

Chapter 1

Problem Solutions

1.1

- (a) fcc: 8 corner atoms $\times 1/8 = 1$ atom
6 face atoms $\times 1/2 = 3$ atoms
Total of 4 atoms per unit cell

- (b) bcc: 8 corner atoms $\times 1/8 = 1$ atom
1 enclosed atom = 1 atom
Total of 2 atoms per unit cell

- (c) Diamond: 8 corner atoms $\times 1/8 = 1$ atom
6 face atoms $\times 1/2 = 3$ atoms
4 enclosed atoms = 4 atoms
Total of 8 atoms per unit cell

1.2

- (a) 4 Ga atoms per unit cell

$$\text{Density} = \frac{4}{(5.65 \times 10^{-8})^3} \Rightarrow$$

$$\text{Density of Ga} = 2.22 \times 10^{22} \text{ cm}^{-3}$$

4 As atoms per unit cell, so that

$$\text{Density of As} = 2.22 \times 10^{22} \text{ cm}^{-3}$$

- (b)

8 Ge atoms per unit cell

$$\text{Density} = \frac{8}{(5.65 \times 10^{-8})^3} \Rightarrow$$

$$\text{Density of Ge} = 4.44 \times 10^{22} \text{ cm}^{-3}$$

1.3

- (a) Simple cubic lattice; $a = 2r$

$$\text{Unit cell vol} = a^3 = (2r)^3 = 8r^3$$

$$1 \text{ atom per cell, so atom vol.} = (1) \left(\frac{4\pi r^3}{3} \right)$$

Then

$$\text{Ratio} = \frac{\left(\frac{4\pi r^3}{3} \right)}{8r^3} \times 100\% \Rightarrow \text{Ratio} = 52.4\%$$

- (b) Face-centered cubic lattice

$$d = 4r = a\sqrt{2} \Rightarrow a = \frac{d}{\sqrt{2}} = 2\sqrt{2} r$$

$$\text{Unit cell vol} = a^3 = (2\sqrt{2} r)^3 = 16\sqrt{2} r^3$$

$$4 \text{ atoms per cell, so atom vol.} = 4 \left(\frac{4\pi r^3}{3} \right)$$

Then

$$\text{Ratio} = \frac{4 \left(\frac{4\pi r^3}{3} \right)}{16\sqrt{2} r^3} \times 100\% \Rightarrow \text{Ratio} = 74\%$$

- (c) Body-centered cubic lattice

$$d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}} r$$

$$\text{Unit cell vol.} = a^3 = \left(\frac{4}{\sqrt{3}} r \right)^3$$

$$2 \text{ atoms per cell, so atom vol.} = 2 \left(\frac{4\pi r^3}{3} \right)$$

Then

$$\text{Ratio} = \frac{2 \left(\frac{4\pi r^3}{3} \right)}{\left(\frac{4r}{\sqrt{3}} \right)^3} \times 100\% \Rightarrow \text{Ratio} = 68\%$$

- (d) Diamond lattice

$$\text{Body diagonal} = d = 8r = a\sqrt{3} \Rightarrow a = \frac{8}{\sqrt{3}} r$$

$$\text{Unit cell vol.} = a^3 = \left(\frac{8r}{\sqrt{3}} \right)^3$$

$$8 \text{ atoms per cell, so atom vol.} = 8 \left(\frac{4\pi r^3}{3} \right)$$

Then

$$\text{Ratio} = \frac{8 \left(\frac{4\pi r^3}{3} \right)}{\left(\frac{8r}{\sqrt{3}} \right)^3} \times 100\% \Rightarrow \text{Ratio} = 34\%$$

1.4

From Problem 1.3, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

$$\text{Volume} = 0.74 \text{ cm}^3$$

1.5

(a) $a = 5.43 \text{ \AA}$ From 1.3d, $a = \frac{8}{\sqrt{3}} r$

so that $r = \frac{a\sqrt{3}}{8} = \frac{(5.43)\sqrt{3}}{8} = 1.18 \text{ \AA}$

Center of one silicon atom to center of nearest neighbor $= 2r \Rightarrow \underline{2.36 \text{ \AA}}$

(b) Number density

$$= \frac{8}{(5.43 \times 10^{-8})^3} \Rightarrow \text{Density} = 5 \times 10^{22} \text{ cm}^{-3}$$

(c) Mass density

$$= \rho = \frac{N(\text{At. Wt.})}{N_A} = \frac{(5 \times 10^{22})(28.09)}{6.02 \times 10^{23}} \Rightarrow$$

$$\underline{\rho = 2.33 \text{ grams / cm}^3}$$

1.6

(a) $a = 2r_A = 2(1.02) = 2.04 \text{ \AA}$

Now

$$2r_A + 2r_B = a\sqrt{3} \Rightarrow 2r_B = 2.04\sqrt{3} - 2.04$$

so that $r_B = 0.747 \text{ \AA}$

(b) A-type; 1 atom per unit cell

$$\text{Density} = \frac{1}{(2.04 \times 10^{-8})^3} \Rightarrow$$

$$\text{Density(A)} = 1.18 \times 10^{23} \text{ cm}^{-3}$$

B-type: 1 atom per unit cell, so

$$\text{Density(B)} = 1.18 \times 10^{23} \text{ cm}^{-3}$$

1.7

(b)

$$a = 1.8 + 1.0 \Rightarrow \underline{a = 2.8 \text{ \AA}}$$

(c)

$$\text{Na: Density} = \frac{1/2}{(2.8 \times 10^{-8})^3} = 2.28 \times 10^{22} \text{ cm}^{-3}$$

$$\text{Cl: Density (same as Na)} = 2.28 \times 10^{22} \text{ cm}^{-3}$$

(d)

Na: At. Wt. = 22.99

Cl: At. Wt. = 35.45

So, mass per unit cell

$$= \frac{\frac{1}{2}(22.99) + \frac{1}{2}(35.45)}{6.02 \times 10^{23}} = 4.85 \times 10^{-23}$$

Then mass density is

$$\rho = \frac{4.85 \times 10^{-23}}{(2.8 \times 10^{-8})^3} \Rightarrow$$

$$\underline{\rho = 2.21 \text{ gm / cm}^3}$$

1.8

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8 \text{ \AA}$

so that

$$\underline{a = 4.62 \text{ \AA}}$$

$$\text{Density of A} = \frac{1}{(4.62 \times 10^{-8})^3} \Rightarrow \underline{1.01 \times 10^{22} \text{ cm}^{-3}}$$

$$\text{Density of B} = \frac{1}{(4.62 \times 10^{-8})^3} \Rightarrow \underline{1.01 \times 10^{22} \text{ cm}^{-3}}$$

(b) Same as (a)

(c) Same material

1.9

(a) Surface density

$$= \frac{1}{a^2 \sqrt{2}} = \frac{1}{(4.62 \times 10^{-8})^2 \sqrt{2}} \Rightarrow$$

$$\underline{3.31 \times 10^{14} \text{ cm}^{-2}}$$

Same for A atoms and B atoms

(b) Same as (a)

(c) Same material

1.10

(a) Vol density $= \frac{1}{a_o^3}$

$$\text{Surface density} = \frac{1}{a_o^2 \sqrt{2}}$$

(b) Same as (a)

1.11

Sketch

1.12

(a)

$$\left(\frac{1}{1}, \frac{1}{3}, \frac{1}{1} \right) \Rightarrow \underline{(313)}$$

(b)

$$\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4} \right) \Rightarrow \underline{(121)}$$

1.13

(a) Distance between nearest (100) planes is:

$$d = a = 5.63 \text{ \AA}$$

(b) Distance between nearest (110) planes is:

$$d = \frac{1}{2} a \sqrt{2} = \frac{a}{\sqrt{2}} = \frac{5.63}{\sqrt{2}}$$

or

$$d = 3.98 \text{ \AA}$$

(c) Distance between nearest (111) planes is:

$$d = \frac{1}{3} a \sqrt{3} = \frac{a}{\sqrt{3}} = \frac{5.63}{\sqrt{3}}$$

or

$$d = 3.25 \text{ \AA}$$

1.14

(a)

Simple cubic: $a = 4.50 \text{ \AA}$

(i) (100) plane, surface density,

$$= \frac{1 \text{ atom}}{(4.50 \times 10^{-8})^2} \Rightarrow 4.94 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane, surface density,

$$= \frac{1 \text{ atom}}{\sqrt{2} (4.50 \times 10^{-8})^2} \Rightarrow 3.49 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane, surface density,

$$\begin{aligned} &= \frac{3 \left(\frac{1}{6} \right) \text{ atoms}}{\frac{1}{2} (a \sqrt{2}) (x)} = \frac{\frac{1}{2}}{\frac{1}{2} \cdot a \sqrt{2} \cdot \frac{a \sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3} a^2} \\ &= \frac{1}{\sqrt{3} (4.50 \times 10^{-8})^2} \Rightarrow 2.85 \times 10^{14} \text{ cm}^{-2} \end{aligned}$$

(b)

Body-centered cubic

(i) (100) plane, surface density,

Same as (a),(i); surface density $4.94 \times 10^{14} \text{ cm}^{-2}$

(ii) (110) plane, surface density,

$$= \frac{2 \text{ atoms}}{\sqrt{2} (4.50 \times 10^{-8})^2} \Rightarrow 6.99 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane, surface density,

Same as (a),(iii), surface density $2.85 \times 10^{14} \text{ cm}^{-2}$

(c)

Face centered cubic

(i) (100) plane, surface density

$$= \frac{2 \text{ atoms}}{(4.50 \times 10^{-8})^2} \Rightarrow 9.88 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane, surface density,

$$= \frac{2 \text{ atoms}}{\sqrt{2} (4.50 \times 10^{-8})^2} \Rightarrow 6.99 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane, surface density,

$$= \frac{\left(3 \cdot \frac{1}{6} + 3 \cdot \frac{1}{2} \right)}{\frac{\sqrt{3}}{2} a^2} = \frac{4}{\sqrt{3} (4.50 \times 10^{-8})^2}$$

or $1.14 \times 10^{15} \text{ cm}^{-2}$

1.15

(a)

(100) plane of silicon – similar to a fcc,

$$\begin{aligned} \text{surface density} &= \frac{2 \text{ atoms}}{(5.43 \times 10^{-8})^2} \Rightarrow \\ &6.78 \times 10^{14} \text{ cm}^{-2} \end{aligned}$$

(b)

(110) plane, surface density,

$$= \frac{4 \text{ atoms}}{\sqrt{2} (5.43 \times 10^{-8})^2} \Rightarrow 9.59 \times 10^{14} \text{ cm}^{-2}$$

(c)

(111) plane, surface density,

$$= \frac{4 \text{ atoms}}{\sqrt{3} (5.43 \times 10^{-8})^2} \Rightarrow 7.83 \times 10^{14} \text{ cm}^{-2}$$

1.16

$$d = 4r = a \sqrt{2}$$

then

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.25)}{\sqrt{2}} = 6.364 \text{ \AA}$$

(a)

$$\begin{aligned} \text{Volume Density} &= \frac{4 \text{ atoms}}{(6.364 \times 10^{-8})^3} \Rightarrow \\ &1.55 \times 10^{22} \text{ cm}^{-3} \end{aligned}$$

(b)

Distance between (110) planes,

$$= \frac{1}{2} a \sqrt{2} = \frac{a}{\sqrt{2}} = \frac{6.364}{\sqrt{2}} \Rightarrow$$

or

$$\frac{4.50 \text{ \AA}}{\text{(c) Surface density}}$$

$$= \frac{2 \text{ atoms}}{\sqrt{2} a^2} = \frac{2}{\sqrt{2} (6.364 \times 10^{-8})^2}$$

or

$$\frac{3.49 \times 10^{14} \text{ cm}^{-2}}$$

1.17

Density of silicon atoms = $5 \times 10^{22} \text{ cm}^{-3}$ and 4 valence electrons per atom, so

$$\text{Density of valence electrons } \underline{2 \times 10^{23} \text{ cm}^{-3}}$$

1.18

Density of GaAs atoms

$$= \frac{8 \text{ atoms}}{(5.65 \times 10^{-8})^3} = 4.44 \times 10^{22} \text{ cm}^{-3}$$

An average of 4 valence electrons per atom,

$$\text{Density of valence electrons } \underline{1.77 \times 10^{23} \text{ cm}^{-3}}$$

1.19

$$\text{(a) Percentage} = \frac{2 \times 10^{16}}{5 \times 10^{22}} \times 100\% \Rightarrow$$

$$\underline{4 \times 10^{-5} \%}$$

$$\text{(b) Percentage} = \frac{1 \times 10^{15}}{5 \times 10^{22}} \times 100\% \Rightarrow$$

$$\underline{2 \times 10^{-6} \%}$$

1.20

$$\text{(a) Fraction by weight} \approx \frac{(5 \times 10^{16})(30.98)}{(5 \times 10^{22})(28.06)} \Rightarrow$$

$$\frac{1.10 \times 10^{-6}}{\text{(b) Fraction by weight}}$$

$$\approx \frac{(10^{18})(10.82)}{(5 \times 10^{16})(30.98) + (5 \times 10^{22})(28.06)} \Rightarrow$$

$$\underline{7.71 \times 10^{-6}}$$

1.21

$$\text{Volume density} = \frac{1}{d^3} = 2 \times 10^{15} \text{ cm}^{-3}$$

So

$$d = 7.94 \times 10^{-6} \text{ cm} = 794 \text{ \AA}$$

We have $a_o = 5.43 \text{ \AA}$

So

$$\frac{d}{a_o} = \frac{794}{5.43} \Rightarrow \frac{d}{a_o} = 146$$

Chapter 2

Problem Solutions

2.1 Computer plot

2.2 Computer plot

2.3 Computer plot

2.4

For problem 2.2; Phase = $\frac{2\pi x}{\lambda} - \omega t = \text{constant}$

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0 \quad \text{or} \quad \frac{dx}{dt} = v_p = +\omega \left(\frac{\lambda}{2\pi} \right)$$

