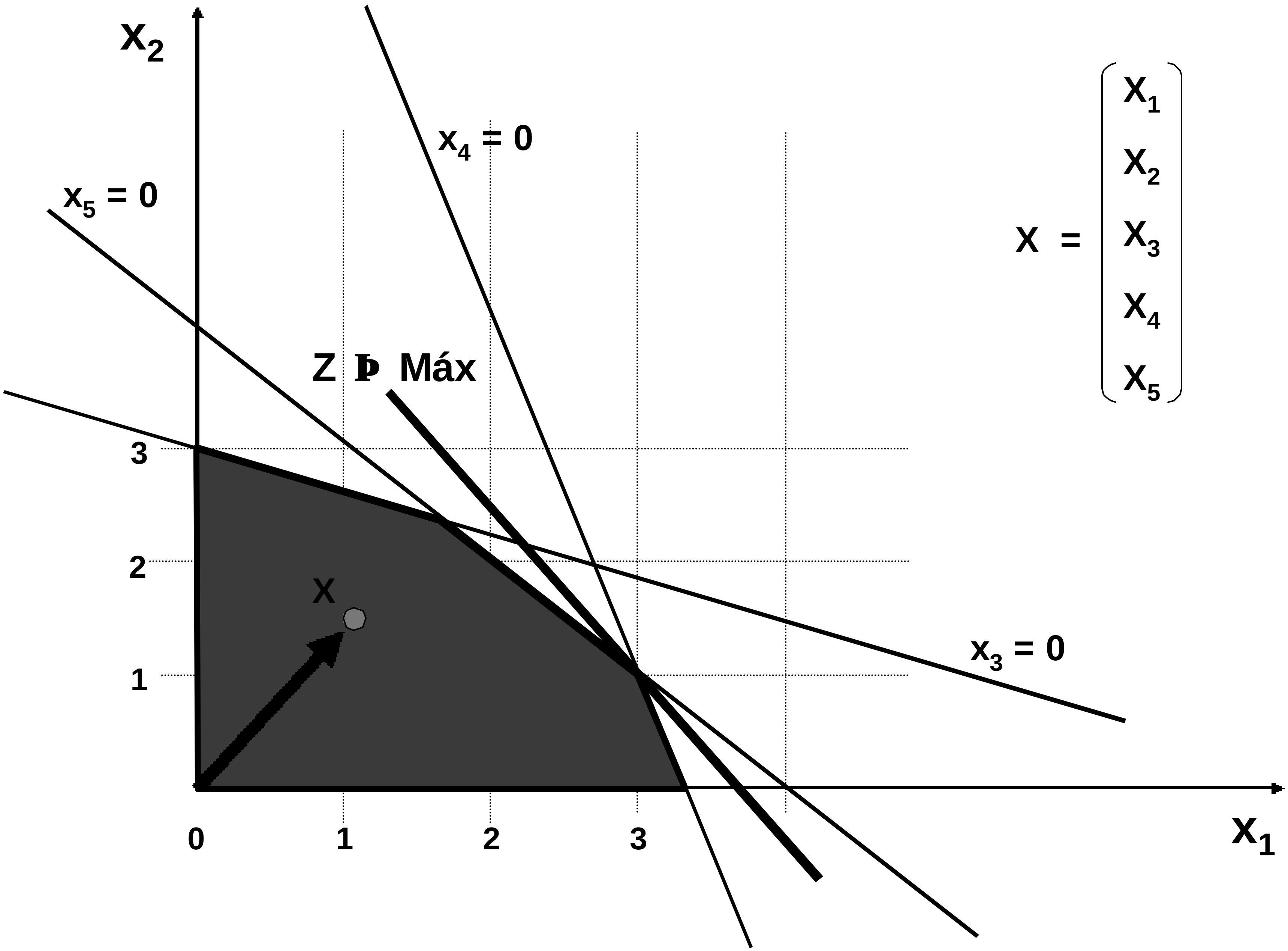
SISTEMA DE ECUACIONES LINEALES

$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

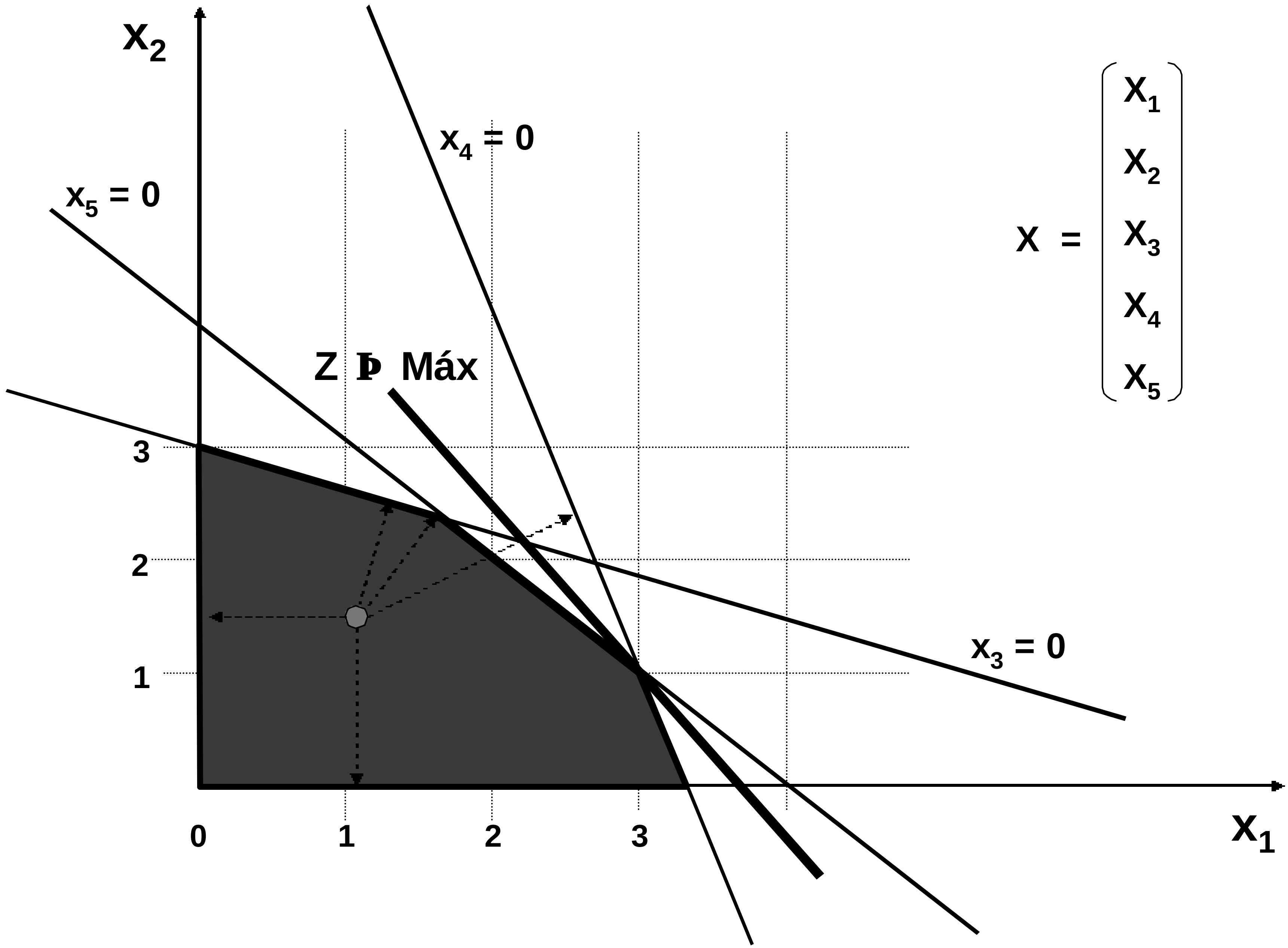
n = número de variables (5)

m = número de restricciones (3)