For problem 2.3; Phase = $\frac{2\pi x}{\lambda} + \omega t = \text{constant}$

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0 \quad \text{or} \quad \frac{dx}{dt} = v_p = -\omega \left(\frac{\lambda}{2\pi} \right)$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

$$\text{Gold: } E = 4.90 \text{ eV} = (4.90)(1.6 \times 10^{-19}) \text{ J}$$

So

$$\lambda = \frac{(6.625 \times 10^{-34})(3 \times 10^{10})}{(4.90)(1.6 \times 10^{-19})} \Rightarrow 2.54 \times 10^{-5} \text{ cm}$$

or

$$\lambda = 0.254 \text{ } \mu\text{m}$$

$$\text{Cesium: } E = 1.90 \text{ eV} = (1.90)(1.6 \times 10^{-19}) \text{ J}$$

So

$$\lambda = \frac{(6.625 \times 10^{-34})(3 \times 10^{10})}{(1.90)(1.6 \times 10^{-19})} \Rightarrow 6.54 \times 10^{-5} \text{ cm}$$

or

$$\lambda = 0.654 \text{ } \mu\text{m}$$

2.6

(a) Electron: (i) K.E. = $T = 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$p = \sqrt{2mT} = \sqrt{2(9.11 \times 10^{-31})(1.6 \times 10^{-19})}$$

or

$$p = 5.4 \times 10^{-25} \text{ kg} \cdot \text{m} / \text{s}$$

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{5.4 \times 10^{-25}} \Rightarrow$$

or

$$\lambda = 12.3 \text{ } \text{\AA}$$

(ii) K.E. = $T = 100 \text{ eV} = 1.6 \times 10^{-17} \text{ J}$

$$p = \sqrt{2mT} \Rightarrow p = 5.4 \times 10^{-24} \text{ kg} \cdot \text{m} / \text{s}$$

$$\lambda = \frac{h}{p} \Rightarrow \lambda = 1.23 \text{ } \text{\AA}$$

(b) Proton: K.E. = $T = 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$p = \sqrt{2mT} = \sqrt{2(1.67 \times 10^{-27})(1.6 \times 10^{-19})}$$

or

$$p = 2.31 \times 10^{-23} \text{ kg} \cdot \text{m} / \text{s}$$

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{2.31 \times 10^{-23}} \Rightarrow$$

or

$$\lambda = 0.287 \text{ } \text{\AA}$$

(c) Tungsten Atom: At. Wt. = 183.92

For $T = 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$p = \sqrt{2mT} = \sqrt{2(183.92)(1.66 \times 10^{-27})(1.6 \times 10^{-19})}$$

or

$$p = 3.13 \times 10^{-22} \text{ kg} \cdot \text{m} / \text{s}$$

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{3.13 \times 10^{-22}} \Rightarrow$$

or

$$\lambda = 0.0212 \text{ } \text{\AA}$$

(d) A 2000 kg traveling at 20 m/s:

$$p = mv = (2000)(20) \Rightarrow$$

or

$$p = 4 \times 10^4 \text{ kg} \cdot \text{m} / \text{s}$$

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{4 \times 10^4} \Rightarrow$$

or

$$\lambda = 1.66 \times 10^{-28} \text{ } \text{\AA}$$

2.7

$$E_{avg} = \frac{3}{2} kT = \frac{3}{2} (0.0259) \Rightarrow$$

or

$$E_{avg} = 0.01727 \text{ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}} \\ = \sqrt{2(9.11 \times 10^{-31})(0.01727)(1.6 \times 10^{-19})}$$

or

$$p_{avg} = 7.1 \times 10^{-26} \text{ kg} \cdot \text{m} / \text{s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{7.1 \times 10^{-26}} \Rightarrow$$

or

$$\lambda = 93.3 \text{ \AA}$$

2.8

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m} \left(\frac{h}{\lambda_e} \right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m} \left(\frac{h}{\lambda_e} \right)^2 = \frac{1}{2m} \left(\frac{10h}{\lambda_p} \right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h} \cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2(9.11 \times 10^{-31})(3 \times 10^8)^2}{100} \Rightarrow$$

So

$$E = 1.64 \times 10^{-15} \text{ J} = 10.3 \text{ keV}$$

2.9

$$(a) \quad E = \frac{1}{2} mv^2 = \frac{1}{2} (9.11 \times 10^{-31})(2 \times 10^4)^2$$

or

$$E = 1.822 \times 10^{-22} \text{ J} \Rightarrow E = 1.14 \times 10^{-3} \text{ eV}$$

Also

$$p = mv = (9.11 \times 10^{-31})(2 \times 10^4) \Rightarrow$$

$$p = 1.822 \times 10^{-26} \text{ kg} \cdot \text{m} / \text{s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{1.822 \times 10^{-26}} \Rightarrow$$

$$\lambda = 364 \text{ \AA}$$

(b)

$$p = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{125 \times 10^{-10}} \Rightarrow$$

$$p = 5.3 \times 10^{-26} \text{ kg} \cdot \text{m} / \text{s}$$

Also

$$v = \frac{p}{m} = \frac{5.3 \times 10^{-26}}{9.11 \times 10^{-31}} = 5.82 \times 10^4 \text{ m} / \text{s}$$

or

$$v = 5.82 \times 10^6 \text{ cm} / \text{s}$$

Now

$$E = \frac{1}{2} mv^2 = \frac{1}{2} (9.11 \times 10^{-31})(5.82 \times 10^4)^2$$

or

$$E = 1.54 \times 10^{-21} \text{ J} \Rightarrow E = 9.64 \times 10^{-3} \text{ eV}$$

2.10

$$(a) \quad E = h\nu = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34})(3 \times 10^8)}{1 \times 10^{-10}}$$

or

$$E = 1.99 \times 10^{-15} \text{ J}$$

Now

$$E = e \cdot V \Rightarrow 1.99 \times 10^{-15} = (1.6 \times 10^{-19})V$$

so

$$V = 12.4 \times 10^3 \text{ V} = 12.4 \text{ kV}$$

$$(b) \quad p = \sqrt{2mE} = \sqrt{2(9.11 \times 10^{-31})(1.99 \times 10^{-15})}$$

$$= 6.02 \times 10^{-23} \text{ kg} \cdot \text{m} / \text{s}$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{6.02 \times 10^{-23}} \Rightarrow \lambda = 0.11 \text{ \AA}$$

2.11

$$(a) \quad \Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-6}} \Rightarrow$$

$$\Delta p = 1.054 \times 10^{-28} \text{ kg} \cdot \text{m} / \text{s}$$

$$(b) \quad E = \frac{hc}{\lambda} = hc \left(\frac{p}{h} \right) = pc$$

So

$$\Delta E = c(\Delta p) = (3 \times 10^8)(1.054 \times 10^{-28}) \Rightarrow$$

or

$$\Delta E = 3.16 \times 10^{-20} \text{ J} \Rightarrow \Delta E = 0.198 \text{ eV}$$

2.12

$$(a) \quad \Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{12 \times 10^{-10}} \Rightarrow$$

$$\Delta p = 8.78 \times 10^{-26} \text{ kg} \cdot \text{m} / \text{s}$$

$$(b) \quad \Delta E = \frac{1}{2} \cdot \frac{(\Delta p)^2}{m} = \frac{1}{2} \cdot \frac{(8.78 \times 10^{-26})^2}{5 \times 10^{-29}} \Rightarrow$$

$$\Delta E = 7.71 \times 10^{-23} \text{ J} \Rightarrow \Delta E = 4.82 \times 10^{-4} \text{ eV}$$

2.13

$$(a) \quad \text{Same as 2.12 (a), } \Delta p = 8.78 \times 10^{-26} \text{ kg} \cdot \text{m} / \text{s}$$

$$(b) \quad \Delta E = \frac{1}{2} \cdot \frac{(\Delta p)^2}{m} = \frac{1}{2} \cdot \frac{(8.78 \times 10^{-26})^2}{5 \times 10^{-26}} \Rightarrow$$

$$\Delta E = 7.71 \times 10^{-26} \text{ J} \Rightarrow \Delta E = 4.82 \times 10^{-7} \text{ eV}$$

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-2}} = 1.054 \times 10^{-32}$$

$$p = mv \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{1.054 \times 10^{-32}}{1500} \Rightarrow$$

or

$$\Delta v = 7 \times 10^{-36} \text{ m} / \text{s}$$

2.15

$$(a) \quad \Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-10}} \Rightarrow$$

$$\Delta p = 1.054 \times 10^{-24} \text{ kg} \cdot \text{m} / \text{s}$$

$$(b) \quad \Delta t = \frac{1.054 \times 10^{-34}}{(1)(1.6 \times 10^{-19})} \Rightarrow$$

or

$$\Delta t = 6.6 \times 10^{-16} \text{ s}$$

2.16

(a) If $\Psi_1(x, t)$ and $\Psi_2(x, t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_1(x, t)}{\partial x^2} + V(x)\Psi_1(x, t) = j\hbar \frac{\partial \Psi_1(x, t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_2(x, t)}{\partial x^2} + V(x)\Psi_2(x, t) = j\hbar \frac{\partial \Psi_2(x, t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} [\Psi_1(x, t) + \Psi_2(x, t)]$$

$$+ V(x)[\Psi_1(x, t) + \Psi_2(x, t)]$$

$$= j\hbar \frac{\partial}{\partial t} [\Psi_1(x, t) + \Psi_2(x, t)]$$

which is Schrodinger's wave equation. So $\Psi_1(x, t) + \Psi_2(x, t)$ is also a solution.

(b)

If $\Psi_1 \cdot \Psi_2$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} (\Psi_1 \cdot \Psi_2) + V(x)(\Psi_1 \cdot \Psi_2)$$

$$= j\hbar \frac{\partial}{\partial t} (\Psi_1 \cdot \Psi_2)$$

which can be written as

$$\frac{-\hbar^2}{2m} \left[\Psi_1 \frac{\partial^2 \Psi_2}{\partial x^2} + \Psi_2 \frac{\partial^2 \Psi_1}{\partial x^2} + 2 \frac{\partial \Psi_1}{\partial x} \cdot \frac{\partial \Psi_2}{\partial x} \right]$$

$$+ V(x)\Psi_1 \cdot \Psi_2 = j\hbar \left[\Psi_1 \frac{\partial \Psi_2}{\partial t} + \Psi_2 \frac{\partial \Psi_1}{\partial t} \right]$$

Dividing by $\Psi_1 \cdot \Psi_2$ we find

$$\frac{-\hbar^2}{2m} \left[\frac{1}{\Psi_2} \cdot \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{1}{\Psi_1} \cdot \frac{\partial^2 \Psi_1}{\partial x^2} + \frac{1}{\Psi_1 \Psi_2} \frac{\partial \Psi_1}{\partial x} \frac{\partial \Psi_2}{\partial x} \right]$$

$$+ V(x) = j\hbar \left[\frac{1}{\Psi_2} \frac{\partial \Psi_2}{\partial t} + \frac{1}{\Psi_1} \frac{\partial \Psi_1}{\partial t} \right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m} \cdot \frac{1}{\Psi_1} \cdot \frac{\partial^2 \Psi_1}{\partial x^2} + V(x) = j\hbar \cdot \frac{1}{\Psi_1} \cdot \frac{\partial \Psi_1}{\partial t}$$

Subtracting these last two equations, we are left with

$$\begin{aligned} \frac{-\hbar^2}{2m} \left[\frac{1}{\Psi_2} \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{2}{\Psi_1 \Psi_2} \frac{\partial \Psi_1}{\partial x} \frac{\partial \Psi_2}{\partial x} \right] \\ = j\hbar \frac{1}{\Psi_2} \frac{\partial \Psi_2}{\partial t} \end{aligned}$$

Since Ψ_2 is also a solution, we may write

$$\frac{-\hbar^2}{2m} \frac{1}{\Psi_2} \frac{\partial^2 \Psi_2}{\partial x^2} + V(x) = j\hbar \frac{1}{\Psi_2} \frac{\partial \Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m} \cdot \frac{2}{\Psi_1 \Psi_2} \cdot \frac{\partial \Psi_1}{\partial x} \cdot \frac{\partial \Psi_2}{\partial x} - V(x) = 0$$

This equation is not necessarily valid, which means that $\Psi_1 \Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\Psi(x, t) = A[\sin(\pi x)] \exp(-j\omega t)$$

$$\int_{-1}^{+1} |\Psi(x, t)|^2 dx = 1 = |A|^2 \int_{-1}^{+1} \sin^2(\pi x) dx$$

or

$$|A|^2 \cdot \left[\frac{1}{2} x - \frac{1}{4\pi} \sin(2\pi x) \right]_{-1}^{+1} = 1$$

which yields

$$|A|^2 = 1 \quad \text{or} \quad A = +1, -1, +j, -j$$

2.18

$$\Psi(x, t) = A[\sin(n\pi x)] \exp(-j\omega t)$$

$$\int_0^{+1} |\Psi(x, t)|^2 dx = 1 = |A|^2 \int_0^{+1} \sin^2(n\pi x) dx$$

or

$$|A|^2 \cdot \left[\frac{1}{2} x - \frac{1}{4n\pi} \sin(2n\pi x) \right]_0^{+1} = 1$$

which yields

$$|A|^2 = 2 \quad \text{or}$$

$$A = +\sqrt{2}, -\sqrt{2}, +j\sqrt{2}, -j\sqrt{2}$$

2.19

$$\text{Note that } \int_0^{\infty} \Psi \cdot \Psi^* dx = 1$$

Function has been normalized

(a) Now

$$\begin{aligned} P &= \int_0^{a_o/4} \left[\sqrt{\frac{2}{a_o}} \exp\left(\frac{-x}{a_o}\right) \right]^2 dx \\ &= \frac{2}{a_o} \int_0^{a_o/4} \exp\left(\frac{-2x}{a_o}\right) dx \\ &= \frac{2}{a_o} \left(\frac{-a_o}{2} \right) \exp\left(\frac{-2x}{a_o}\right) \Big|_0^{a_o/4} \end{aligned}$$

or

$$P = -1 \left[\exp\left(\frac{-2a_o}{4a_o}\right) - 1 \right] = 1 - \exp\left(\frac{-1}{2}\right)$$

which yields

$$P = 0.393$$

(b)

$$\begin{aligned} P &= \int_{a_o/4}^{a_o/2} \left(\sqrt{\frac{2}{a_o}} \exp\left(\frac{-x}{a_o}\right) \right)^2 dx \\ &= \frac{2}{a_o} \int_{a_o/4}^{a_o/2} \exp\left(\frac{-2x}{a_o}\right) dx \\ &= \frac{2}{a_o} \left(\frac{-a_o}{2} \right) \exp\left(\frac{-2x}{a_o}\right) \Big|_{a_o/4}^{a_o/2} \end{aligned}$$

or

$$P = -1 \left[\exp(-1) - \exp\left(\frac{-1}{2}\right) \right]$$

which yields

$$P = 0.239$$

(c)

$$\begin{aligned} P &= \int_0^{a_o} \left(\sqrt{\frac{2}{a_o}} \exp\left(\frac{-x}{a_o}\right) \right)^2 dx \\ &= \frac{2}{a_o} \int_0^{a_o} \exp\left(\frac{-2x}{a_o}\right) dx = \frac{2}{a_o} \left(\frac{-a_o}{2} \right) \exp\left(\frac{-2x}{a_o}\right) \Big|_0^{a_o} \end{aligned}$$

or

$$P = -1 [\exp(-2) - 1]$$

which yields

$$P = 0.865$$

2.20

(a) $kx - \omega t = \text{constant}$

Then

$$k \frac{dx}{dt} - \omega = 0 \Rightarrow \frac{dx}{dt} = v_p = + \frac{\omega}{k}$$

or

$$v_p = \frac{1.5 \times 10^{13}}{1.5 \times 10^9} = 10^4 \text{ m/s}$$

$$v_p = 10^6 \text{ cm/s}$$

(b)

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5 \times 10^9}$$

or

$$\lambda = 41.9 \text{ \AA}$$

Also

$$p = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{41.9 \times 10^{-10}} \Rightarrow$$

or

$$p = 1.58 \times 10^{-25} \text{ kg-m/s}$$

Now

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34})(3 \times 10^8)}{41.9 \times 10^{-10}}$$

or

$$E = 4.74 \times 10^{-17} \text{ J} \Rightarrow E = 2.96 \times 10^2 \text{ eV}$$

2.21

$$\psi(x) = A \exp[-j(kx + \omega t)]$$

where

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$= \frac{\sqrt{2(9.11 \times 10^{-31})(0.015)(1.6 \times 10^{-19})}}{1.054 \times 10^{-34}}$$

or

$$k = 6.27 \times 10^8 \text{ m}^{-1}$$

Now

$$\omega = \frac{E}{\hbar} = \frac{(0.015)(1.6 \times 10^{-19})}{1.054 \times 10^{-34}}$$

or

$$\omega = 2.28 \times 10^{13} \text{ rad/s}$$

2.22

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054 \times 10^{-34})^2 \pi^2 n^2}{2(9.11 \times 10^{-31})(100 \times 10^{-10})^2}$$

so

$$E = 6.018 \times 10^{-22} n^2 \text{ (J)}$$

or

$$E = 3.76 \times 10^{-3} n^2 \text{ (eV)}$$

Then

$$n = 1 \Rightarrow E_1 = 3.76 \times 10^{-3} \text{ eV}$$

$$n = 2 \Rightarrow E_2 = 1.50 \times 10^{-2} \text{ eV}$$

$$n = 3 \Rightarrow E_3 = 3.38 \times 10^{-2} \text{ eV}$$

2.23

$$(a) \quad E = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

$$= \frac{(1.054 \times 10^{-34})^2 \pi^2 n^2}{2(9.11 \times 10^{-31})(12 \times 10^{-10})^2}$$

$$= 4.81 \times 10^{-20} n^2 \text{ (J)}$$

So

$$E_1 = 4.18 \times 10^{-20} \text{ J} \Rightarrow E_1 = 0.261 \text{ eV}$$

$$E_2 = 1.67 \times 10^{-19} \text{ J} \Rightarrow E_2 = 1.04 \text{ eV}$$

(b)

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

or

$$\lambda = \frac{(6.625 \times 10^{-34})(3 \times 10^8)}{1.67 \times 10^{-19} - 4.18 \times 10^{-20}} \Rightarrow$$

$$\lambda = 1.59 \times 10^{-6} \text{ m}$$

or

$$\lambda = 1.59 \text{ }\mu\text{m}$$

2.24

(a) For the infinite potential well

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \Rightarrow n^2 = \frac{2ma^2 E}{\hbar^2 \pi^2}$$

so

$$n^2 = \frac{2(10^{-5})(10^{-2})^2(10^{-2})}{(1.054 \times 10^{-34})^2 \pi^2} = 1.82 \times 10^{56}$$

or

$$(b) \quad n = 1.35 \times 10^{28}$$

$$\Delta E = \frac{\hbar^2 \pi^2}{2ma^2} [(n+1)^2 - n^2]$$

$$= \frac{\hbar^2 \pi^2}{2ma^2} (2n+1)$$

or

$$\Delta E = \frac{(1.054 \times 10^{-34})^2 \pi^2 (2)(1.35 \times 10^{28})}{2(10^{-5})(10^{-2})^2}$$

$$\Delta E = 1.48 \times 10^{-30} \text{ J}$$

Energy in the (n+1) state is 1.48×10^{-30} Joules larger than 10 mJ.

(c)
Quantum effects would not be observable.

2.25

For a neutron and $n = 1$:

$$E_1 = \frac{\hbar^2 \pi^2}{2ma^2} = \frac{(1.054 \times 10^{-34})^2 \pi^2}{2(1.66 \times 10^{-27})(10^{-14})^2}$$

or

$$E_1 = 2.06 \times 10^6 \text{ eV}$$

For an electron in the same potential well:

$$E_1 = \frac{(1.054 \times 10^{-34})^2 \pi^2}{2(9.11 \times 10^{-31})(10^{-14})^2}$$

or

$$E_1 = 3.76 \times 10^9 \text{ eV}$$

2.26

Schrodinger's wave equation

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} (E - V(x)) \psi(x) = 0$$

We know that

$$\psi(x) = 0 \text{ for } x \geq \frac{a}{2} \text{ and } x \leq -\frac{a}{2}$$

$$V(x) = 0 \text{ for } -\frac{a}{2} \leq x \leq \frac{a}{2}$$

so in this region

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \psi(x) = 0$$

Solution is of the form

$$\psi(x) = A \cos Kx + B \sin Kx$$

$$\text{where } K = \sqrt{\frac{2mE}{\hbar^2}}$$

Boundary conditions:

$$\psi(x) = 0 \text{ at } x = \frac{+a}{2}, x = \frac{-a}{2}$$

So, first mode:

$$\psi_1(x) = A \cos Kx$$

$$\text{where } K = \frac{\pi}{a} \text{ so } E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$$

Second mode:

$$\psi_2(x) = B \sin Kx$$

$$\text{where } K = \frac{2\pi}{a} \text{ so } E_2 = \frac{4\pi^2 \hbar^2}{2ma^2}$$

Third mode:

$$\psi_3(x) = A \cos Kx$$

$$\text{where } K = \frac{3\pi}{a} \text{ so } E_3 = \frac{9\pi^2 \hbar^2}{2ma^2}$$

Fourth mode:

$$\psi_4(x) = B \sin Kx$$

$$\text{where } K = \frac{4\pi}{a} \text{ so } E_4 = \frac{16\pi^2 \hbar^2}{2ma^2}$$

2.27

The 3-D wave equation in cartesian coordinates, for $V(x,y,z) = 0$

$$\frac{\partial^2 \psi(x,y,z)}{\partial x^2} + \frac{\partial^2 \psi(x,y,z)}{\partial y^2} + \frac{\partial^2 \psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2} \psi(x,y,z) = 0$$

Use separation of variables, so let

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we get

$$YZ \frac{\partial^2 X}{\partial x^2} + XZ \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} XYZ = 0$$

Dividing by XYZ and letting $k^2 = \frac{2mE}{\hbar^2}$, we

obtain

$$(1) \quad \frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

We may set

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2 \text{ so } \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form

$$X(x) = A \sin(k_x x) + B \cos(k_x x)$$

Boundary conditions: $X(0) = 0 \Rightarrow B = 0$

$$\text{and } X(x=a) = 0 \Rightarrow k_x = \frac{n_x \pi}{a}$$

where $n_x = 1, 2, 3, \dots$

Similarly, let

$$\frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} = -k_y^2 \text{ and } \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

Applying the boundary conditions, we find

$$k_y = \frac{n_y \pi}{a}, n_y = 1, 2, 3, \dots$$

$$k_z = \frac{n_z \pi}{a}, n_z = 1, 2, 3, \dots$$

From Equation (1) above, we have

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0$$

or

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \frac{2mE}{\hbar^2}$$

so that

$$E \Rightarrow E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2} (n_x^2 + n_y^2 + n_z^2)$$

2.28

For the 2-dimensional infinite potential well:

$$\frac{\partial^2 \psi(x, y)}{\partial x^2} + \frac{\partial^2 \psi(x, y)}{\partial y^2} + \frac{2mE}{\hbar^2} \psi(x, y) = 0$$

$$\text{Let } \psi(x, y) = X(x)Y(y)$$

Then substituting,

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2} XY = 0$$

Divide by XY

So

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2$$

or

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form:

$$X = A \sin(k_x x) + B \cos(k_x x)$$

But $X(x=0) = 0 \Rightarrow B = 0$

So

$$X = A \sin(k_x x)$$

Also, $X(x=a) = 0 \Rightarrow k_x a = n_x \pi$

Where $n_x = 1, 2, 3, \dots$

$$\text{So that } k_x = \frac{n_x \pi}{a}$$

We can also define

$$\frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

Solution is of the form

$$Y = C \sin(k_y y) + D \cos(k_y y)$$

But

$$Y(y=0) = 0 \Rightarrow D = 0$$

and

$$Y(y=b) = 0 \Rightarrow k_y b = n_y \pi$$

so that

$$k_y = \frac{n_y \pi}{b}$$

Now

$$-k_x^2 - k_y^2 + \frac{2mE}{\hbar^2} = 0$$

which yields

$$E \Rightarrow E_{n_x n_y} = \frac{\hbar^2 \pi^2}{2m} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} \right)$$

Similarities: energy is quantized

Difference: now a function of 2 integers

2.29

(a) Derivation of energy levels exactly the same as in the text.

$$(b) \Delta E = \frac{\hbar^2 \pi^2}{2ma^2} (n_2^2 - n_1^2)$$

For $n_2 = 2, n_1 = 1$

Then

$$\Delta E = \frac{3\hbar^2 \pi^2}{2ma^2}$$

(i) $a = 4 \text{ \AA}$

$$\Delta E = \frac{3(1.054 \times 10^{-34})^2 \pi^2}{2(1.67 \times 10^{-27})(4 \times 10^{-10})^2} \Rightarrow$$

$$\Delta E = 3.85 \times 10^{-3} \text{ eV}$$

(ii) $a = 0.5 \text{ cm}$

$$\Delta E = \frac{3(1.054 \times 10^{-34})^2 \pi^2}{2(1.67 \times 10^{-27})(0.5 \times 10^{-2})^2} \Rightarrow$$

$$\Delta E = 2.46 \times 10^{-17} \text{ eV}$$

2.30

(a) For region II, $x > 0$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2} (E - V_o) \psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

where

$$K_2 = \sqrt{\frac{2m}{\hbar^2} (E - V_o)}$$

Term with B_2 represents incident wave, and term with A_2 represents the reflected wave.

Region I, $x < 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_1(x) = 0$$

The general solution is of the form

$$\psi_1(x) = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

where

$$K_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave, and the term involving A_1 represents the reflected wave; but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1 \exp(-jK_1 x)$$

$$\psi_2(x) = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

(b)

Boundary conditions:

(1) $\psi_1(x=0) = \psi_2(x=0)$

(2) $\left. \frac{\partial \psi_1(x)}{\partial x} \right|_{x=0} = \left. \frac{\partial \psi_2(x)}{\partial x} \right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$

$$K_2 A_2 - K_2 B_2 = -K_1 B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{K_2 - K_1}{K_2 + K_1} \right) B_2 \quad \text{and} \quad B_1 = \left(\frac{2K_2}{K_2 + K_1} \right) B_2$$

The reflection coefficient is

$$R = \frac{A_2 A_2^*}{B_2 B_2^*} \Rightarrow R = \left(\frac{K_2 - K_1}{K_2 + K_1} \right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow T = \frac{4K_1 K_2}{(K_1 + K_2)^2}$$

2.31

In region II, $x > 0$, we have

$$\psi_2(x) = A_2 \exp(-K_2 x)$$

where

$$K_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$$

For $V_o = 2.4 \text{ eV}$ and $E = 2.1 \text{ eV}$

$$K_2 = \left\{ \frac{2(9.11 \times 10^{-31})(2.4 - 2.1)(1.6 \times 10^{-19})}{(1.054 \times 10^{-34})^2} \right\}^{1/2}$$

or

$$K_2 = 2.81 \times 10^9 \text{ m}^{-1}$$

Probability at x compared to $x = 0$, given by

$$P = \left| \frac{\psi_2(x)}{\psi_2(0)} \right|^2 = \exp(-2K_2 x)$$

(a) For $x = 12 \text{ \AA}$

$$P = \exp[-2(2.81 \times 10^9)(12 \times 10^{-10})] \Rightarrow$$

$$P = 1.18 \times 10^{-3} = 0.118 \%$$

(b) For $x = 48 \text{ \AA}$

$$P = \exp[-2(2.81 \times 10^9)(48 \times 10^{-10})] \Rightarrow$$

$$P = 1.9 \times 10^{-10} \%$$

2.32

For $V_o = 6 \text{ eV}$, $E = 2.2 \text{ eV}$

We have that

$$T = 16 \left(\frac{E}{V_o} \right) \left(1 - \frac{E}{V_o} \right) \exp(-2K_2 a)$$

where

$$K_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}} = \left\{ \frac{2(9.11 \times 10^{-31})(6 - 2.2)(1.6 \times 10^{-19})}{(1.054 \times 10^{-34})^2} \right\}^{1/2}$$

or

$$K_2 = 9.98 \times 10^9 \text{ m}^{-1}$$

For $a = 10^{-10} \text{ m}$

$$T = 16 \left(\frac{2.2}{6} \right) \left(1 - \frac{2.2}{6} \right) \exp[-2(9.98 \times 10^9)(10^{-10})]$$

or

$$T = 0.50$$

For $a = 10^{-9} \text{ m}$

$$T = 7.97 \times 10^{-9}$$

2.33

Assume that Equation [2.62] is valid:

$$T = 16 \left(\frac{E}{V_o} \right) \left(1 - \frac{E}{V_o} \right) \exp(-2K_2 a)$$

(a) For $m = (0.067)m_o$

$$K_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}} = \left\{ \frac{2(0.067)(9.11 \times 10^{-31})(0.8 - 0.2)(1.6 \times 10^{-19})}{(1.054 \times 10^{-34})^2} \right\}^{1/2}$$

or

$$K_2 = 1.027 \times 10^9 \text{ m}^{-1}$$

Then

$$T = 16 \left(\frac{0.2}{0.8} \right) \left(1 - \frac{0.2}{0.8} \right) \exp[-2(1.027 \times 10^9)(15 \times 10^{-10})]$$

or

$$T = 0.138$$

(b) For $m = (1.08)m_o$

$$K_2 = \left\{ \frac{2(1.08)(9.11 \times 10^{-31})(0.8 - 0.2)(1.6 \times 10^{-19})}{(1.054 \times 10^{-34})^2} \right\}^{1/2}$$

or

$$K_2 = 4.124 \times 10^9 \text{ m}^{-1}$$

Then

$$T = 3 \exp[-2(4.124 \times 10^9)(15 \times 10^{-10})]$$

or

$$T = 1.27 \times 10^{-5}$$

2.34

$V_o = 10 \times 10^6 \text{ eV}$, $E = 3 \times 10^6 \text{ eV}$, $a = 10^{-14} \text{ m}$

and $m = 1.67 \times 10^{-27} \text{ kg}$

Now

$$K_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}} = \left\{ \frac{2(1.67 \times 10^{-27})(10 - 3)(10^6)(1.6 \times 10^{-19})}{(1.054 \times 10^{-34})^2} \right\}^{1/2}$$

or

$$K_2 = 5.80 \times 10^{14} \text{ m}^{-1}$$

So

$$T = 16 \left(\frac{3}{10} \right) \left(1 - \frac{3}{10} \right) \exp[-2(5.80 \times 10^{14})(10^{-14})]$$

or

$$T = 3.06 \times 10^{-5}$$

2.35

Region I, $V = 0$ ($x < 0$); Region II,

$V = V_o$ ($0 < x < a$); Region III, $V = 0$ ($x > a$).

(a) Region I;

$$\psi_1(x) = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

(incident) (reflected)

Region II;

$$\psi_2(x) = A_2 \exp(K_2 x) + B_2 \exp(-K_2 x)$$

Region III;

$$\psi_3(x) = A_3 \exp(jK_1 x) + B_3 \exp(-jK_1 x)$$

(b)

In region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into region III, there will not be a reflected wave which means that $B_3 = 0$.