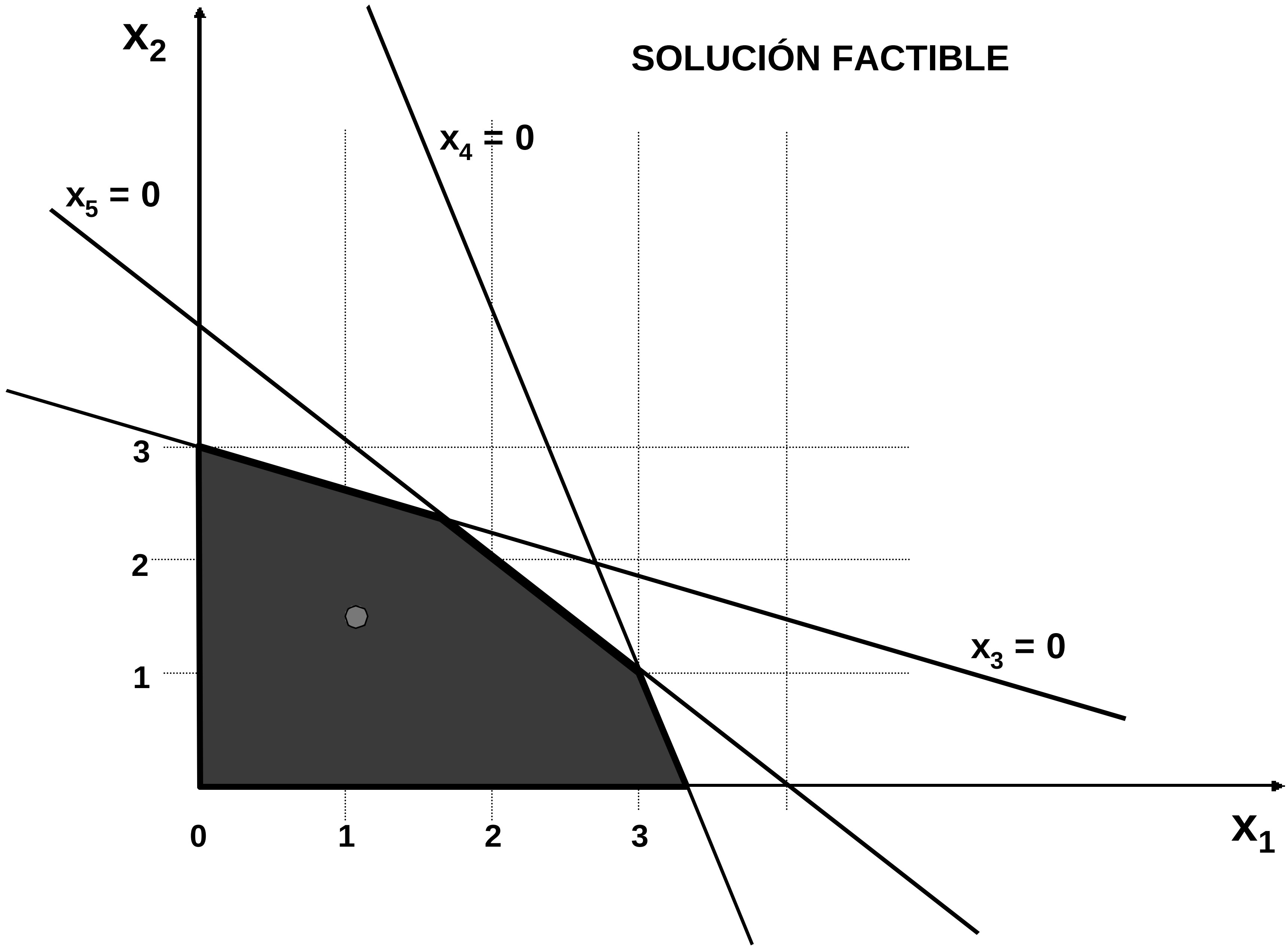

x_2
$x_4 = 0$
$x_5 = 0$
SOLUCIÓN
3
2
1
$x_3 = 0$
0
1
2
3
x_1



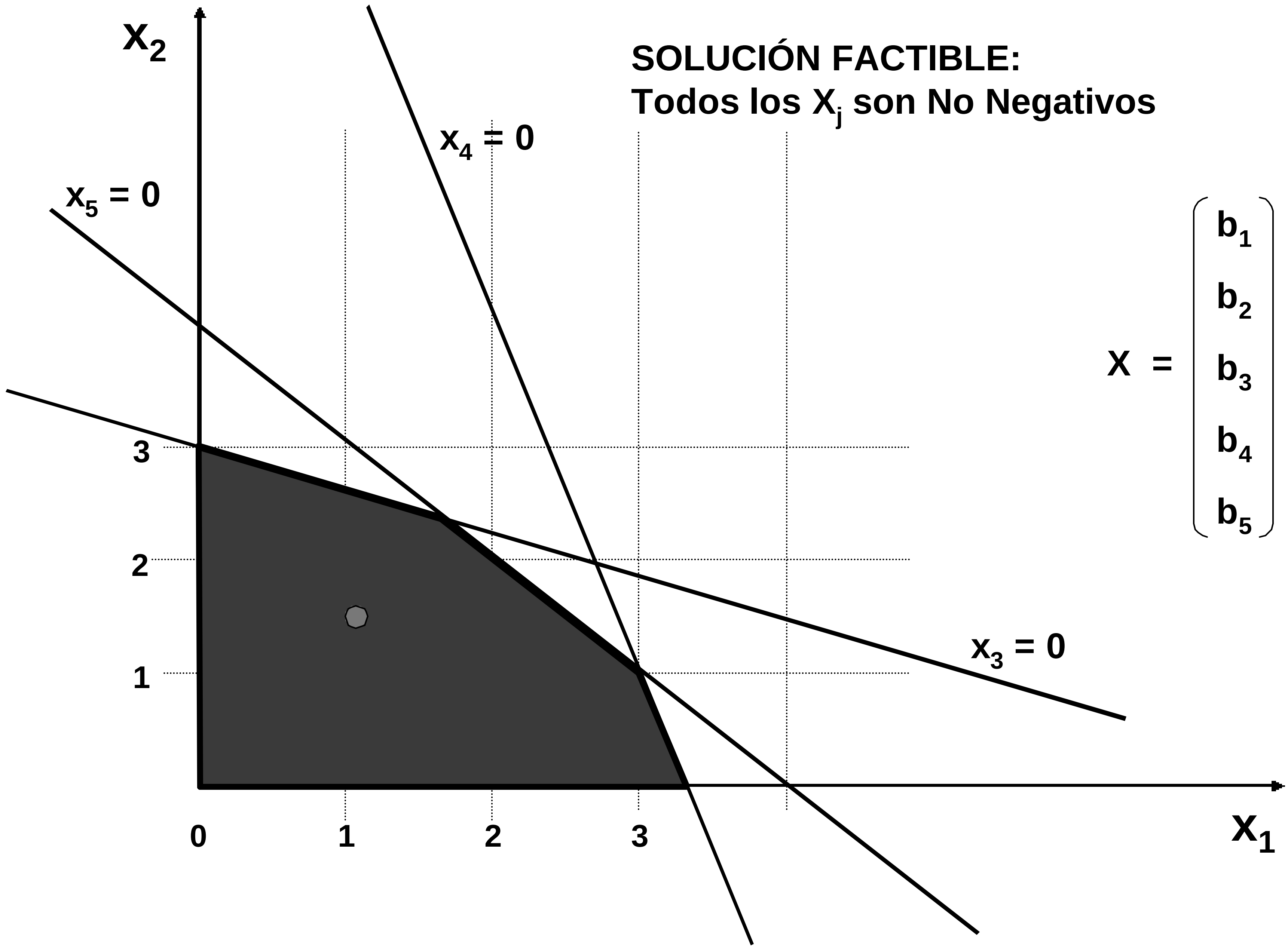
x_2
$x_4 = 0$
$x_5 = 0$
$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$
Z Þ Máx
3
2
1
X
$x_3 = 0$
0
1
2
3
x_1



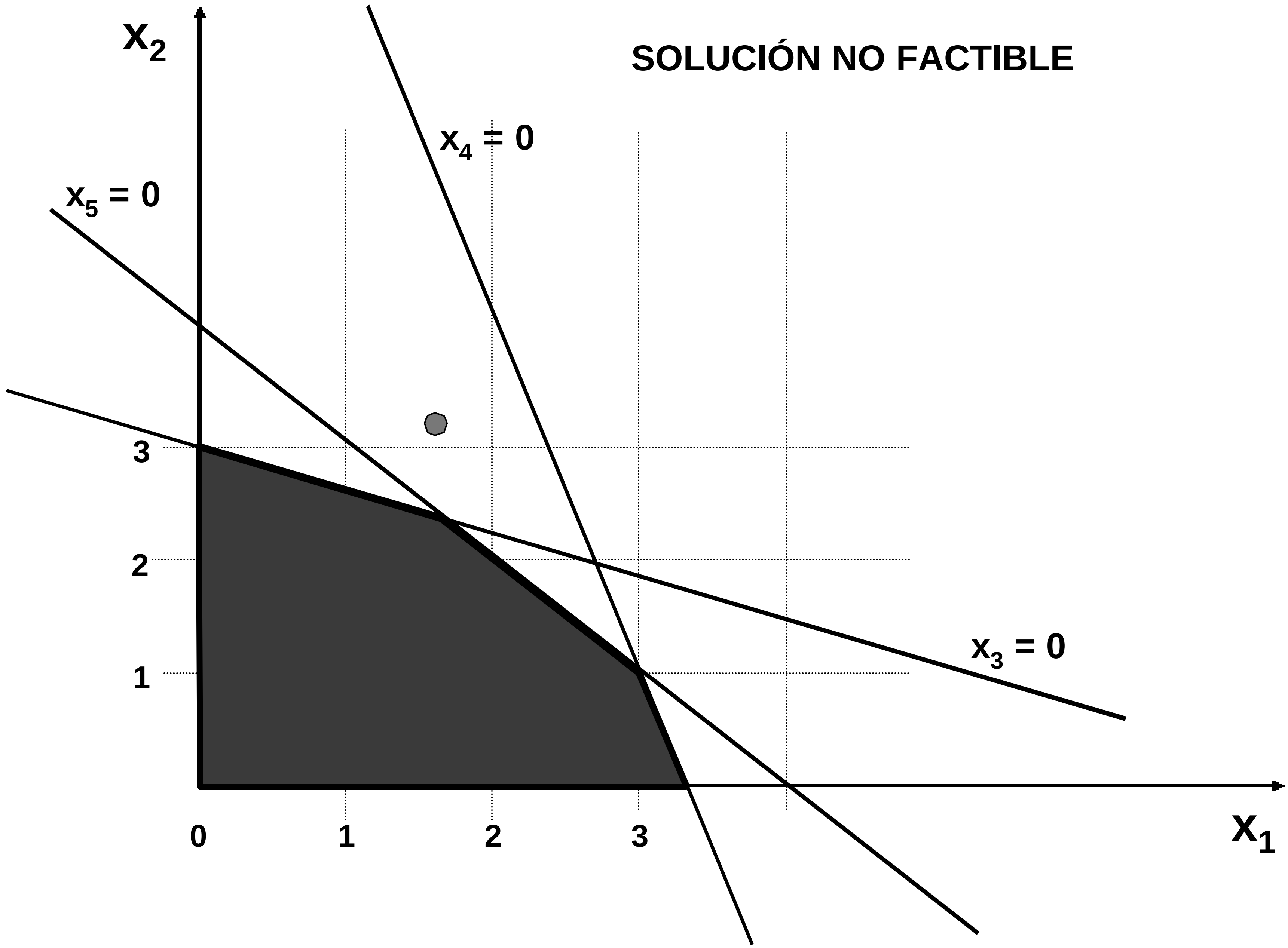
x_2
$x_4 = 0$
$x_5 = 0$
Z Þ Máx
$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$
3
2
1
$x_3 = 0$
0
1
2
3
x_1


SOLUCIÓN FACTIBLE
x_2
x_1
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
0
1
2
3
1
2
3

SOLUCIÓN FACTIBLE:
Todos los X_j son No Negativos

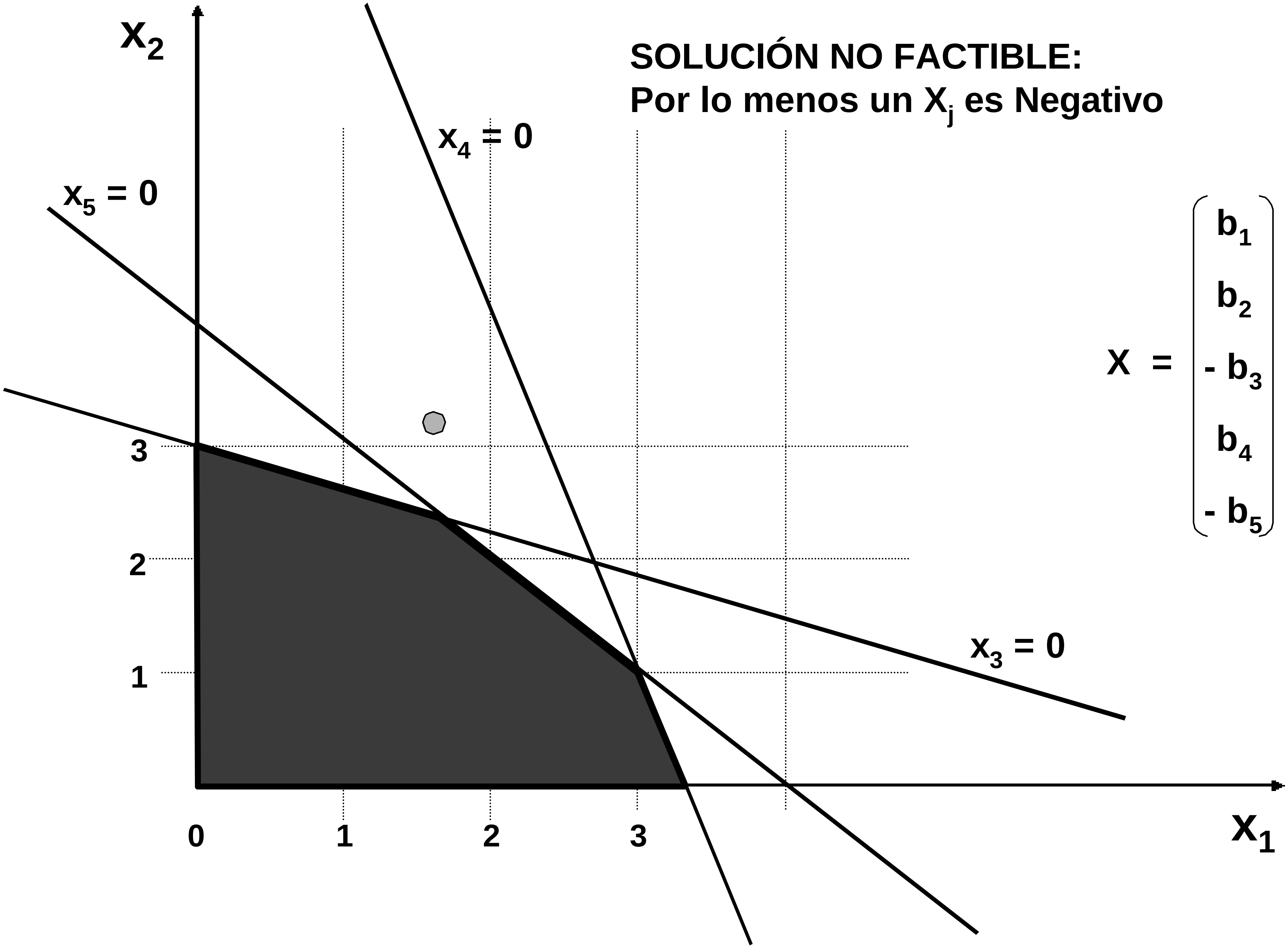


$$X = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}$$



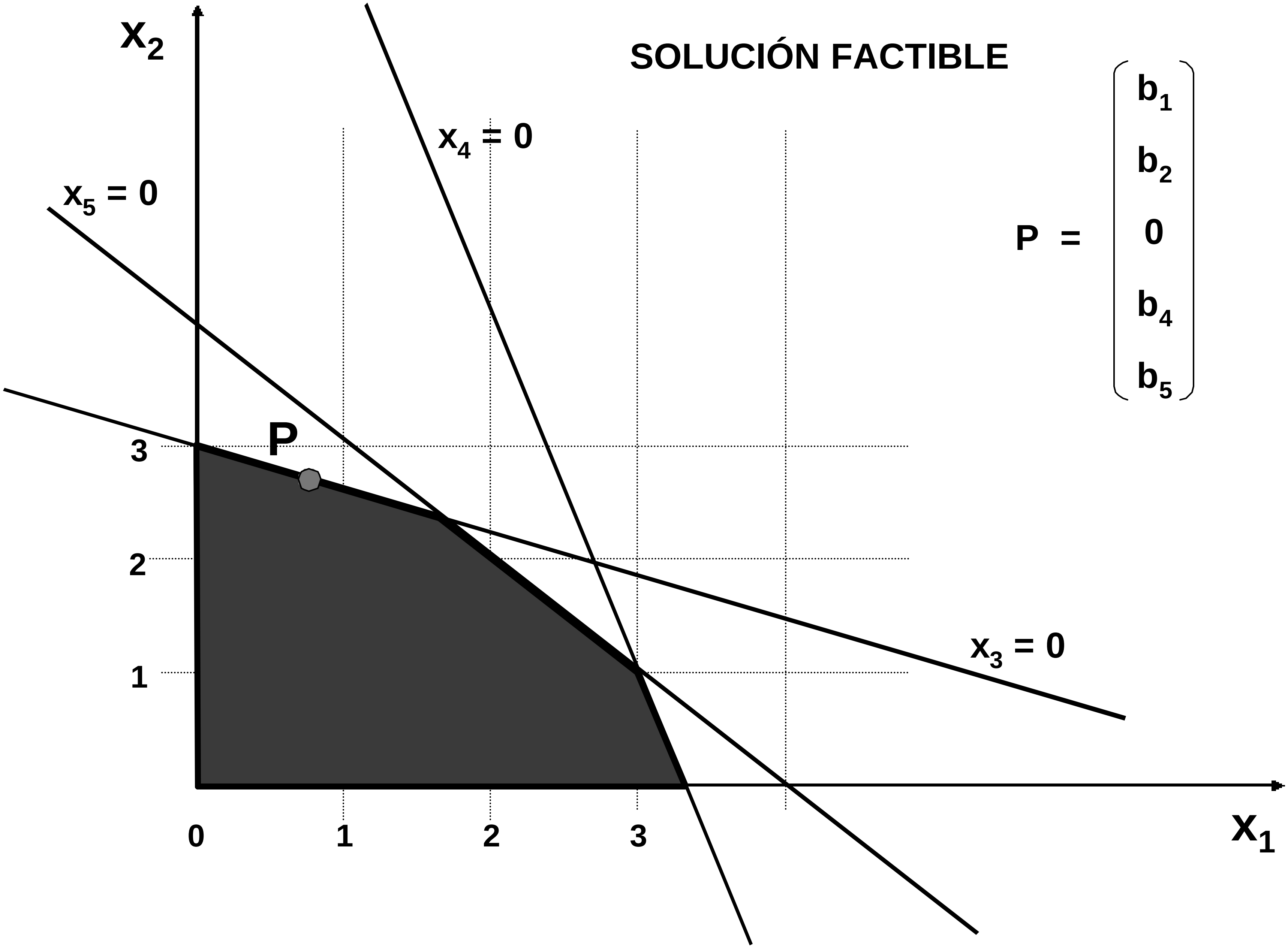
SOLUCIÓN NO FACTIBLE
x_2
x_1
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
3
2
1
0
1
2
3