(c)

Boundary conditions:

For $x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow jK_1 A_1 - jK_1 B_1 = K_2 A_2 - K_2 B_2$$

For $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(K_2 a) + B_2 \exp(-K_2 a) = A_3 \exp(jK_1 a)$$

And also

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$K_2 A_2 \exp(K_2 a) - K_2 B_2 \exp(-K_2 a)$$

$$= jK_1 A_3 \exp(jK_1 a)$$

Transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1 . Solving for A_1 in terms of A_3 , we find

$$A_1 = \frac{+jA_3}{4K_1 K_2} \left\{ (K_2^2 - K_1^2) [\exp(K_2 a) - \exp(-K_2 a)] \right.$$

$$\left. - 2jK_1 K_2 [\exp(K_2 a) + \exp(-K_2 a)] \right\} \exp(jK_2 a)$$

We then find that

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1 K_2)^2} \left\{ (K_2^2 - K_1^2) [\exp(K_2 a) \right.$$

$$\left. - \exp(-K_2 a)]^2 \right.$$

$$\left. + 4K_1^2 K_2^2 [\exp(K_2 a) + \exp(-K_2 a)]^2 \right\}$$

We have

$$K_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$$

and since $V_o \gg E$, then $K_2 a$ will be large so that

$$\exp(K_2 a) \gg \exp(-K_2 a)$$

Then we can write

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1 K_2)^2} \left\{ (K_2^2 - K_1^2) [\exp(K_2 a)]^2 \right.$$

$$\left. + 4K_1^2 K_2^2 [\exp(K_2 a)]^2 \right\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1 K_2)^2} (K_2^2 + K_1^2) \exp(2K_2 a)$$

Substituting the expressions for K_1 and K_2 , we find

$$K_1^2 + K_2^2 = \frac{2mV_o}{\hbar^2}$$

and

$$K_1^2 K_2^2 = \left[\frac{2m(V_o - E)}{\hbar^2} \right] \left[\frac{2mE}{\hbar^2} \right]$$

$$= \left(\frac{2m}{\hbar^2} \right) (V_o - E)(E)$$

or

$$K_1^2 K_2^2 = \left(\frac{2m}{\hbar^2} \right)^2 V_o \left(1 - \frac{E}{V_o} \right) (E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^* \left(\frac{2mV_o}{\hbar^2} \right)^2 \exp(2K_2 a)}{16 \left[\left(\frac{2m}{\hbar^2} \right)^2 V_o \left(1 - \frac{E}{V_o} \right) (E) \right]}$$

$$= \frac{A_3 A_3^*}{16 \left(\frac{E}{V_o} \right) \left(1 - \frac{E}{V_o} \right) \exp(-2K_2 a)}$$

or finally

$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16 \left(\frac{E}{V_o} \right) \left(1 - \frac{E}{V_o} \right) \exp(-2K_2 a)$$

2.36

Region I: $V = 0$

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_1 = 0 \Rightarrow$$

$$\psi_1 = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

(incident wave) (reflected wave)

$$\text{where } K_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II: $V = V_1$

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2} \psi_2 = 0 \Rightarrow$$

$$\psi_2 = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

(transmitted wave) (reflected wave)

$$\text{where } K_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$$

Region III: $V = V_2$

$$\frac{\partial^2 \psi_3}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2} \psi_3 = 0 \Rightarrow$$

$$\psi_3 = A_3 \exp(jK_3 x)$$

(transmitted wave)

$$\text{where } K_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$$

There is no reflected wave in region III.

The transmission coefficient is defined as

$$T = \frac{v_3}{v_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*} = \frac{K_3}{K_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*}$$

From boundary conditions, solve for A_3 in terms of A_1 . The boundary conditions are:

$$x = 0: \psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow K_1 A_1 - K_1 B_1 = K_2 A_2 - K_2 B_2$$

$$x = a: \psi_2 = \psi_3 \Rightarrow$$

$$A_2 \exp(jK_2 a) + B_2 \exp(-jK_2 a) = A_3 \exp(jK_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$K_2 A_2 \exp(jK_2 a) - K_2 B_2 \exp(-jK_2 a) = K_3 A_3 \exp(jK_3 a)$$

$$\text{But } K_2 a = 2n\pi \Rightarrow$$

$$\exp(jK_2 a) = \exp(-jK_2 a) = 1$$

Then, eliminating B_1 , A_2 , B_2 from the above equations, we have

$$T = \frac{K_3}{K_1} \cdot \frac{4K_1^2}{(K_1 + K_3)^2} \Rightarrow T = \frac{4K_1 K_3}{(K_1 + K_3)^2}$$

2.37

(a) Region I: Since $V_o > E$, we can write

$$\frac{\partial^2 \psi_1}{\partial x^2} - \frac{2m(V_o - E)}{\hbar^2} \psi_1 = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_2 = 0$$

Region III: $V \rightarrow \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1 = B_1 \exp(+K_1 x)$$

$$\psi_2 = A_2 \sin(K_2 x) + B_2 \cos(K_2 x)$$

$$\psi_3 = 0$$

where

$$K_1 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}} \quad \text{and} \quad K_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b)

Boundary conditions:

$$x = 0: \psi_1 = \psi_2 \Rightarrow B_1 = B_2$$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow K_1 B_1 = K_2 A_2$$

$$x = a: \psi_2 = \psi_3 \Rightarrow$$

$$A_2 \sin K_2 a + B_2 \cos K_2 a = 0$$

or

$$B_2 = -A_2 \tan K_2 a$$

(c)

$$K_1 B_1 = K_2 A_2 \Rightarrow A_2 = \left(\frac{K_1}{K_2} \right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{K_1}{K_2} \right) B_2$$

From $B_2 = -A_2 \tan K_2 a$, we can write

$$B_2 = -\left(\frac{K_1}{K_2} \right) B_2 \tan K_2 a$$

which gives

$$1 = -\left(\frac{K_1}{K_2} \right) \tan K_2 a$$

In turn, this equation can be written as

$$1 = -\sqrt{\frac{V_o - E}{E}} \tan \left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a \right]$$

or

$$\sqrt{\frac{E}{V_o - E}} = -\tan \left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a \right]$$

This last equation is valid only for specific values of the total energy E . The energy levels are quantized.

2.38

$$\begin{aligned} E_n &= \frac{-m_o e^4}{(4\pi \epsilon_o)^2 2\hbar^2 n^2} (J) \\ &= \frac{m_o e^3}{(4\pi \epsilon_o)^2 2\hbar^2 n^2} (eV) \\ &= \frac{-(9.11 \times 10^{-31})(1.6 \times 10^{-19})^3}{[4\pi(8.85 \times 10^{-12})]^2 2(1.054 \times 10^{-34})^2 n^2} \Rightarrow \\ E_n &= \frac{-13.58}{n^2} (eV) \end{aligned}$$

Then

$$\begin{aligned} n=1 &\Rightarrow E_1 = -13.58 \text{ eV} \\ n=2 &\Rightarrow E_2 = -3.395 \text{ eV} \\ n=3 &\Rightarrow E_3 = -1.51 \text{ eV} \\ n=4 &\Rightarrow E_4 = -0.849 \text{ eV} \end{aligned}$$

2.39

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100} \psi_{100}^* = 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\begin{aligned} \frac{dP(r)}{dr} &= 0 \\ &= \frac{4}{(a_o)^3} \left\{ r^2 \left(\frac{-2}{a_o}\right) \exp\left(\frac{-2r}{a_o}\right) + 2r \exp\left(\frac{-2r}{a_o}\right) \right\} \end{aligned}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow r = a_o$$

or $r = a_o$ is the radius that gives the greatest probability.

2.40

ψ_{100} is independent of θ and ϕ , so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{2m_o}{\hbar^2} (E - V(r)) \psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi \epsilon_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\begin{aligned} \psi_{100} &= \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) \Rightarrow \\ \frac{d\psi_{100}}{dr} &= \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left(\frac{-1}{a_o}\right) \exp\left(\frac{-r}{a_o}\right) \end{aligned}$$

Then

$$r^2 \frac{d\psi_{100}}{dr} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

so that

$$\begin{aligned} \frac{d}{dr} \left(r^2 \frac{d\psi_{100}}{dr} \right) \\ = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} \left[2r \exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right) \exp\left(\frac{-r}{a_o}\right) \right] \end{aligned}$$

Substituting into the wave equation, we have

$$\begin{aligned} \frac{-1}{r^2 \sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} \left[2r \exp\left(\frac{-r}{a_o}\right) - \frac{r^2}{a_o} \exp\left(\frac{-r}{a_o}\right) \right] \\ + \frac{2m_o}{\hbar^2} \left[E + \frac{\hbar^2}{m_o a_o r} \right] \cdot \left(\frac{1}{\sqrt{\pi}}\right) \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) = 0 \end{aligned}$$

where

$$E = E_1 = \frac{-m_o e^4}{(4\pi \epsilon_o)^2 \cdot 2\hbar^2} \Rightarrow E_1 = \frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\begin{aligned} \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left[\exp\left(\frac{-r}{a_o}\right) \right] \left\{ \frac{-1}{r^2 a_o} \left[2r - \frac{r^2}{a_o} \right] \right. \\ \left. + \frac{2m_o}{\hbar^2} \left(\frac{-\hbar^2}{2m_o a_o} + \frac{\hbar^2}{m_o a_o r} \right) \right\} = 0 \end{aligned}$$

or

$$\begin{aligned} \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left[\exp\left(\frac{-r}{a_o}\right) \right] \\ \times \left\{ \frac{-2}{a_o r} + \frac{1}{a_o^2} + \left(\frac{-1}{a_o^2} + \frac{2}{a_o r} \right) \right\} = 0 \end{aligned}$$

which gives $0 = 0$, and shows that ψ_{100} is indeed a solution of the wave equation.

2.41

All elements from Group I column of the periodic table. All have one valence electron in the outer shell.

Chapter 3

Problem Solutions

3.1 If a_o were to increase, the bandgap energy would decrease and the material would begin to behave less like a semiconductor and more like a metal. If a_o were to decrease, the bandgap energy would increase and the material would begin to behave more like an insulator.

3.2

Schrodinger's wave equation

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x) \cdot \Psi(x,t) = j\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

Let the solution be of the form

$$\Psi(x,t) = u(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right]$$

Region I, $V(x) = 0$, so substituting the proposed solution into the wave equation, we obtain

$$\begin{aligned} \frac{-\hbar^2}{2m} \cdot \frac{\partial}{\partial x} \left\{ jku(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right. \\ \left. + \frac{\partial u(x)}{\partial x} \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right\} \\ = j\hbar \left(\frac{-jE}{\hbar} \right) \cdot u(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \end{aligned}$$

which becomes

$$\begin{aligned} \frac{-\hbar^2}{2m} \left\{ (jk)^2 u(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right. \\ \left. + 2jk \frac{\partial u(x)}{\partial x} \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right. \\ \left. + \frac{\partial^2 u(x)}{\partial x^2} \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right\} \\ = +Eu(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \end{aligned}$$

This equation can then be written as

$$-k^2 u(x) + 2jk \frac{\partial u(x)}{\partial x} + \frac{\partial^2 u(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \cdot u(x) = 0$$

Setting $u(x) = u_1(x)$ for region I, this equation becomes

$$\frac{d^2 u_1(x)}{dx^2} + 2jk \frac{du_1(x)}{dx} - (k^2 - \alpha^2) u_1(x) = 0$$

where

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

Q.E.D.

In region II, $V(x) = V_o$. Assume the same form of the solution

$$\Psi(x,t) = u(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right]$$

Substituting into Schrodinger's wave equation, we obtain

$$\begin{aligned} \frac{-\hbar^2}{2m} \left\{ (jk)^2 u(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right. \\ \left. + 2jk \frac{\partial u(x)}{\partial x} \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right. \\ \left. + \frac{\partial^2 u(x)}{\partial x^2} \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \right\} \\ + V_o u(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \\ = Eu(x) \exp \left[j \left(kx - \left(\frac{E}{\hbar} \right) t \right) \right] \end{aligned}$$

This equation can be written as

$$\begin{aligned} -k^2 u(x) + 2jk \frac{\partial u(x)}{\partial x} + \frac{\partial^2 u(x)}{\partial x^2} \\ - \frac{2mV_o}{\hbar^2} u(x) + \frac{2mE}{\hbar^2} u(x) = 0 \end{aligned}$$

Setting $u(x) = u_2(x)$ for region II, this equation becomes

$$\frac{d^2 u_2(x)}{dx^2} + 2jk \frac{du_2(x)}{dx} - \left(k^2 - \alpha^2 + \frac{2mV_o}{\hbar^2} \right) u_2(x) = 0$$

where

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

Q.E.D.

3.3

We have

$$\frac{d^2 u_1(x)}{dx^2} + 2jk \frac{du_1(x)}{dx} - (k^2 - \alpha^2) u_1(x) = 0$$

The proposed solution is

$$u_1(x) = A \exp[j(\alpha - k)x] + B \exp[-j(\alpha + k)x]$$

The first derivative is

$$\begin{aligned} \frac{du_1(x)}{dx} &= j(\alpha - k) A \exp[j(\alpha - k)x] \\ &\quad - j(\alpha + k) B \exp[-j(\alpha + k)x] \end{aligned}$$

and the second derivative becomes

$$\begin{aligned} \frac{d^2 u_1(x)}{dx^2} &= [j(\alpha - k)]^2 A \exp[j(\alpha - k)x] \\ &\quad + [j(\alpha + k)]^2 B \exp[-j(\alpha + k)x] \end{aligned}$$

Substituting these equations into the differential equation, we find

$$\begin{aligned} &-(\alpha - k)^2 A \exp[j(\alpha - k)x] \\ &-(\alpha + k)^2 B \exp[-j(\alpha + k)x] \\ &+ 2jk \{ j(\alpha - k) A \exp[j(\alpha - k)x] \\ &\quad - j(\alpha + k) B \exp[-j(\alpha + k)x] \} \\ &- (k^2 - \alpha^2) \{ A \exp[j(\alpha - k)x] \\ &\quad + B \exp[-j(\alpha + k)x] \} = 0 \end{aligned}$$

Combining terms, we have

$$\begin{aligned} &\{ -(\alpha^2 - 2\alpha k + k^2) - 2k(\alpha - k) \\ &\quad - (k^2 - \alpha^2) \} A \exp[j(\alpha - k)x] \\ &+ \{ -(\alpha^2 + 2\alpha k + k^2) + 2k(\alpha + k) \\ &\quad - (k^2 - \alpha^2) \} B \exp[-j(\alpha + k)x] = 0 \end{aligned}$$

We find that

$$0 = 0 \quad \text{Q.E.D.}$$

For the differential equation in $u_2(x)$ and the proposed solution, the procedure is exactly the same as above.