SOLUCIÓN NO FACTIBLE:
Por lo menos un X_j es Negativo

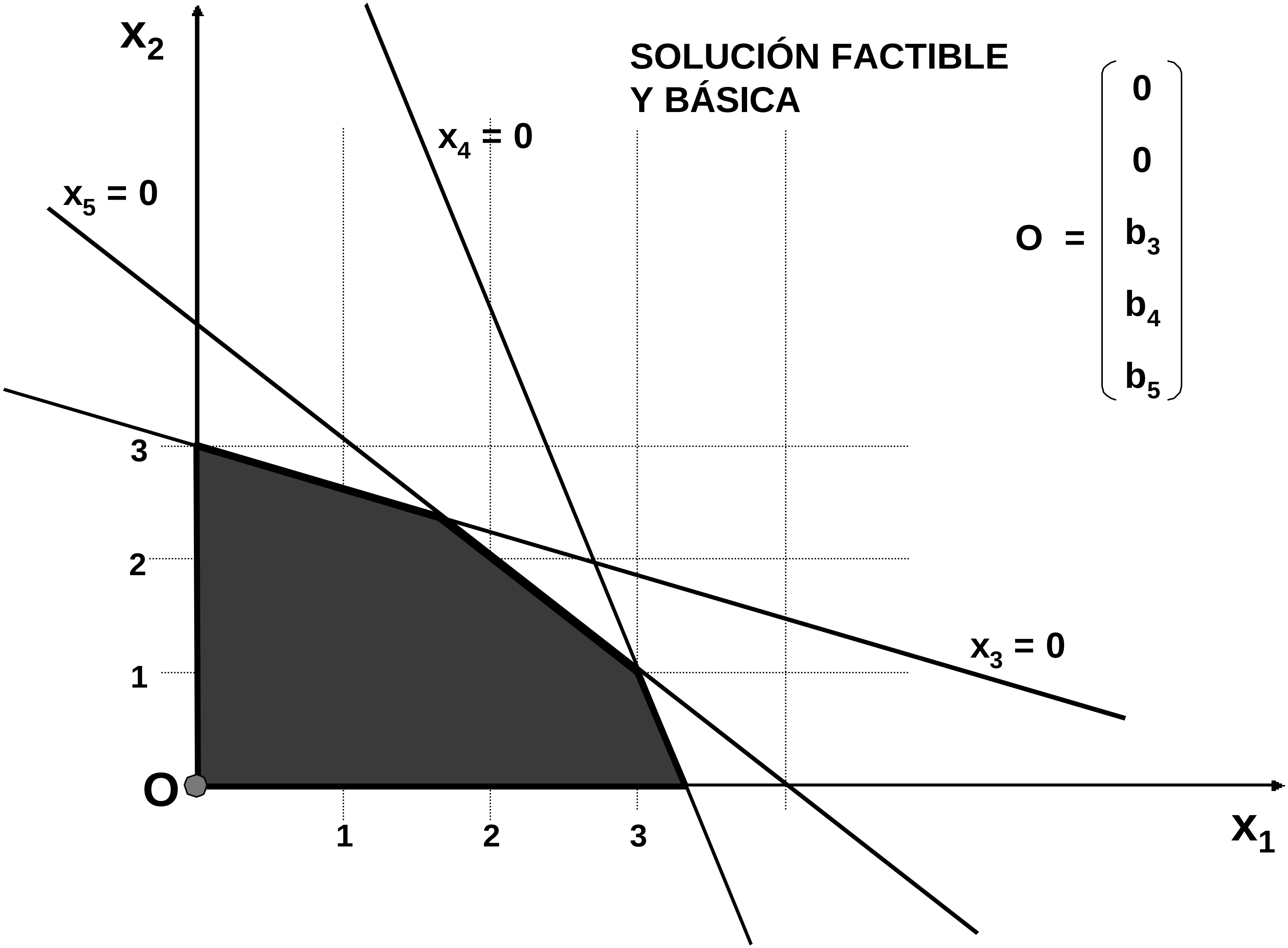


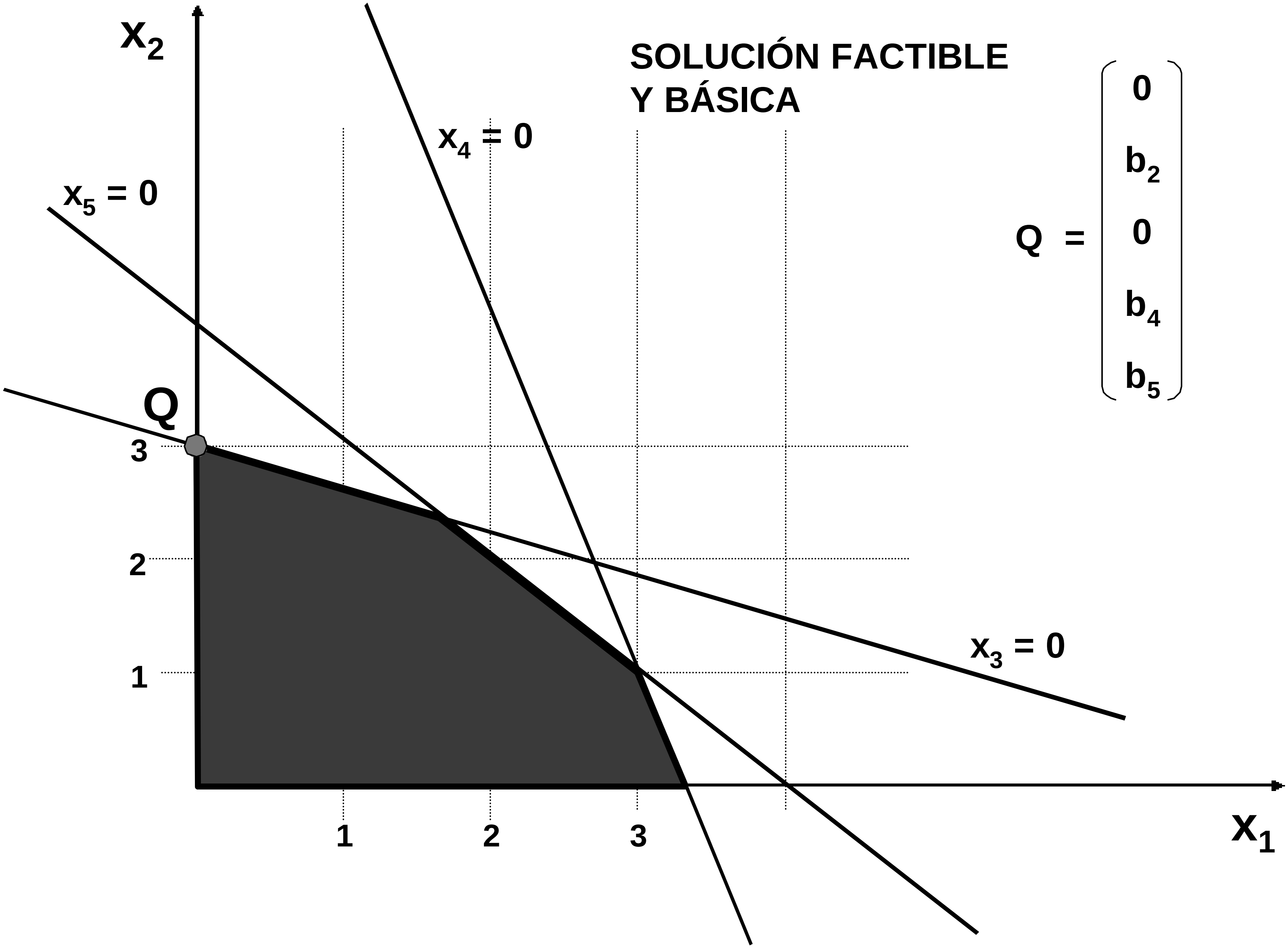
$$X = \begin{pmatrix} b_1 \\ b_2 \\ -b_3 \\ b_4 \\ -b_5 \end{pmatrix}$$

SOLUCIÓN FACTIBLE

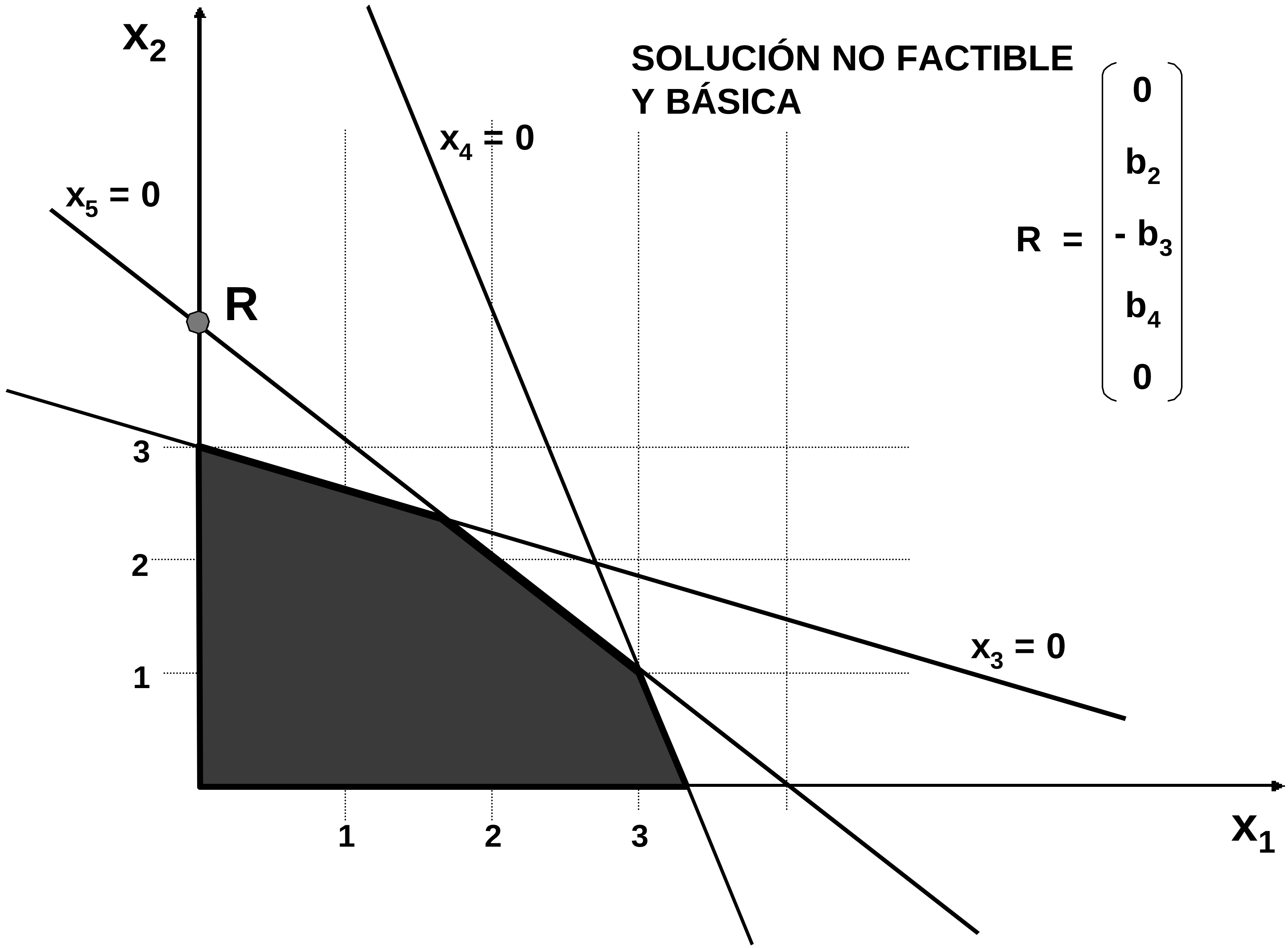


$$P = \begin{pmatrix} b_1 \\ b_2 \\ 0 \\ b_4 \\ b_5 \end{pmatrix}$$

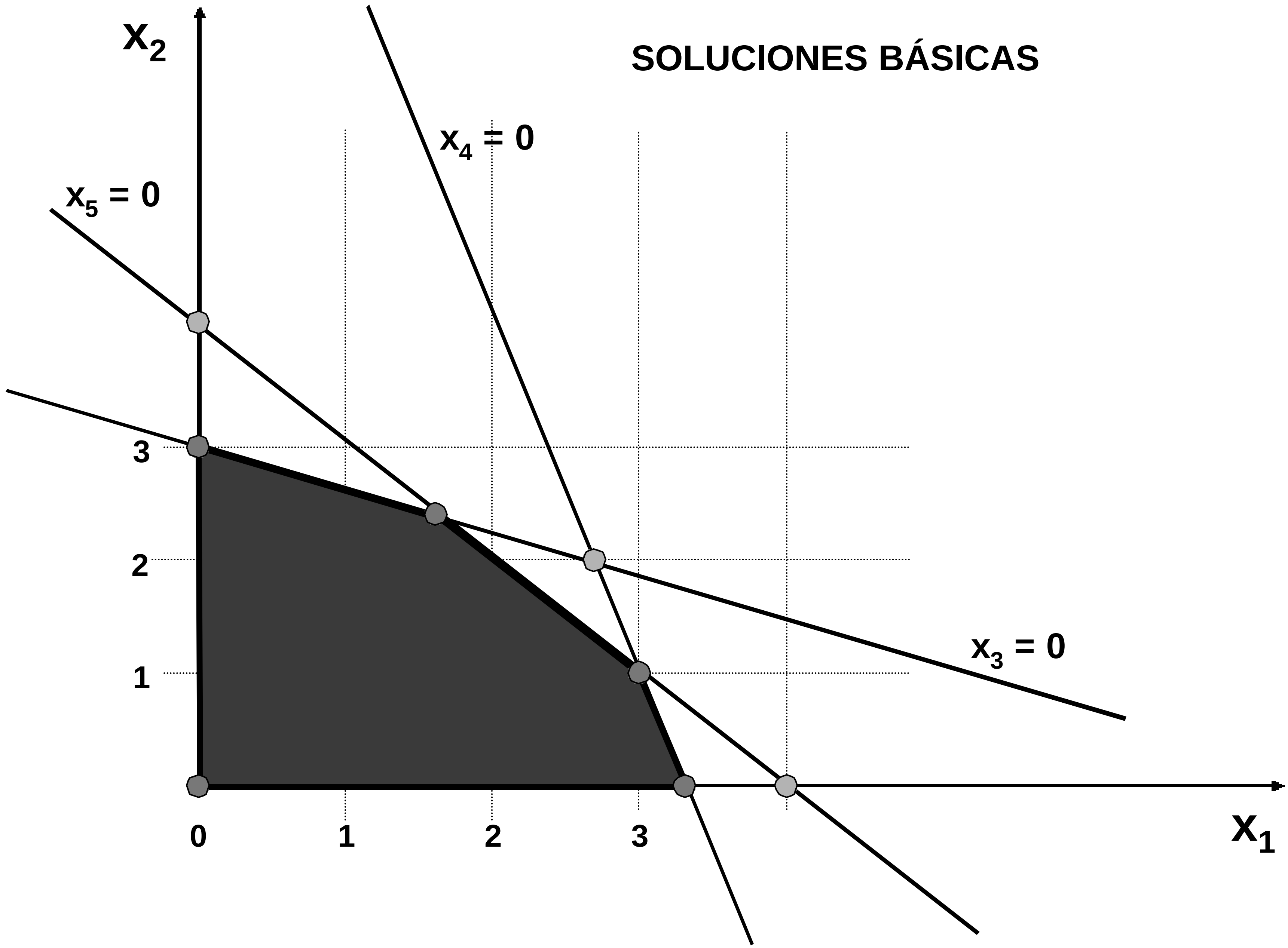

SOLUCIÓN FACTIBLE
Y BÁSICA
O = (0, 0, b3, b4, b5)
x2
x1
x4 = 0
x5 = 0
x3 = 0
O
1
2
3
1
2
3