3.4

We have the solutions

$$u_1(x) = A \exp[j(\alpha - k)x] + B \exp[-j(\alpha + k)x] \quad \text{for } 0 < x < a$$

$$u_2(x) = C \exp[j(\beta - k)x] + D \exp[-j(\beta + k)x] \quad \text{for } -b < x < 0$$

The boundary conditions:

$$u_1(0) = u_2(0)$$

which yields

$$A + B - C - D = 0$$

Also

$$\left. \frac{du_1}{dx} \right|_{x=0} = \left. \frac{du_2}{dx} \right|_{x=0}$$

which yields

$$(\alpha - k)A - (\alpha + k)B - (\beta - k)C + (\beta + k)D = 0$$

The third boundary condition is

$$u_1(a) = u_2(-b)$$

which gives

$$\begin{aligned} &A \exp[j(\alpha - k)a] + B \exp[-j(\alpha + k)a] \\ &= C \exp[j(\beta - k)(-b)] + D \exp[-j(\beta + k)(-b)] \end{aligned}$$

This becomes

$$\begin{aligned} &A \exp[j(\alpha - k)a] + B \exp[-j(\alpha + k)a] \\ &- C \exp[-j(\beta - k)b] - D \exp[j(\beta + k)b] = 0 \end{aligned}$$

The last boundary condition is

$$\left. \frac{du_1}{dx} \right|_{x=a} = \left. \frac{du_2}{dx} \right|_{x=-b}$$

which gives

$$\begin{aligned} &j(\alpha - k)A \exp[j(\alpha - k)a] \\ &- j(\alpha + k)B \exp[-j(\alpha + k)a] \\ &= j(\beta - k)C \exp[j(\beta - k)(-b)] \\ &\quad - j(\beta + k)D \exp[-j(\beta + k)(-b)] \end{aligned}$$

This becomes

$$\begin{aligned} &(\alpha - k)A \exp[j(\alpha - k)a] \\ &- (\alpha + k)B \exp[-j(\alpha + k)a] \\ &- (\beta - k)C \exp[-j(\beta - k)b] \\ &+ (\beta + k)D \exp[j(\beta + k)b] = 0 \end{aligned}$$

3.5 Computer plot

3.6 Computer plot

3.7

$$P' \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

Let $ka = y$, $\alpha a = x$

Then

$$P' \frac{\sin x}{x} + \cos x = \cos y$$

Consider $\frac{d}{dy}$ of this function

$$\frac{d}{dy} \left\{ \left[P' \cdot (x)^{-1} \cdot \sin x \right] + \cos x \right\} = -\sin y$$

We obtain

$$P' \left\{ (-1)(x)^{-2} \sin x \frac{dx}{dy} + (x)^{-1} \cos x \frac{dx}{dy} \right\} \\ - \sin x \frac{dx}{dy} = -\sin y$$

Then

$$\frac{dx}{dy} \left\{ P' \left[\frac{-1}{x^2} \sin x + \frac{\cos x}{x} \right] - \sin x \right\} = -\sin y$$

For $y = ka = n\pi$, $n = 0, 1, 2, \dots$

$$\Rightarrow \sin y = 0$$

So that, in general, then

$$\frac{dx}{dy} = 0 = \frac{d(\alpha a)}{d(ka)} = \frac{d\alpha}{dk}$$

And

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow \frac{d\alpha}{dk} = \frac{1}{2} \left(\frac{2mE}{\hbar^2} \right)^{-1/2} \left(\frac{2m}{\hbar^2} \right) \frac{dE}{dk}$$

This implies that

$$\frac{d\alpha}{dk} = 0 = \frac{dE}{dk} \text{ for } k = \frac{n\pi}{a}$$

3.8

$$f(\alpha a) = 9 \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

(a) $ka = \pi \Rightarrow \cos ka = -1$

1st point: $\alpha a = \pi$; 2nd point: $\alpha a = 1.66\pi$
(2nd point by trial and error)

Now

$$\alpha a = a \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow E = \left(\frac{\alpha a}{a} \right)^2 \cdot \frac{\hbar^2}{2m}$$

So

$$E = \frac{(\alpha a)^2}{(5 \times 10^{-10})^2} \cdot \frac{(1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})} \Rightarrow$$

$$E = (\alpha a)^2 [2.439 \times 10^{-20}] \text{ (J)}$$

or

$$E = (\alpha a)^2 (0.1524) \text{ (eV)}$$

So

$$\alpha a = \pi \Rightarrow E_1 = 1.504 \text{ eV}$$

$$\alpha a = 1.66\pi \Rightarrow E_2 = 4.145 \text{ eV}$$

Then

$$\Delta E = 2.64 \text{ eV}$$

(b)

$$ka = 2\pi \Rightarrow \cos ka = +1$$

1st point: $\alpha a = 2\pi$

2nd point: $\alpha a = 2.54\pi$

Then

$$E_3 = 6.0165 \text{ eV}$$

$$E_4 = 9.704 \text{ eV}$$

so

$$\Delta E = 3.69 \text{ eV}$$

(c)

$$ka = 3\pi \Rightarrow \cos ka = -1$$

1st point: $\alpha a = 3\pi$

2nd point: $\alpha a = 3.44\pi$

Then

$$E_5 = 13.537 \text{ eV}$$

$$E_6 = 17.799 \text{ eV}$$

so

$$\Delta E = 4.26 \text{ eV}$$

(d)

$$ka = 4\pi \Rightarrow \cos ka = +1$$

1st point: $\alpha a = 4\pi$

2nd point: $\alpha a = 4.37\pi$

Then

$$E_7 = 24.066 \text{ eV}$$

$$E_8 = 28.724 \text{ eV}$$

so

$$\Delta E = 4.66 \text{ eV}$$

3.9

(a) $0 < ka < \pi$

For $ka = 0 \Rightarrow \cos ka = +1$

By trial and error: 1st point: $\alpha a = 0.822\pi$

2nd point: $\alpha a = \pi$

From Problem 3.8, $E = (\alpha a)^2 (0.1524) \text{ (eV)}$

Then

$$E_1 = 1.0163 \text{ eV}$$

$$E_2 = 1.5041 \text{ eV}$$

so

$$\Delta E = 0.488 \text{ eV}$$

(b)

$\pi < ka < 2\pi$

Using results of Problem 3.8

1st point: $\alpha a = 1.66\pi$

2nd point: $\alpha a = 2\pi$

Then

$$E_3 = 4.145 \text{ eV}$$

$$E_4 = 6.0165 \text{ eV}$$

so

$$\Delta E = 1.87 \text{ eV}$$

(c)

$$2\pi < ka < 3\pi$$

$$1^{\text{st}} \text{ point: } \alpha a = 2.54\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 3\pi$$

Then

$$E_5 = 9.704 \text{ eV}$$

$$E_6 = 13.537 \text{ eV}$$

so

$$\Delta E = 3.83 \text{ eV}$$

(d)

$$3\pi < ka < 4\pi$$

$$1^{\text{st}} \text{ point: } \alpha a = 3.44\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 4\pi$$

Then

$$E_7 = 17.799 \text{ eV}$$

$$E_8 = 24.066 \text{ eV}$$

so

$$\Delta E = 6.27 \text{ eV}$$

3.10

$$6 \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

Forbidden energy bands

$$(a) \quad ka = \pi \Rightarrow \cos ka = -1$$

$$1^{\text{st}} \text{ point: } \alpha a = \pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 1.56\pi \text{ (By trial and error)}$$

From Problem 3.8, $E = (\alpha a)^2 (0.1524) \text{ eV}$

Then

$$E_1 = 1.504 \text{ eV}$$

$$E_2 = 3.660 \text{ eV}$$

so

$$\Delta E = 2.16 \text{ eV}$$

(b)

$$ka = 2\pi \Rightarrow \cos ka = +1$$

$$1^{\text{st}} \text{ point: } \alpha a = 2\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 2.42\pi$$

Then

$$E_3 = 6.0165 \text{ eV}$$

$$E_4 = 8.809 \text{ eV}$$

so

$$\Delta E = 2.79 \text{ eV}$$

(c)

$$ka = 3\pi \Rightarrow \cos ka = -1$$

$$1^{\text{st}} \text{ point: } \alpha a = 3\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 3.33\pi$$

Then

$$E_5 = 13.537 \text{ eV}$$

$$E_6 = 16.679 \text{ eV}$$

so

$$\Delta E = 3.14 \text{ eV}$$

(d)

$$ka = 4\pi \Rightarrow \cos ka = +1$$

$$1^{\text{st}} \text{ point: } \alpha a = 4\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 4.26\pi$$

Then

$$E_7 = 24.066 \text{ eV}$$

$$E_8 = 27.296 \text{ eV}$$

so

$$\Delta E = 3.23 \text{ eV}$$

3.11

Allowed energy bands

Use results from Problem 3.10.

(a)

$$0 < ka < \pi$$

$$1^{\text{st}} \text{ point: } \alpha a = 0.759\pi \text{ (By trial and error)}$$

$$2^{\text{nd}} \text{ point: } \alpha a = \pi$$

We have

$$E = (\alpha a)^2 (0.1524) \text{ eV}$$

Then

$$E_1 = 0.8665 \text{ eV}$$

$$E_2 = 1.504 \text{ eV}$$

so

$$\Delta E = 0.638 \text{ eV}$$

(b)

$$\pi < ka < 2\pi$$

$$1^{\text{st}} \text{ point: } \alpha a = 1.56\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 2\pi$$

Then

$$E_3 = 3.660 \text{ eV}$$

$$E_4 = 6.0165 \text{ eV}$$

so

$$\Delta E = 2.36 \text{ eV}$$

(c)

$$2\pi < ka < 3\pi$$

$$1^{\text{st}} \text{ point: } \alpha a = 2.42\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 3\pi$$

Then

$$E_5 = 8.809 \text{ eV}$$

$$E_6 = 13.537 \text{ eV}$$

so

$$\Delta E = 4.73 \text{ eV}$$

(d)

$$3\pi < ka < 4\pi$$

$$1^{\text{st}} \text{ point: } \alpha a = 3.33\pi$$

$$2^{\text{nd}} \text{ point: } \alpha a = 4\pi$$

Then

$$E_7 = 16.679 \text{ eV}$$

$$E_8 = 24.066 \text{ eV}$$

so

$$\Delta E = 7.39 \text{ eV}$$

3.12

$$T = 100 \text{ K}; \quad E_g = 1.170 - \frac{(4.73 \times 10^{-4})(100)^2}{636 + 100} \Rightarrow$$

$$E_g = 1.164 \text{ eV}$$

$$T = 200 \text{ K} \Rightarrow E_g = 1.147 \text{ eV}$$

$$T = 300 \text{ K} \Rightarrow E_g = 1.125 \text{ eV}$$

$$T = 400 \text{ K} \Rightarrow E_g = 1.097 \text{ eV}$$

$$T = 500 \text{ K} \Rightarrow E_g = 1.066 \text{ eV}$$

$$T = 600 \text{ K} \Rightarrow E_g = 1.032 \text{ eV}$$

3.13

The effective mass is given by

$$m^* = \left(\frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2} \right)^{-1}$$

We have that

$$\frac{d^2 E}{dk^2}(\text{curve A}) > \frac{d^2 E}{dk^2}(\text{curve B})$$

so that

$$m^*(\text{curve A}) < m^*(\text{curve B})$$

3.14

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2} \cdot \left| \frac{d^2 E}{dk^2} \right| \right)^{-1}$$

We have that

$$\left| \frac{d^2 E}{dk^2} \right|(\text{curve A}) > \left| \frac{d^2 E}{dk^2} \right|(\text{curve B})$$

so that

$$m_p^*(\text{curve A}) < m_p^*(\text{curve B})$$

3.15

$$\text{Points A, B: } \frac{\partial E}{\partial k} < 0 \Rightarrow \text{velocity in } -x \text{ direction;}$$

$$\text{Points C, D: } \frac{\partial E}{\partial x} > 0 \Rightarrow \text{velocity in } +x \text{ direction;}$$

$$\text{Points A, D: } \frac{\partial^2 E}{\partial k^2} < 0 \Rightarrow \text{negative effective mass;}$$

$$\text{Points B, C: } \frac{\partial^2 E}{\partial k^2} > 0 \Rightarrow \text{positive effective mass;}$$

3.16

$$E - E_c = \frac{k^2 \hbar^2}{2m}$$

$$\text{At } k = 0.1 \text{ (}\AA^{-1}\text{)} \Rightarrow \frac{1}{k} = 10 \text{ }\AA = 10^{-9} \text{ m}$$

So

$$k = 10^{+9} \text{ m}^{-1}$$

For A:

$$(0.07)(1.6 \times 10^{-19}) = \frac{(10^9)^2 (1.054 \times 10^{-34})^2}{2m}$$

which yields

$$m = 4.96 \times 10^{-31} \text{ kg}$$

so

$$\text{curve A: } \frac{m}{m_0} = 0.544$$

For B:

$$(0.7)(1.6 \times 10^{-19}) = \frac{(10^9)^2 (1.054 \times 10^{-34})^2}{2m}$$

which yields

$$m = 4.96 \times 10^{-32} \text{ kg}$$

so

$$\text{Curve B: } \frac{m}{m_0} = 0.0544$$

3.17

$$E_v - E = \frac{k^2 \hbar^2}{2m}$$

$$k = 0.1 (A^*)^{-1} \Rightarrow 10^9 m^{-1}$$

For Curve A:

$$(0.08)(1.6 \times 10^{-19}) = \frac{(10^9)^2 (1.054 \times 10^{-34})^2}{2m}$$

which yields

$$m = 4.34 \times 10^{-31} kg \Rightarrow \frac{m}{m_o} = 0.476$$

For Curve B:

$$(0.4)(1.6 \times 10^{-19}) = \frac{(10^9)^2 (1.054 \times 10^{-34})^2}{2m}$$

which yields

$$m = 8.68 \times 10^{-32} kg \Rightarrow \frac{m}{m_o} = 0.0953$$

3.18

(a) $E = h\nu$

Then

$$\nu = \frac{E}{h} = \frac{(1.42)(1.6 \times 10^{-19})}{(6.625 \times 10^{-34})} \Rightarrow$$

$$\nu = 3.43 \times 10^{14} Hz$$

(b)

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.43 \times 10^{14}} = 8.75 \times 10^{-7} m$$

or

$$\lambda = 0.875 \mu m$$

3.19

(c) Curve A: Effective mass is a constant

Curve B: Effective mass is positive around

$$k = 0, \text{ and is negative around } k = \pm \frac{\pi}{2}.$$

3.20

$$E = E_o - E_1 \cos[\alpha(k - k_o)]$$

$$\frac{dE}{dk} = (-E_1)(-\alpha) \sin[\alpha(k - k_o)]$$

$$= +E_1 \alpha \sin[\alpha(k - k_o)]$$

So

$$\frac{d^2 E}{dk^2} = E_1 \alpha^2 \cos[\alpha(k - k_o)]$$

Then

$$\left. \frac{d^2 E}{dk^2} \right|_{k=k_o} = E_1 \alpha^2$$

We have

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2} = \frac{E_1 \alpha^2}{\hbar^2}$$

or

$$m^* = \frac{\hbar^2}{E_1 \alpha^2}$$

3.21

For the 3-dimensional infinite potential well,

$V(x) = 0$ when $0 < x < a$, $0 < y < a$, and

$0 < z < a$. In this region, the wave equation is

$$\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2} + \frac{2mE}{\hbar^2} \psi(x, y, z) = 0$$

Use separation of variables technique, so let

$$\psi(x, y, z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we have

$$YZ \frac{\partial^2 X}{\partial x^2} + XZ \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} \cdot XYZ = 0$$

Dividing by XYZ , we obtain

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2 \Rightarrow \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

The solution is of the form

$$X(x) = A \sin k_x x + B \cos k_x x$$

Since $\psi(x, y, z) = 0$ at $x = 0$, then $X(0) = 0$ so that $B \equiv 0$.