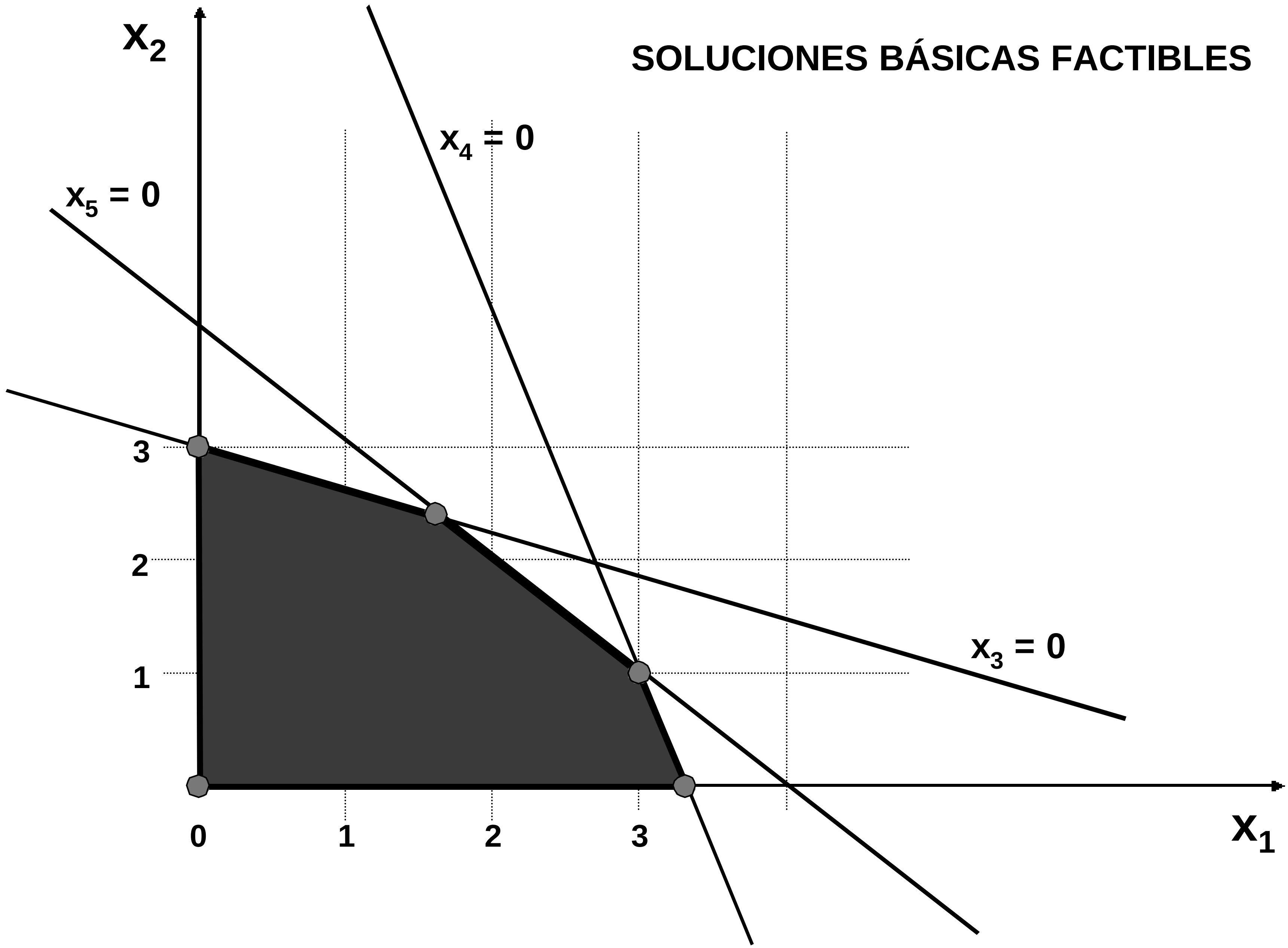


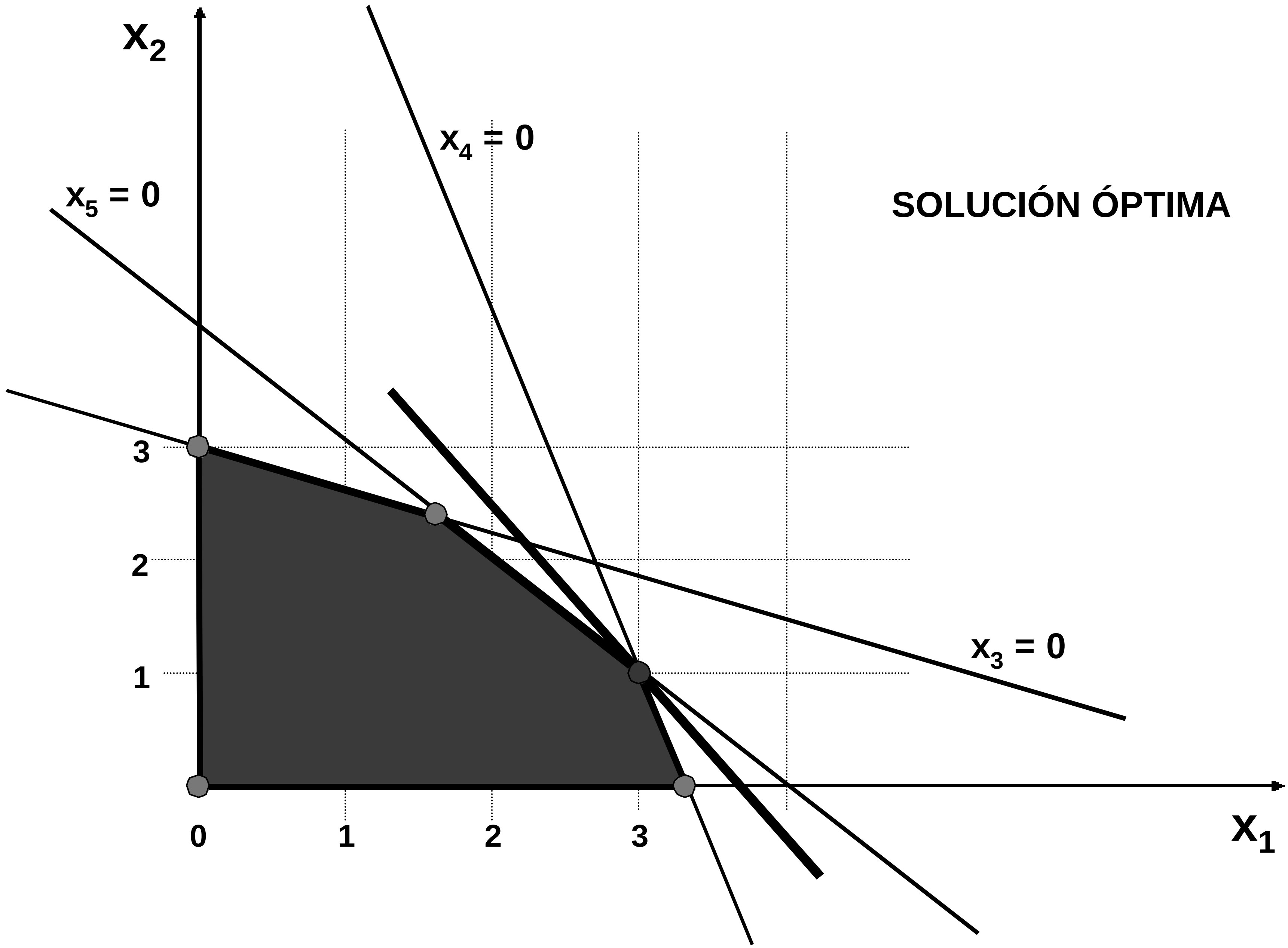
SOLUCIÓN FACTIBLE
Y BÁSICA
$Q = \begin{pmatrix} 0 \\ b_2 \\ 0 \\ b_4 \\ b_5 \end{pmatrix}$
x_2
x_1
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
Q
1
2
3
1
2
3

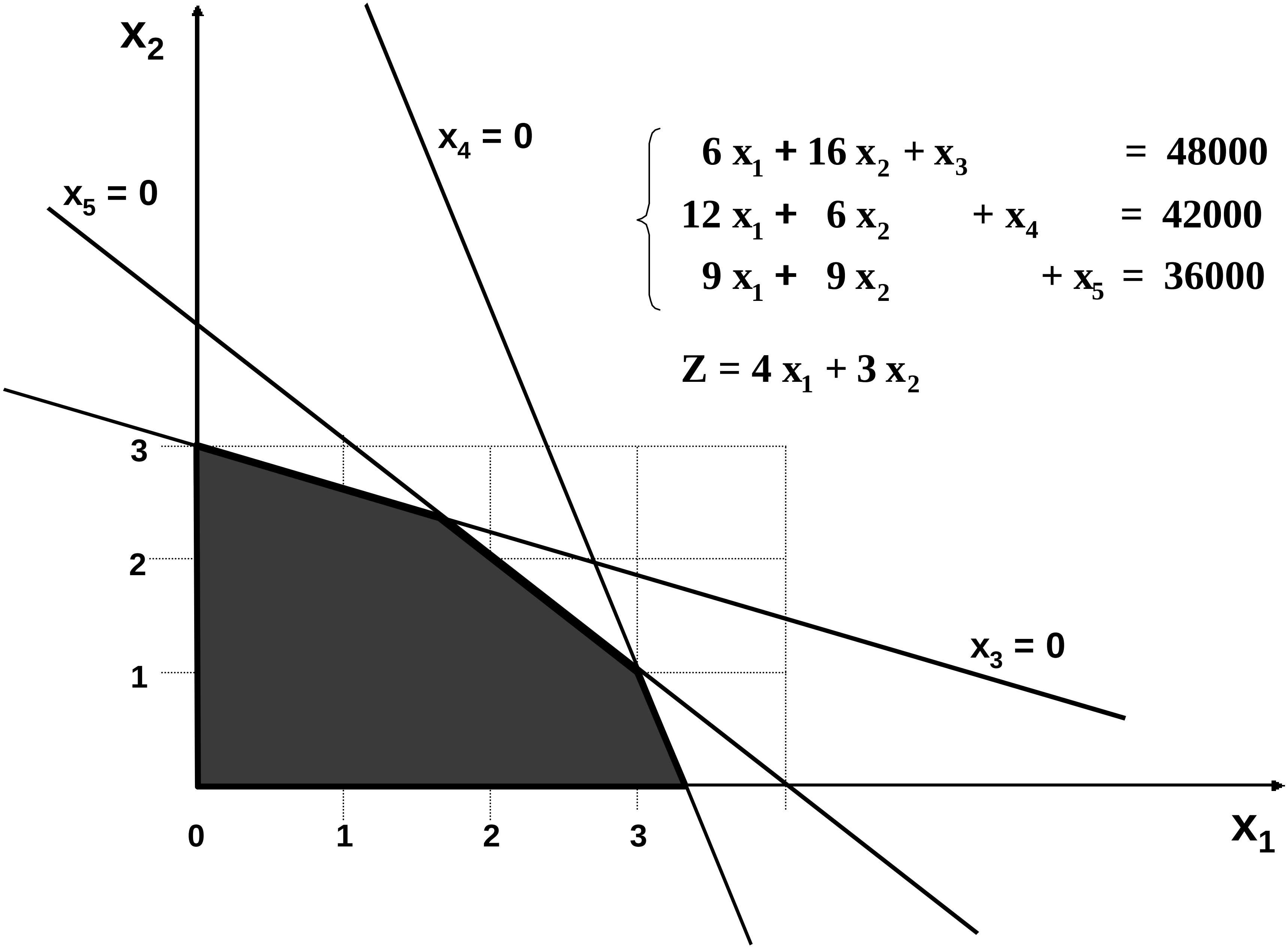


SOLUCIÓN NO FACTIBLE
Y BÁSICA
x_2
x_1
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
R
$R = \begin{pmatrix} 0 \\ b_2 \\ -b_3 \\ b_4 \\ 0 \end{pmatrix}$
1
2
3
1
2
3


SOLUCIONES BÁSICAS
x_2
x_1
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
3
2
1
0
1
2
3


SOLUCIONES BÁSICAS FACTIBLES
x_2
x_1
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
3
2
1
0
1
2
3
3


x_2
$x_4 = 0$
$x_5 = 0$
SOLUCIÓN ÓPTIMA
3
2
1
$x_3 = 0$
0
1
2
3
x_1



$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

$$Z = 4x_1 + 3x_2$$

$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

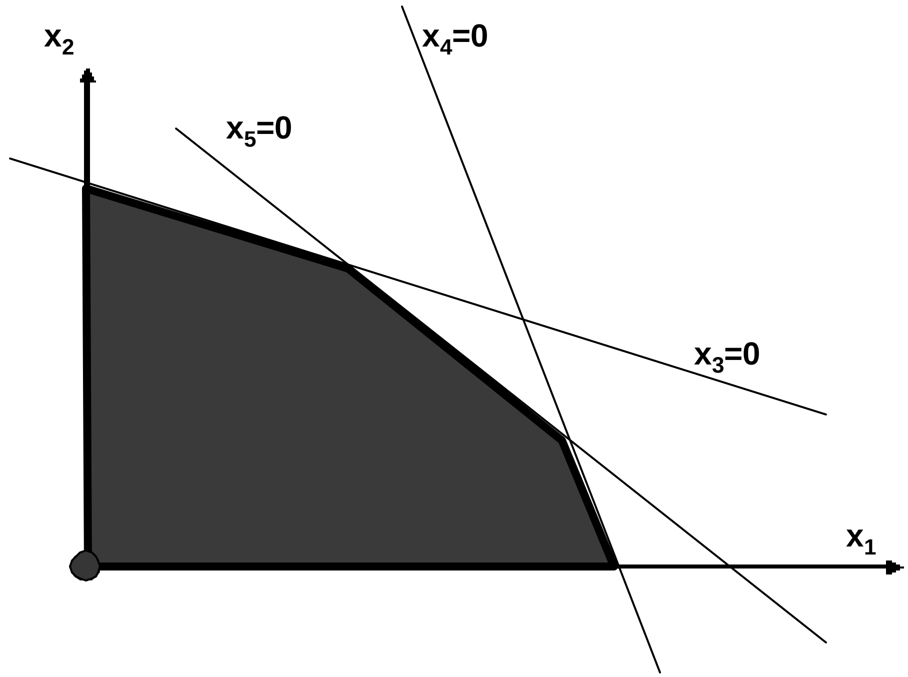
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \qquad Z = 4x_1 + 3x_2$$

$$\begin{cases} x_3 = 48000 \\ x_4 = 42000 \\ x_5 = 36000 \end{cases}$$

$$X = \begin{pmatrix} 0 \\ 0 \\ 48000 \\ 42000 \\ 36000 \end{pmatrix} \qquad Z = 0$$

$$\begin{cases} x_3 = 48000 \\ x_4 = 42000 \\ x_5 = 36000 \end{cases}$$

$$X = \begin{pmatrix} 0 \\ 0 \\ 48000 \\ 42000 \\ 36000 \end{pmatrix} \qquad Z = 0$$



$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

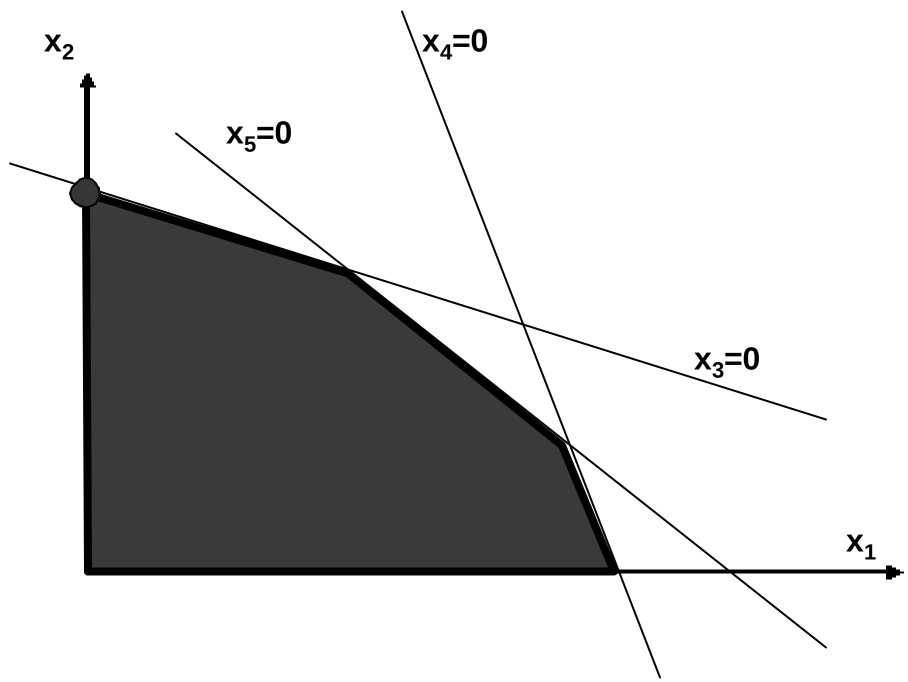
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \qquad Z = 4x_1 + 3x_2$$

$$\begin{cases} 16\,x_2 & & & = 48000 \\ 6\,x_2 & + x_4 & & = 42000 \\ 9\,x_2 & & + x_5 & = 36000 \end{cases}$$

$$X = \begin{pmatrix} 0 \\ 3000 \\ 0 \\ 24000 \\ 9000 \end{pmatrix} \qquad Z = 9000$$

$$\left\{\begin{array}{lllll} 16\,x_2 & & & = & 48000 \\ 6\,x_2 & + x_4 & & = & 42000 \\ 9\,x_2 & & + x_5 & = & 36000 \end{array}\right.$$

$$X = \begin{pmatrix} 0 \\ 3000 \\ 0 \\ 24000 \\ 9000 \end{pmatrix} \qquad Z = 9000$$



$$\begin{cases} 6\,x_1 + 16\,x_2 + x_3 = 48000 \\ 12\,x_1 + 6\,x_2 + x_4 = 42000 \\ 9\,x_1 + 9\,x_2 + x_5 = 36000 \end{cases}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \qquad Z = 4\,x_1 + 3\,x_2$$

$$\begin{cases} 6x_1 + x_3 = 48000 \\ 12x_1 + x_4 = 42000 \\ 9x_1 = 36000 \end{cases}$$

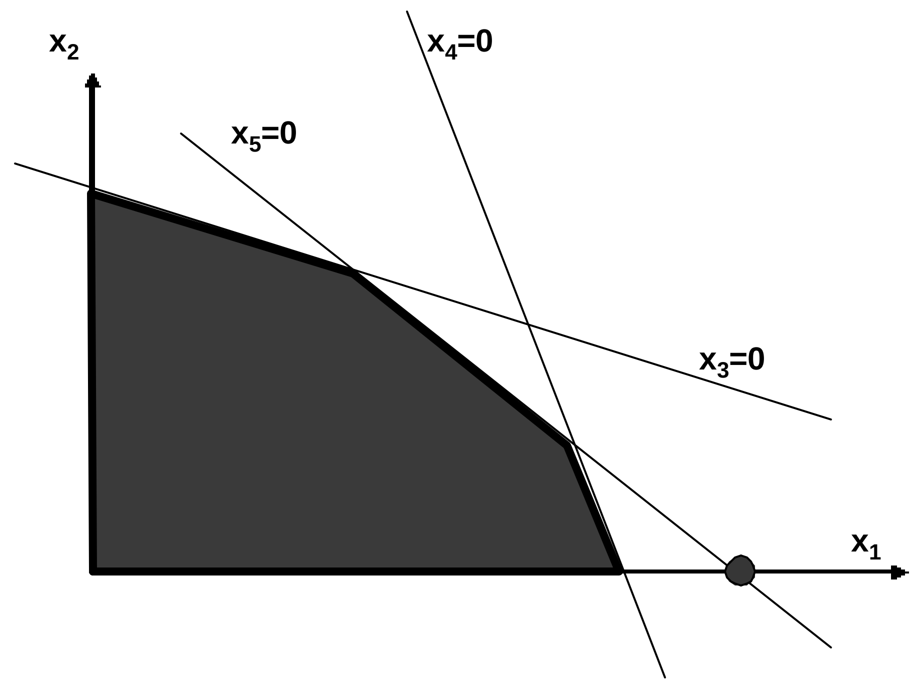
$$X = \begin{pmatrix} 4000 \\ 0 \\ 24000 \\ -6000 \\ 0 \end{pmatrix}$$