Also, $\psi(x, y, z) = 0$ at $x = a$, then $X(a) = 0$ so

we must have $k_x a = n_x \pi$, where

$$n_x = 1, 2, 3, \dots$$

Similarly, we have

$$\frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} = -k_y^2 \text{ and } \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

From the boundary conditions, we find

$$k_y a = n_y \pi \text{ and } k_z a = n_z \pi$$

where $n_y = 1, 2, 3, \dots$ and $n_z = 1, 2, 3, \dots$

From the wave equation, we have

$$-k_x^2 - k_y^2 - k_z^2 + \frac{2mE}{\hbar^2} = 0$$

The energy can then be written as

$$E = \frac{\hbar^2}{2m} (n_x^2 + n_y^2 + n_z^2) \left(\frac{\pi}{a} \right)^2$$

3.22

The total number of quantum states in the 3-dimensional potential well is given (in k-space) by

$$g_T(k) dk = \frac{\pi k^2 dk}{\pi^3} \cdot a^3$$

where

$$k^2 = \frac{2mE}{\hbar^2}$$

We can then write

$$k = \frac{1}{\hbar} \cdot \sqrt{2mE}$$

Taking the differential, we obtain

$$dk = \frac{1}{\hbar} \cdot \sqrt{2m} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{E}} \cdot dE = \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Substituting these expressions into the density of states function, we obtain

$$g_T(E) dE = \frac{\pi a^3}{\pi^3} \left(\frac{2mE}{\hbar^2} \right) \cdot \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Noting that

$$\hbar = \frac{h}{2\pi}$$

this density of states function can be simplified and written as

$$g_T(E) dE = \frac{4\pi a^3}{h^3} (2m)^{3/2} \cdot \sqrt{E} \cdot dE$$

Dividing by a^3 will yield the density of states, so that

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \sqrt{E}$$

3.23

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c}$$

Now

$$\begin{aligned} g_T &= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \int_{E_c}^{E_c + kT} \sqrt{E - E_c} \cdot dE \\ &= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \left(\frac{2}{3} \right) (E - E_c)^{3/2} \Big|_{E_c}^{E_c + kT} \\ &= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \left(\frac{2}{3} \right) (kT)^{3/2} \end{aligned}$$

Then

$$\begin{aligned} g_T &= \frac{4\pi [2(0.067)(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} \left(\frac{2}{3} \right) \\ &\quad \times [(0.0259)(1.6 \times 10^{-19})]^{3/2} \end{aligned}$$

or

$$g_T = 3.28 \times 10^{23} \text{ m}^{-3} = 3.28 \times 10^{17} \text{ cm}^{-3}$$

3.24

$$g_V(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_V - E}$$

Now

$$\begin{aligned} g_T &= \frac{4\pi(2m_p^*)^{3/2}}{h^3} \int_{E_V - kT}^{E_V} \sqrt{E_V - E} \cdot dE \\ &= \frac{4\pi(2m_p^*)^{3/2}}{h^3} \left(\frac{-2}{3} \right) (E_V - E)^{3/2} \Big|_{E_V - kT}^{E_V} \\ &= \frac{4\pi(2m_p^*)^{3/2}}{h^3} \left(\frac{2}{3} \right) (kT)^{3/2} \\ g_T &= \frac{4\pi [2(0.48)(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} \left(\frac{2}{3} \right) \\ &\quad \times [(0.0259)(1.6 \times 10^{-19})]^{3/2} \end{aligned}$$

or

$$g_T = 6.29 \times 10^{24} \text{ m}^{-3} = 6.29 \times 10^{18} \text{ cm}^{-3}$$

3.25

$$\begin{aligned} \text{(a) } g_c(E) &= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \\ &= \frac{4\pi [2(1.08)(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} (1.6 \times 10^{-19})^{1/2} \sqrt{E - E_c} \\ &= 4.77 \times 10^{46} \sqrt{E - E_c} \text{ m}^{-3} \text{ J}^{-1} \end{aligned}$$

or

$$g_c(E) = 7.63 \times 10^{21} \sqrt{E - E_c} \text{ cm}^{-3} \text{ eV}^{-1}$$

Then

E	g_c
$E_c + 0.05 \text{ eV}$	$1.71 \times 10^{21} \text{ cm}^{-3} \text{ eV}^{-1}$
$E_c + 0.10 \text{ eV}$	2.41×10^{21}
$E_c + 0.15 \text{ eV}$	2.96×10^{21}
$E_c + 0.20 \text{ eV}$	3.41×10^{21}

$$\begin{aligned} \text{(b)} \quad g_v(E) &= \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \\ &= \frac{4\pi[2(0.56)(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} (1.6 \times 10^{-19})^{1/2} \sqrt{E_v - E} \\ &= 1.78 \times 10^{46} \sqrt{E_v - E} \text{ m}^{-3} \text{ J}^{-1} \\ g_v(E) &= 2.85 \times 10^{21} \sqrt{E_v - E} \text{ cm}^{-3} \text{ eV}^{-1} \end{aligned}$$

E	$g_v(E)$
$E_v - 0.05 \text{ eV}$	$0.637 \times 10^{21} \text{ cm}^{-3} \text{ eV}^{-1}$
$E_v - 0.10 \text{ eV}$	0.901×10^{21}
$E_v - 0.15 \text{ eV}$	1.10×10^{21}
$E_v - 0.20 \text{ eV}$	1.27×10^{21}

3.26

$$\frac{g_c}{g_v} = \frac{(m_n^*)^{3/2}}{(m_p^*)^{3/2}} \Rightarrow \frac{g_c}{g_v} = \left(\frac{m_n^*}{m_p^*} \right)^{3/2}$$

3.27

Computer Plot

3.28

$$\begin{aligned} \frac{g_i!}{N_i!(g_i - N_i)!} &= \frac{10!}{8!(10-8)!} \\ &= \frac{(10)(9)(8!)}{(8!)(2!)} = \frac{(10)(9)}{(2)(1)} \Rightarrow \underline{45} \end{aligned}$$

3.29

$$\text{(a)} \quad f(E) = \frac{1}{1 + \exp\left[\frac{(E_c + kT) - E_c}{kT}\right]}$$

$$= \frac{1}{1 + \exp(1)} \Rightarrow \underline{f(E) = 0.269}$$

(b)

$$\begin{aligned} 1 - f(E) &= 1 - \frac{1}{1 + \exp\left[\frac{(E_v - kT) - E_v}{kT}\right]} \\ &= 1 - \frac{1}{1 + \exp(-1)} \Rightarrow \underline{1 - f(E) = 0.269} \end{aligned}$$

3.30

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$\text{(a)} \quad E - E_F = kT, \quad f(E) = \frac{1}{1 + \exp(1)} \Rightarrow$$

$$\underline{f(E) = 0.269}$$

$$\text{(b)} \quad E - E_F = 5kT, \quad f(E) = \frac{1}{1 + \exp(5)} \Rightarrow$$

$$\underline{f(E) = 6.69 \times 10^{-3}}$$

$$\text{(c)} \quad E - E_F = 10kT, \quad f(E) = \frac{1}{1 + \exp(10)} \Rightarrow$$

$$\underline{f(E) = 4.54 \times 10^{-5}}$$

3.31

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

or

$$1 - f(E) = \frac{1}{1 + \exp\left(\frac{E_F - E}{kT}\right)}$$

$$\text{(a)} \quad E_F - E = kT, \quad 1 - f(E) = 0.269$$

$$\text{(b)} \quad E_F - E = 5kT, \quad 1 - f(E) = 6.69 \times 10^{-3}$$

$$\text{(c)} \quad E_F - E = 10kT, \quad 1 - f(E) = 4.54 \times 10^{-5}$$

3.32

$$\text{(a)} \quad T = 300 \text{ K} \Rightarrow kT = 0.0259 \text{ eV}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \approx \exp\left[\frac{-(E - E_F)}{kT}\right]$$

E	$f(E)$
E_c	6.43×10^{-5}
$E_c + (1/2)kT$	3.90×10^{-5}
$E_c + kT$	2.36×10^{-5}
$E_c + (3/2)kT$	1.43×10^{-5}
$E_c + 2kT$	0.87×10^{-5}

(b) $T = 400K \Rightarrow kT = 0.03453$

E	$f(E)$
E_c	7.17×10^{-4}
$E_c + (1/2)kT$	4.35×10^{-4}
$E_c + kT$	2.64×10^{-4}
$E_c + (3/2)kT$	1.60×10^{-4}
$E_c + 2kT$	0.971×10^{-4}

3.33

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054 \times 10^{-34})^2 n^2 \pi^2}{2(9.11 \times 10^{-31})(10 \times 10^{-10})^2}$$

or

$$E_n = 6.018 \times 10^{-20} n^2 \text{ J} = 0.376 n^2 \text{ eV}$$

$$\text{For } n = 4 \Rightarrow E_4 = 6.02 \text{ eV},$$

$$\text{For } n = 5 \Rightarrow E_5 = 9.40 \text{ eV}.$$

As a 1st approximation for $T > 0$, assume the probability of $n = 5$ state being occupied is the same as the probability of $n = 4$ state being empty. Then

$$1 - \frac{1}{1 + \exp\left(\frac{E_4 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_5 - E_F}{kT}\right)}$$

$$\Rightarrow \frac{1}{1 + \exp\left(\frac{E_F - E_4}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_5 - E_F}{kT}\right)}$$

or

$$E_F - E_4 = E_5 - E_F \Rightarrow E_F = \frac{E_4 + E_5}{2}$$

Then

$$E_F = \frac{6.02 + 9.40}{2} \Rightarrow E_F = 7.71 \text{ eV}$$

3.34

(a) For 3-Dimensional infinite potential well,

$$E = \frac{\hbar^2 \pi^2}{2ma^2} (n_x^2 + n_y^2 + n_z^2)$$

$$= \frac{(1.054 \times 10^{-34})^2 \pi^2}{2(9.11 \times 10^{-31})(10^{-9})^2} (n_x^2 + n_y^2 + n_z^2)$$

$$= 0.376 (n_x^2 + n_y^2 + n_z^2) \text{ eV}$$

For 5 electrons, energy state corresponding to $n_x n_y n_z = 221 = 122$ contains both an electron and an empty state, so

$$E_F = (0.376)(2^2 + 2^2 + 1^2) \Rightarrow$$

$$E_F = 3.384 \text{ eV}$$

(b) For 13 electrons, energy state corresponding to $n_x n_y n_z = 323 = 233$ contains both an electron and an empty state, so

$$E_F = (0.376)(2^2 + 3^2 + 3^2) \Rightarrow$$

$$E_F = 8.272 \text{ eV}$$

3.35

The probability of a state at $E_1 = E_F + \Delta E$ being occupied is

$$f_1(E_1) = \frac{1}{1 + \exp\left(\frac{E_1 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{\Delta E}{kT}\right)}$$

The probability of a state at $E_2 = E_F - \Delta E$ being empty is

$$1 - f_2(E_2) = 1 - \frac{1}{1 + \exp\left(\frac{E_2 - E_F}{kT}\right)}$$

$$= 1 - \frac{1}{1 + \exp\left(\frac{-\Delta E}{kT}\right)} = \frac{\exp\left(\frac{-\Delta E}{kT}\right)}{1 + \exp\left(\frac{-\Delta E}{kT}\right)}$$

or

$$1 - f_2(E_2) = \frac{1}{1 + \exp\left(\frac{+\Delta E}{kT}\right)}$$

Hence, we have that

$$f_1(E_1) = 1 - f_2(E_2) \quad \text{Q.E.D.}$$

3.36

(a) At energy E_i , we want

$$\frac{1}{\exp\left(\frac{E_i - E_F}{kT}\right)} - \frac{1}{1 + \exp\left(\frac{E_i - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_i - E_F}{kT}\right)} = 0.01$$

This expression can be written as

$$\frac{1 + \exp\left(\frac{E_i - E_F}{kT}\right)}{\exp\left(\frac{E_i - E_F}{kT}\right)} - 1 = 0.01$$

$$\Rightarrow 1 = (0.01) \exp\left(\frac{E_i - E_F}{kT}\right)$$

or

$$E_i = E_F + kT \ln(100)$$

Then

$$E_i = E_F + 4.6kT$$

(b)

$$\text{At } E_i = E_F + 4.6kT,$$

$$f(E_i) = \frac{1}{1 + \exp\left(\frac{E_i - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{4.6kT}{kT}\right)}$$

which yields

$$f(E_i) = 0.00990 \approx 0.01$$

3.37

(a) $E_F = 6.25 \text{ eV}$, $T = 300 \text{ K}$, At $E = 6.50 \text{ eV}$

$$f(E) = \frac{1}{1 + \exp\left(\frac{6.50 - 6.25}{0.0259}\right)} = 6.43 \times 10^{-5}$$

or

$$6.43 \times 10^{-3} \%$$

(b)

$$T = 950 \text{ K} \Rightarrow kT = (0.0259) \left(\frac{950}{300} \right)$$

or

$$kT = 0.0820 \text{ eV}$$

Then

$$f(E) = \frac{1}{1 + \exp\left(\frac{6.50 - 6.25}{0.0820}\right)} = 0.0453$$

or

$$4.53 \%$$

$$(c) 1 - 0.01 = \frac{1}{1 + \exp\left(\frac{-0.30}{kT}\right)} = 0.99$$

Then

$$1 + \exp\left(\frac{-0.30}{kT}\right) = \frac{1}{0.99} = 1.0101$$

which can be written as

$$\exp\left(\frac{+0.30}{kT}\right) = \frac{1}{0.0101} = 99$$

Then

$$\frac{0.30}{kT} = \ln(99) \Rightarrow kT = \frac{0.30}{\ln(99)} = 0.06529$$

So

$$T = 756 \text{ K}$$

3.38

(a)

$$f(E) = \frac{1}{1 + \exp\left(\frac{7.15 - 7.0}{0.0259}\right)} = 0.00304$$

or

$$0.304 \%$$

(b)

$$\text{At } T = 1000 \text{ K} \Rightarrow kT = 0.08633 \text{ eV}$$

Then

$$f(E) = \frac{1}{1 + \exp\left(\frac{7.15 - 7.0}{0.08633}\right)} = 0.1496$$

$$\text{or } 14.96 \%$$

(c)

$$f(E) = \frac{1}{1 + \exp\left(\frac{6.85 - 7.0}{0.0259}\right)} = 0.997$$

or

$$99.7 \%$$

(d)

$$\text{At } E = E_F, f(E) = \frac{1}{2} \text{ for all temperatures.}$$

3.39

For $E = E_i$,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E_i - E_F}{kT}\right)} \approx \exp\left[\frac{-(E_i - E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) \Rightarrow f(E_1) = 9.3 \times 10^{-6}$$

For $E = E_2$, $E_F - E_2 = 1.12 - 0.3 = 0.82 \text{ eV}$

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{-0.82}{0.0259}\right)}$$

or

$$1 - f(E) \approx 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right)\right] \\ = \exp\left(\frac{-0.82}{0.0259}\right) \Rightarrow 1 - f(E) = 1.78 \times 10^{-14}$$

(b)

For $E_F - E_2 = 0.4 \Rightarrow E_1 - E_F = 0.72 \text{ eV}$

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

so

$$f(E) = 8.45 \times 10^{-13}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-0.4}{0.0259}\right)$$

so

$$1 - f(E) = 1.96 \times 10^{-7}$$

3.40

(a) At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left(\frac{-0.30}{0.0259}\right)$$

or

$$f(E) = 9.3 \times 10^{-6}$$

At $E = E_2$, then

$$E_F - E_2 = 1.42 - 0.3 = 1.12 \text{ eV},$$

So

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-1.12}{0.0259}\right)$$

or

$$1 - f(E) = 1.66 \times 10^{-19}$$

(b)

For $E_F - E_2 = 0.4 \Rightarrow E_1 - E_F = 1.02 \text{ eV}$,

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-1.02}{0.0259}\right)$$

or

$$f(E) = 7.88 \times 10^{-18}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$1 - f(E) = 1.96 \times 10^{-7}$$

3.41

$$f(E) = \left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-1}$$

so

$$\frac{df(E)}{dE} = (-1) \left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-2} \\ \times \left(\frac{1}{kT}\right) \exp\left(\frac{E - E_F}{kT}\right)$$

or

$$\frac{df(E)}{dE} = \frac{\frac{-1}{kT} \exp\left(\frac{E - E_F}{kT}\right)}{\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^2}$$

(a) $T = 0$, For

$$E < E_F \Rightarrow \exp(-\infty) = 0 \Rightarrow \frac{df}{dE} = 0$$

$$E > E_F \Rightarrow \exp(+\infty) = +\infty \Rightarrow \frac{df}{dE} = 0$$

$$\text{At } E = E_F \Rightarrow \frac{df}{dE} \rightarrow -\infty$$

3.42

(a) At $E = E_{midgap}$,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12 \text{ eV}$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{1.12}{2(0.0259)}\right]}$$

or

$$f(E) = 4.07 \times 10^{-10}$$

Ge: $E_g = 0.66 \text{ eV}$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{0.66}{2(0.0259)}\right]}$$

or

$$f(E) = 2.93 \times 10^{-6}$$

GaAs: $E_g = 1.42 \text{ eV}$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{1.42}{2(0.0259)}\right]}$$

or

$$f(E) = 1.24 \times 10^{-12}$$

(b)

Using results of Problem 3.35, the answers to part (b) are exactly the same as those given in part (a).