NO FACTIBLE

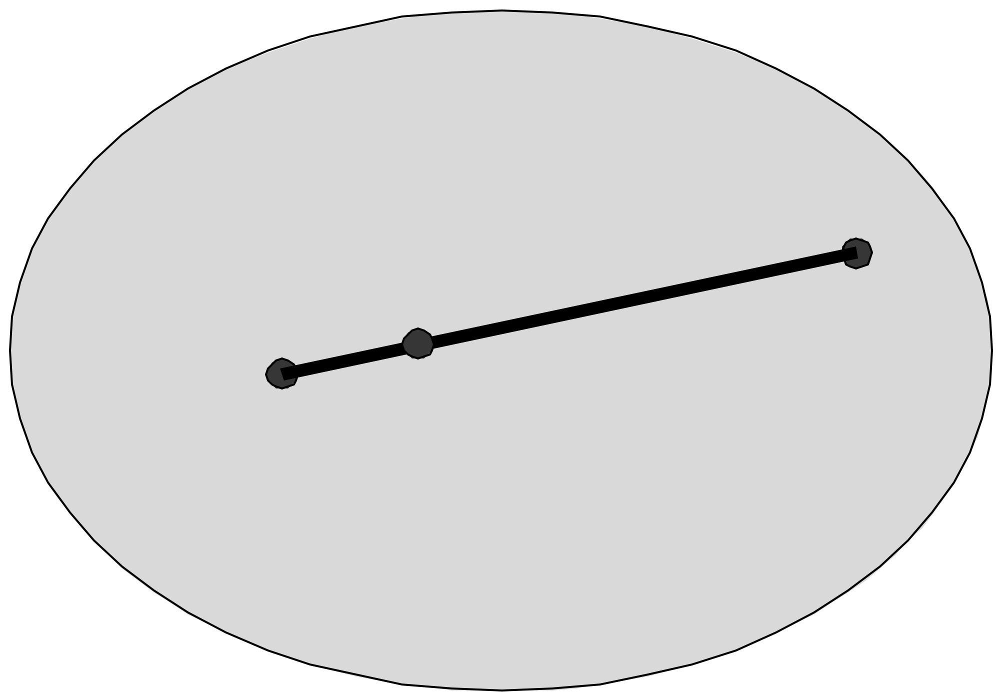
$$\begin{cases} 6\,x_1 + x_3 = 48000 \\ 12\,x_1 + x_4 = 42000 \\ 9\,x_1 = 36000 \end{cases}$$

$$X = \begin{pmatrix} 4000 \\ 0 \\ 24000 \\ -6000 \\ 0 \end{pmatrix}$$

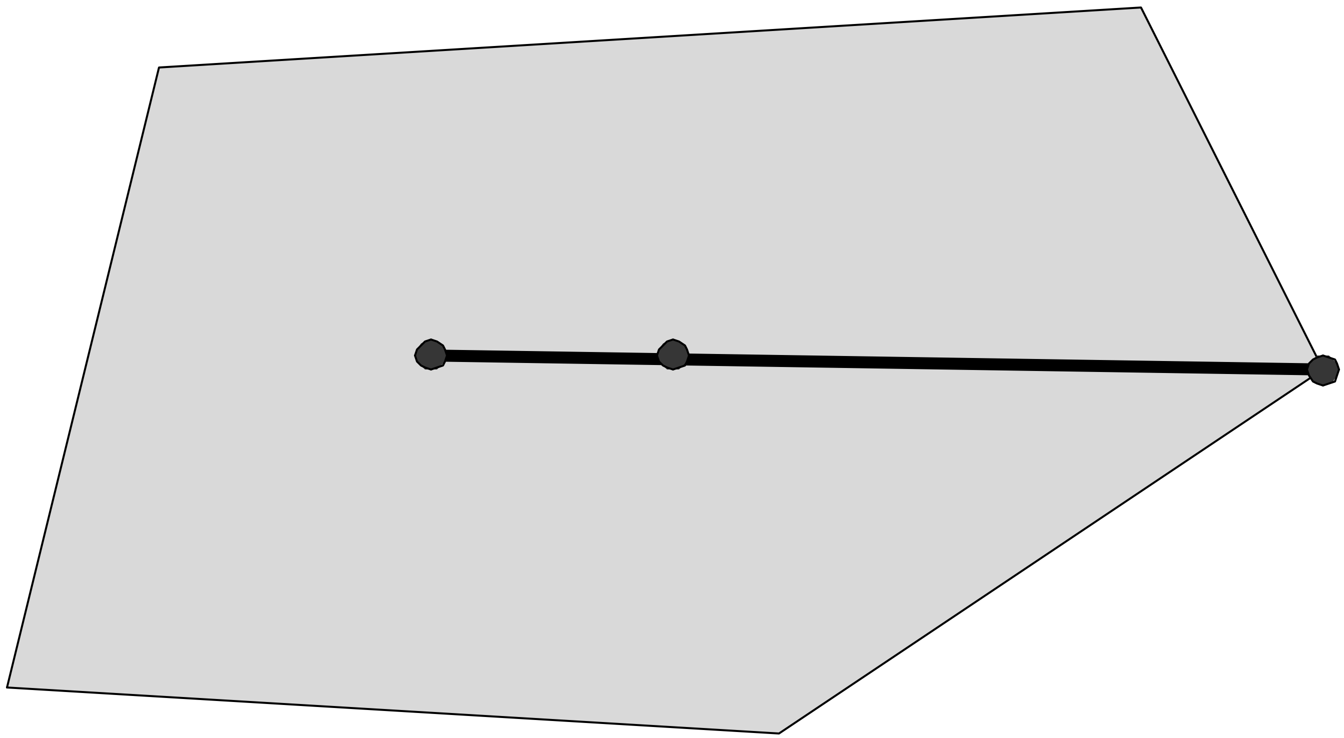
NO FACTIBLE



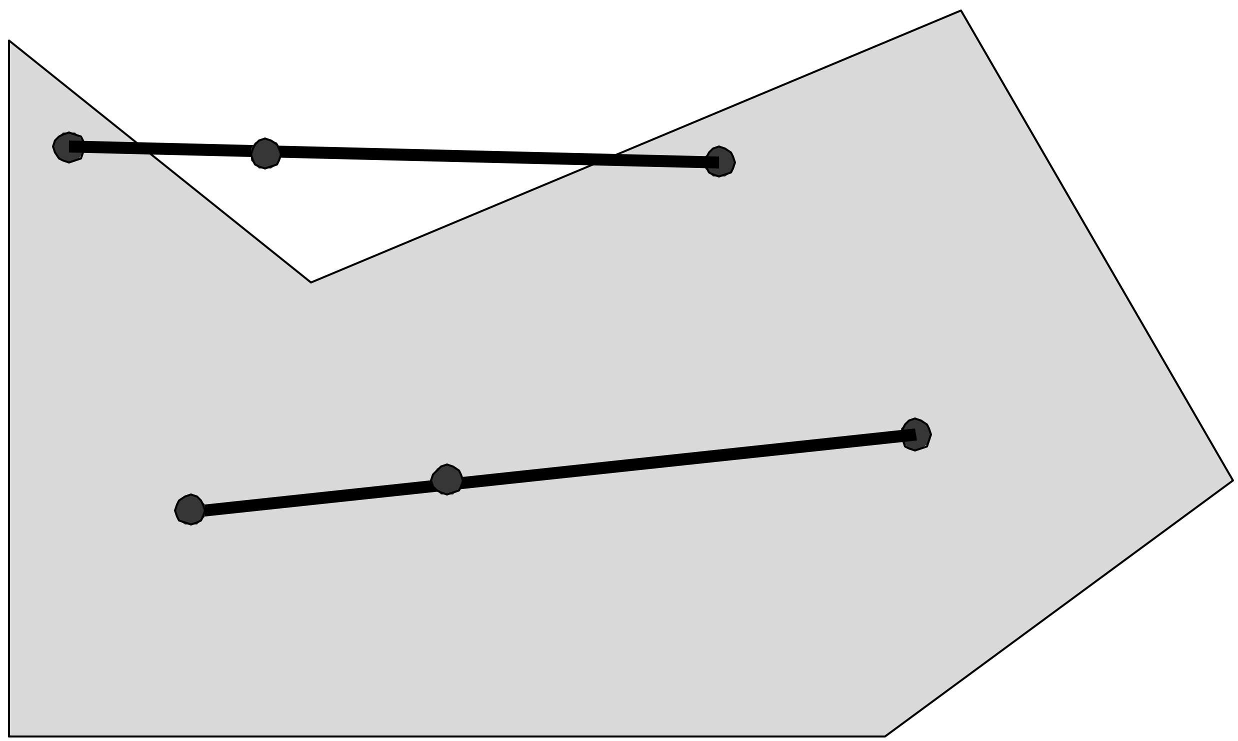
CONJUNTO CONVEXO



CONJUNTO CONVEXO



CONJUNTO NO CONVEXO



CONJUNTO NO CONVEXO