3.43

$$f(E) = 10^{-6} = \frac{1}{1 + \exp\left(\frac{0.55}{kT}\right)}$$

Then

$$1 + \exp\left(\frac{0.55}{kT}\right) = \frac{1}{10^{-6}} = 10^{+6} \Rightarrow$$

$$\exp\left(\frac{0.55}{kT}\right) \approx 10^{+6} \Rightarrow \left(\frac{0.55}{kT}\right) = \ln(10^6)$$

or

$$kT = \frac{0.55}{\ln(10^6)} \Rightarrow T = 461K$$

3.44

At $E = E_2$, $f(E_2) = 0.05$

So

$$0.05 = \frac{1}{1 + \exp\left(\frac{E_2 - E_F}{kT}\right)}$$

Then

$$\frac{E_2 - E_F}{kT} = \ln(19)$$

By symmetry, at $E = E_1$, $1 - f(E_1) = 0.05$,

So

$$\frac{E_F - E_1}{kT} = \ln(19)$$

Then

$$\frac{E_2 - E_1}{kT} = 2 \ln(19)$$

(a)

At $T = 300K$, $kT = 0.0259 \text{ eV}$

$$E_2 - E_1 = \Delta E = (0.0259)(2) \ln(19) \Rightarrow$$

$$\Delta E = 0.1525 \text{ eV}$$

(b)

At $T = 500K$, $kT = 0.04317 \text{ eV}$

$$E_2 - E_1 = \Delta E = (0.04317)(2) \ln(19) \Rightarrow$$

$$\Delta E = 0.254 \text{ eV}$$

Chapter 4

Problem Solutions

4.1

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

(a) Silicon

$T(^{\circ}K)$	kT (eV)	n_i (cm^{-3})
200	0.01727	7.68×10^4
400	0.03453	2.38×10^{12}
600	0.0518	9.74×10^{14}

(b) Germanium (c) GaAs

$T(^{\circ}K)$	n_i (cm^{-3})	n_i (cm^{-3})
200	2.16×10^{10}	1.38
400	8.60×10^{14}	3.28×10^9
600	3.82×10^{16}	5.72×10^{12}

4.2

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

$$(10^{12})^2 = (2.8 \times 10^{19})(1.04 \times 10^{19}) \left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.12}{kT}\right)$$

Then

$$\exp\left(\frac{1.12}{kT}\right) = (2.912 \times 10^{14}) \left(\frac{T}{300}\right)^3$$

By trial and error

$$T = 381K$$

4.3

Computer Plot

4.4

$$n_i^2 = N_{co} N_{vo} \cdot (T)^3 \cdot \exp\left(\frac{-E_g}{kT}\right)$$

So

$$\frac{n_i^2(T_2)}{n_i^2(T_1)} = \left(\frac{T_2}{T_1}\right)^3 \exp\left[-E_g \left(\frac{1}{kT_2} - \frac{1}{kT_1}\right)\right]$$

$$\text{At } T_2 = 300K \Rightarrow kT = 0.0259 \text{ eV}$$

$$\text{At } T_1 = 200K \Rightarrow kT = 0.01727 \text{ eV}$$

Then

$$\left(\frac{5.83 \times 10^7}{1.82 \times 10^2}\right)^2 = \left(\frac{300}{200}\right)^3 \exp\left[-E_g \left(\frac{1}{0.0259} - \frac{1}{0.01727}\right)\right]$$

or

$$1.026 \times 10^{11} = 3.375 \exp[(19.29)E_g]$$

which yields

$$E_g = 1.25 \text{ eV}$$

For $T = 300K$,

$$(5.83 \times 10^7)^2 = (N_{co} N_{vo})(300)^3 \exp\left(\frac{-1.25}{0.0259}\right)$$

or

$$N_{co} N_{vo} = 1.15 \times 10^{29}$$

4.5

$$\begin{aligned} \text{(a)} \quad g_c f_F &\propto \sqrt{E - E_c} \exp\left[\frac{-(E - E_F)}{kT}\right] \\ &\propto \sqrt{E - E_c} \exp\left[\frac{-(E - E_c)}{kT}\right] \exp\left[\frac{-(E_c - E_F)}{kT}\right] \end{aligned}$$

Let $E - E_c \equiv x$

Then

$$g_c f_F \propto \sqrt{x} \exp\left(\frac{-x}{kT}\right)$$

Now, to find the maximum value

$$\begin{aligned} \frac{d(g_c f_F)}{dx} &\propto \frac{1}{2} x^{-1/2} \exp\left(\frac{-x}{kT}\right) \\ &\quad - \frac{1}{kT} \cdot x^{1/2} \exp\left(\frac{-x}{kT}\right) = 0 \end{aligned}$$

This yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

Then the maximum value occurs at

$$E = E_c + \frac{kT}{2}$$

(b)

$$\begin{aligned} g_v (1 - f_F) &\propto \sqrt{E_v - E} \exp\left[\frac{-(E_F - E)}{kT}\right] \\ &\propto \sqrt{E_v - E} \exp\left[\frac{-(E_F - E_v)}{kT}\right] \exp\left[\frac{-(E_v - E)}{kT}\right] \end{aligned}$$

Let $E_v - E \equiv x$

Then

$$g_v(1 - f_v) \propto \sqrt{x} \exp\left(\frac{-x}{kT}\right)$$

To find the maximum value

$$\frac{d[g_v(1 - f_v)]}{dx} \propto \frac{d}{dx} \left[\sqrt{x} \exp\left(\frac{-x}{kT}\right) \right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2} = E_v - E$$

or

$$E = E_v - \frac{kT}{2}$$

4.6

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_c} \exp\left[\frac{-(E_1 - E_c)}{kT}\right]}{\sqrt{E_2 - E_c} \exp\left[\frac{-(E_2 - E_c)}{kT}\right]}$$

where

$$E_1 = E_c + 4kT \quad \text{and} \quad E_2 = E_c + \frac{kT}{2}$$

Then

$$\begin{aligned} \frac{n(E_1)}{n(E_2)} &= \frac{\sqrt{4kT}}{\sqrt{\frac{kT}{2}}} \exp\left[\frac{-(E_1 - E_2)}{kT}\right] \\ &= 2\sqrt{2} \exp\left[-\left(4 - \frac{1}{2}\right)\right] = 2\sqrt{2} \exp(-3.5) \end{aligned}$$

or

$$\frac{n(E_1)}{n(E_2)} = 0.0854$$

4.7

Computer Plot

4.8

$$\frac{n_i^2(A)}{n_i^2(B)} = \frac{\exp\left(\frac{-E_{gA}}{kT}\right)}{\exp\left(\frac{-E_{gB}}{kT}\right)} = \exp\left[\frac{-(E_{gA} - E_{gB})}{kT}\right]$$

or

$$\frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(E_{gA} - E_{gB})}{2kT}\right]$$

$$= \exp\left[\frac{-(1 - 1.2)}{2(0.0259)}\right] = \exp\left[\frac{+0.20}{2(0.0259)}\right]$$

or

$$\frac{n_i(A)}{n_i(B)} = 47.5$$

4.9

Computer Plot

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_o$, $m_n^* = 1.08m_o$

$$E_{Fi} - E_{midgap} = -0.0128 \text{ eV}$$

Germanium: $m_p^* = 0.37m_o$, $m_n^* = 0.55m_o$

$$E_{Fi} - E_{midgap} = -0.0077 \text{ eV}$$

Gallium Arsenide: $m_p^* = 0.48m_o$, $m_n^* = 0.067m_o$

$$E_{Fi} - E_{midgap} = +0.038 \text{ eV}$$

4.11

$$(a) \quad E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4} (0.0259) \ln\left(\frac{1.4}{0.62}\right) \Rightarrow$$

$$E_{Fi} - E_{midgap} = +0.0158 \text{ eV}$$

(b)

$$E_{Fi} - E_{midgap} = \frac{3}{4} (0.0259) \ln\left(\frac{0.25}{1.10}\right) \Rightarrow$$

$$E_{Fi} - E_{midgap} = -0.0288 \text{ eV}$$

4.12

$$E_{Fi} - E_{midgap} = \frac{1}{2} (kT) \ln\left(\frac{N_v}{N_c}\right)$$

$$= \frac{1}{2} (kT) \ln\left(\frac{1.04 \times 10^{19}}{2.8 \times 10^{19}}\right) = -0.495(kT)$$

$T(^{\circ}K)$	kT (eV)	$E_{Fi} - E_{midgap}$ (eV)
200	0.01727	-0.0085
400	0.03453	-0.017
600	0.0518	-0.0256

4.13

Computer Plot

4.14

Let $g_c(E) = K = \text{constant}$

Then,

$$\begin{aligned} n_o &= \int_{E_c}^{\infty} g_c(E) f_F(E) dE \\ &= K \int_{E_c}^{\infty} \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE \\ &\approx K \int_{E_c}^{\infty} \exp\left[\frac{-(E - E_F)}{kT}\right] dE \end{aligned}$$

Let

$$\eta = \frac{E - E_F}{kT} \quad \text{so that} \quad dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E_c - E_F) - (E_c - E)$$

so that

$$\exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left[\frac{-(E_c - E_F)}{kT}\right] \cdot \exp(-\eta)$$

The integral can then be written as

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_c - E_F)}{kT}\right] \int_0^{\infty} \exp(-\eta) d\eta$$

which becomes

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

4.15

Let $g_c(E) = C_1(E - E_c)$ for $E \geq E_c$

$$n_o = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= C_1 \int_{E_c}^{\infty} \frac{(E - E_c)}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE$$

or

$$n_o \approx C_1 \int_{E_c}^{\infty} (E - E_c) \exp\left[\frac{-(E - E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E - E_c}{kT} \quad \text{so that} \quad dE = kT \cdot d\eta$$

We can write

$$(E - E_F) = (E - E_c) + (E_c - E_F)$$

Then

$$\begin{aligned} n_o &= C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right] \\ &\quad \times \int_{E_c}^{\infty} (E - E_c) \exp\left[\frac{-(E - E_c)}{kT}\right] dE \end{aligned}$$

or

$$\begin{aligned} &= C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right] \\ &\quad \times \int_0^{\infty} (kT) \eta [\exp(-\eta)] (kT) d\eta \end{aligned}$$

We find that

$$\int_0^{\infty} \eta \exp(-\eta) d\eta = \frac{e^{-\eta}}{1} (-\eta - 1) \Big|_0^{\infty} = +1$$

So

$$n_o = C_1 (kT)^2 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

4.16

$$\text{We have } \frac{r_1}{a_o} = \epsilon_r \left(\frac{m_o}{m^*} \right)$$

For Germanium, $\epsilon_r = 16$, $m^* = 0.55m_o$

Then

$$r_1 = (16) \left(\frac{1}{0.55} \right) a_o = 29(0.53)$$

so

$$r_1 = 15.4 \text{ \AA}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_o} \right) \left(\frac{\epsilon_o}{\epsilon_s} \right)^2 (13.6) \text{ eV}$$

$$= \frac{0.55}{(16)^2} (13.6) \Rightarrow E = 0.029 \text{ eV}$$

4.17

We have $\frac{r_i}{a_o} = \epsilon_r \left(\frac{m_o}{m^*} \right)$

For GaAs, $\epsilon_r = 13.1$, $m^* = 0.067 m_o$

Then

$$r_i = (13.1) \left(\frac{1}{0.067} \right) (0.53)$$

or

$$r_i = 104 \text{ \AA}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_o} \right) \left(\frac{\epsilon_o}{\epsilon_s} \right)^2 (13.6) = \frac{0.067}{(13.1)^2} (13.6)$$

or

$$E = 0.0053 \text{ eV}$$

4.18

$$(a) \quad p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^4} \Rightarrow$$

$$p_o = 4.5 \times 10^{15} \text{ cm}^{-3}, \quad p_o > n_o \Rightarrow \text{p-type}$$

(b)

$$\begin{aligned} E_{Fi} - E_F &= kT \ln \left(\frac{p_o}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{4.5 \times 10^{15}}{1.5 \times 10^{10}} \right) \end{aligned}$$

or

$$E_{Fi} - E_F = 0.3266 \text{ eV}$$

4.19

$$\begin{aligned} p_o &= N_v \exp \left[\frac{-(E_F - E_v)}{kT} \right] \\ &= 1.04 \times 10^{19} \exp \left(\frac{-0.22}{0.0259} \right) \end{aligned}$$

so

$$p_o = 2.13 \times 10^{15} \text{ cm}^{-3}$$

Assuming

$$E_c - E_F = 1.12 - 0.22 = 0.90 \text{ eV}$$

Then

$$\begin{aligned} n_o &= N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right] \\ &= 2.8 \times 10^{18} \exp \left(\frac{-0.90}{0.0259} \right) \end{aligned}$$

or

$$n_o = 2.27 \times 10^4 \text{ cm}^{-3}$$

4.20

$$(a) \quad T = 400 \text{ K} \Rightarrow kT = 0.03453 \text{ eV}$$

$$N_c = 4.7 \times 10^{17} \left(\frac{400}{300} \right)^{3/2} = 7.24 \times 10^{17} \text{ cm}^{-3}$$

Then

$$\begin{aligned} n_o &= N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right] \\ &= 7.24 \times 10^{17} \exp \left(\frac{-0.25}{0.03453} \right) \end{aligned}$$

or

$$n_o = 5.19 \times 10^{14} \text{ cm}^{-3}$$

Also

$$N_v = 7 \times 10^{18} \left(\frac{400}{300} \right)^{3/2} = 1.08 \times 10^{19} \text{ cm}^{-3}$$

and

$$E_F - E_v = 1.42 - 0.25 = 1.17 \text{ eV}$$

Then

$$p_o = 1.08 \times 10^{19} \exp \left(\frac{-1.17}{0.03453} \right)$$

or

$$p_o = 2.08 \times 10^4 \text{ cm}^{-3}$$

(b)

$$\begin{aligned} E_c - E_F &= kT \ln \left(\frac{N_c}{n_o} \right) \\ &= (0.0259) \ln \left(\frac{7.24 \times 10^{17}}{5.19 \times 10^{14}} \right) \end{aligned}$$

$$\text{or } E_c - E_F = 0.176 \text{ eV}$$

Then

$$E_F - E_v = 1.42 - 0.176 = 1.244 \text{ eV}$$

and

$$p_o = (7 \times 10^{18}) \exp \left(\frac{-1.244}{0.0259} \right)$$

$$\text{or } p_o = 9.67 \times 10^{-3} \text{ cm}^{-3}$$

4.21

$$p_o = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

or

$$\begin{aligned} E_F - E_v &= kT \ln\left(\frac{N_v}{p_o}\right) \\ &= (0.0259) \ln\left(\frac{1.04 \times 10^{19}}{10^{15}}\right) = 0.24 \text{ eV} \end{aligned}$$

Then

$$E_c - E_F = 1.12 - 0.24 = 0.88 \text{ eV}$$

So

$$\begin{aligned} n_o &= N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] \\ &= 2.8 \times 10^{19} \exp\left(\frac{-0.88}{0.0259}\right) \end{aligned}$$

or

$$\underline{n_o = 4.9 \times 10^4 \text{ cm}^{-3}}$$

4.22

$$\begin{aligned} \text{(a)} \quad p_o &= n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) \\ &= 1.5 \times 10^{10} \exp\left(\frac{0.35}{0.0259}\right) \end{aligned}$$

or

$$\underline{p_o = 1.11 \times 10^{16} \text{ cm}^{-3}}$$

(b)

From Problem 4.1, $n_i(400K) = 2.38 \times 10^{12} \text{ cm}^{-3}$

$$kT = (0.0259) \left(\frac{400}{300}\right) = 0.03453 \text{ eV}$$

Then

$$\begin{aligned} E_{Fi} - E_F &= kT \ln\left(\frac{p_o}{n_i}\right) \\ &= (0.03453) \ln\left(\frac{1.11 \times 10^{16}}{2.38 \times 10^{12}}\right) \end{aligned}$$

or

$$\underline{E_{Fi} - E_F = 0.292 \text{ eV}}$$

(c)

From (a)

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{1.11 \times 10^{16}}$$

or

$$\underline{n_o = 2.03 \times 10^4 \text{ cm}^{-3}}$$

From (b)

$$n_o = \frac{n_i^2}{p_o} = \frac{(2.38 \times 10^{12})^2}{1.11 \times 10^{16}}$$

or

$$\underline{n_o = 5.10 \times 10^8 \text{ cm}^{-3}}$$

4.23

$$\begin{aligned} \text{(a)} \quad p_o &= n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) \\ &= (1.8 \times 10^6) \exp\left(\frac{0.35}{0.0259}\right) \end{aligned}$$

or

$$\underline{p_o = 1.33 \times 10^{12} \text{ cm}^{-3}}$$

(b) From Problem 4.1,

$$n_i(400K) = 3.28 \times 10^9 \text{ cm}^{-3}, \quad kT = 0.03453 \text{ eV}$$

Then

$$\begin{aligned} E_{Fi} - E_F &= kT \ln\left(\frac{p_o}{n_i}\right) \\ &= (0.03453) \ln\left(\frac{1.33 \times 10^{12}}{3.28 \times 10^9}\right) \end{aligned}$$

or

$$\underline{E_{Fi} - E_F = 0.207 \text{ eV}}$$

(c) From (a)

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{1.33 \times 10^{12}}$$

or

$$\underline{n_o = 2.44 \text{ cm}^{-3}}$$

From (b)

$$n_o = \frac{(3.28 \times 10^9)^2}{1.33 \times 10^{12}}$$

or

$$\underline{n_o = 8.09 \times 10^6 \text{ cm}^{-3}}$$

4.24For silicon, $T = 300K$, $E_F = E_v$

$$\eta' = \frac{E_v - E_F}{kT} = 0 \Rightarrow F_{1/2}(\eta') = 0.60$$

We can write

$$p_o = \frac{2}{\sqrt{\pi}} N_v F_{1/2}(\eta') = \frac{2}{\sqrt{\pi}} (1.04 \times 10^{19}) (0.60)$$

or

$$p_o = 7.04 \times 10^{18} \text{ cm}^{-3}$$

4.25

Silicon, $T = 300 \text{ K}$, $n_o = 5 \times 10^{19} \text{ cm}^{-3}$

We have

$$n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$

or

$$5 \times 10^{19} = \frac{2}{\sqrt{\pi}} (2.8 \times 10^{19}) F_{1/2}(\eta_F)$$

which gives

$$F_{1/2}(\eta_F) = 1.58$$

Then

$$\eta_F = 1.3 = \frac{E_F - E_c}{kT}$$

$$\text{or } E_F - E_c = (1.3)(0.0259) \Rightarrow$$

$$E_c - E_F = -0.034 \text{ eV}$$

4.26

For the electron concentration

$$n(E) = g_c(E) f_F(E)$$

The Boltzmann approximation applies so

$$n(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \exp\left[\frac{-(E - E_F)}{kT}\right]$$

or

$$n(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \exp\left[\frac{-(E_c - E_F)}{kT}\right] \times \sqrt{kT} \sqrt{\frac{E - E_c}{kT}} \exp\left[\frac{-(E - E_c)}{kT}\right]$$

Define

$$x = \frac{E - E_c}{kT}$$

Then

$$n(E) \rightarrow n(x) = K \sqrt{x} \exp(-x)$$

To find maximum $n(E) \rightarrow n(x)$, set

$$\frac{dn(x)}{dx} = 0 = K \left[\frac{1}{2} x^{-1/2} \exp(-x) + x^{1/2} (-1) \exp(-x) \right]$$

or

$$0 = K x^{-1/2} \exp(-x) \left[\frac{1}{2} - x \right]$$

which yields

$$x = \frac{1}{2} = \frac{E - E_c}{kT} \Rightarrow E = E_c + \frac{1}{2} kT$$

For the hole concentration

$$p(E) = g_v(E) [1 - f_F(E)]$$

From the text, using the Maxwell-Boltzmann approximation, we can write

$$p(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \exp\left[\frac{-(E_F - E)}{kT}\right]$$

or

$$p(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \exp\left[\frac{-(E_F - E_v)}{kT}\right] \times \sqrt{kT} \sqrt{\frac{E_v - E}{kT}} \exp\left[\frac{-(E_v - E)}{kT}\right]$$

$$\text{Define } x' = \frac{E_v - E}{kT}$$

Then

$$p(x') = K' \sqrt{x'} \exp(-x')$$

To find the maximum of $p(E) \rightarrow p(x')$, set

$$\frac{dp(x')}{dx'} = 0. \text{ Using the results from above, we}$$

find the maximum at

$$E = E_v - \frac{1}{2} kT$$

4.27

(a) Silicon: We have

$$n_o = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

We can write

$$E_c - E_F = (E_c - E_d) + (E_d - E_F)$$

For

$$E_c - E_d = 0.045 \text{ eV}, E_d - E_F = 3kT$$

$$n_o = (2.8 \times 10^{19}) \exp\left[\frac{-0.045}{0.0259} - 3\right] \\ = (2.8 \times 10^{19}) \exp(-4.737)$$

or

$$n_o = 2.45 \times 10^{17} \text{ cm}^{-3}$$

We also have

$$p_o = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

Again, we can write

$$E_F - E_v = (E_F - E_a) + (E_a - E_v)$$

For

$$E_F - E_a = 3kT, E_a - E_v = 0.045 \text{ eV}$$

Then

$$\begin{aligned} p_o &= (1.04 \times 10^{19}) \exp\left[-3 - \frac{0.045}{0.0259}\right] \\ &= (1.04 \times 10^{19}) \exp(-4.737) \end{aligned}$$

or

$$\underline{p_o = 9.12 \times 10^{16} \text{ cm}^{-3}}$$

(b)

GaAs: Assume $E_c - E_d = 0.0058 \text{ eV}$

Then

$$\begin{aligned} n_o &= (4.7 \times 10^{17}) \exp\left[\frac{-0.0058}{0.0259} - 3\right] \\ &= (4.7 \times 10^{17}) \exp(-3.224) \end{aligned}$$

or

$$\underline{n_o = 1.87 \times 10^{16} \text{ cm}^{-3}}$$

Assume $E_a - E_v = 0.0345 \text{ eV}$

Then

$$\begin{aligned} p_o &= (7 \times 10^{18}) \exp\left[\frac{-0.0345}{0.0259} - 3\right] \\ &= (7 \times 10^{18}) \exp(-4.332) \end{aligned}$$

or

$$\underline{p_o = 9.20 \times 10^{16} \text{ cm}^{-3}}$$

4.28

Computer Plot

4.29

(a) Ge:

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Then

$$p_o = \frac{10^{13}}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + (2.4 \times 10^{13})^2}$$

or

$$\underline{p_o = 2.95 \times 10^{13} \text{ cm}^{-3}}$$

and

$$\begin{aligned} n_o &= \frac{n_i^2}{p_o} = \frac{(2.4 \times 10^{13})^2}{2.95 \times 10^{13}} \Rightarrow \\ &= 1.95 \times 10^{13} \text{ cm}^{-3} \end{aligned}$$

(b)

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Then

$$n_o = \frac{5 \times 10^{15}}{2} + \sqrt{\left(\frac{5 \times 10^{15}}{2}\right)^2 + (2.4 \times 10^{13})^2}$$

or

$$\underline{n_o \approx 5 \times 10^{15} \text{ cm}^{-3}}$$

and

$$\begin{aligned} p_o &= \frac{n_i^2}{n_o} = \frac{(2.4 \times 10^{13})^2}{5 \times 10^{15}} \Rightarrow \\ &= 1.15 \times 10^{11} \text{ cm}^{-3} \end{aligned}$$

4.30

For the donor level

$$\begin{aligned} \frac{n_d}{N_d} &= \frac{1}{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)} \\ &= \frac{1}{1 + \frac{1}{2} \exp\left(\frac{0.20}{0.0259}\right)} \end{aligned}$$

or

$$\underline{\frac{n_d}{N_d} = 8.85 \times 10^{-4}}$$

And

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_c) + (E_c - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$\begin{aligned} f_F(E) &= \frac{1}{1 + \exp\left(1 + \frac{0.245}{0.0259}\right)} \Rightarrow \\ &= 2.87 \times 10^{-5} \end{aligned}$$

4.31

(a) $n_o = N_d = 2 \times 10^{15} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{15}} \Rightarrow$$

$$p_o = 1.125 \times 10^5 \text{ cm}^{-3}$$

(b)

$$p_o = N_a = 10^{16} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{10^{16}} \Rightarrow$$

$$n_o = 2.25 \times 10^4 \text{ cm}^{-3}$$

(c)

$$n_o = p_o = n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

(d)

$$T = 400 \text{ K} \Rightarrow kT = 0.03453 \text{ eV}$$

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19}) \left(\frac{400}{300} \right)^3 \exp \left(\frac{-1.12}{0.03453} \right)$$

or

$$n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2} \right)^2 + n_i^2}$$

$$= 5 \times 10^{13} + \sqrt{(5 \times 10^{13})^2 + (2.38 \times 10^{12})^2}$$

or

$$p_o = 1.0 \times 10^{14} \text{ cm}^{-3}$$

Also

$$n_o = \frac{n_i^2}{p_o} = \frac{(2.38 \times 10^{12})^2}{10^{14}} \Rightarrow$$

$$n_o = 5.66 \times 10^{10} \text{ cm}^{-3}$$

(e)

$$T = 500 \text{ K} \Rightarrow kT = 0.04317 \text{ eV}$$

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19}) \left(\frac{500}{300} \right)^3 \exp \left(\frac{-1.12}{0.04317} \right)$$

or

$$n_i = 8.54 \times 10^{13} \text{ cm}^{-3}$$

Now

$$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2} \right)^2 + n_i^2}$$

$$= 5 \times 10^{13} + \sqrt{(5 \times 10^{13})^2 + (8.54 \times 10^{13})^2}$$

or

$$n_o = 1.49 \times 10^{14} \text{ cm}^{-3}$$

Also

$$p_o = \frac{n_i^2}{n_o} = \frac{(8.54 \times 10^{13})^2}{1.49 \times 10^{14}} \Rightarrow$$

$$p_o = 4.89 \times 10^{13} \text{ cm}^{-3}$$

4.32

(a) $n_o = N_d = 2 \times 10^{15} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.8 \times 10^6)^2}{2 \times 10^{15}} \Rightarrow$$

$$p_o = 1.62 \times 10^{-3} \text{ cm}^{-3}$$

(b)

$$p_o = N_a = 10^{16} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{10^{16}} \Rightarrow$$

$$n_o = 3.24 \times 10^{-4} \text{ cm}^{-3}$$

(c)

$$n_o = p_o = n_i = 1.8 \times 10^6 \text{ cm}^{-3}$$

(d)

$$kT = 0.03453 \text{ eV}$$

$$n_i^2 = (4.7 \times 10^{17})(7 \times 10^{18}) \left(\frac{400}{300} \right)^3 \exp \left(\frac{-1.42}{0.03453} \right)$$

or

$$n_i = 3.28 \times 10^9 \text{ cm}^{-3}$$

Now

$$p_o = N_a = 10^{14} \text{ cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{(3.28 \times 10^9)^2}{10^{14}} \Rightarrow$$

$$n_o = 1.08 \times 10^5 \text{ cm}^{-3}$$

(e)

$$kT = 0.04317 \text{ eV}$$

$$n_i^2 = (4.7 \times 10^{17})(7 \times 10^{18}) \left(\frac{500}{300} \right)^3 \exp \left(\frac{-1.42}{0.04317} \right)$$

or

$$n_i = 2.81 \times 10^{11} \text{ cm}^{-3}$$

Now

$$n_o = N_d = 10^{14} \text{ cm}^{-3}$$

Also

$$p_o = \frac{n_i^2}{n_o} = \frac{(2.81 \times 10^{11})^2}{10^{14}} \Rightarrow$$

$$p_o = 7.90 \times 10^8 \text{ cm}^{-3}$$

4.33

(a) $N_a > N_d \Rightarrow$ p-type

(b) Si:

$$p_o = N_a - N_d = 2.5 \times 10^{13} - 1 \times 10^{13}$$

or

$$p_o = 1.5 \times 10^{13} \text{ cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^{13}} \Rightarrow$$

$$n_o = 1.5 \times 10^7 \text{ cm}^{-3}$$

Ge:

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5 \times 10^{13}}{2}\right) + \sqrt{\left(\frac{1.5 \times 10^{13}}{2}\right)^2 + (2.4 \times 10^{13})^2}$$

or

$$p_o = 3.26 \times 10^{13} \text{ cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(2.4 \times 10^{13})^2}{3.26 \times 10^{13}} \Rightarrow$$

$$n_o = 1.77 \times 10^{13} \text{ cm}^{-3}$$

GaAs:

$$p_o = 1.5 \times 10^{13} \text{ cm}^{-3}$$

And

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{1.5 \times 10^{13}} \Rightarrow$$

$$n_o = 0.216 \text{ cm}^{-3}$$

4.34

For $T = 450 \text{ K}$

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19}) \left(\frac{450}{300}\right)^3$$

$$\times \exp\left[\frac{-1.12}{(0.0259)(450/300)}\right]$$

or

$$n_i = 1.72 \times 10^{13} \text{ cm}^{-3}$$

(a)

$N_a > N_d \Rightarrow$ p-type

(b)

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \frac{1.5 \times 10^{15} - 8 \times 10^{14}}{2}$$

$$+ \sqrt{\left(\frac{1.5 \times 10^{15} - 8 \times 10^{14}}{2}\right)^2 + (1.72 \times 10^{13})^2}$$

or

$$p_o \approx N_a - N_d = 7 \times 10^{14} \text{ cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.72 \times 10^{13})^2}{7 \times 10^{14}} \Rightarrow$$

$$n_o = 4.23 \times 10^{11} \text{ cm}^{-3}$$

(c)

Total ionized impurity concentration

$$N_I = N_a + N_d = 1.5 \times 10^{15} + 8 \times 10^{14}$$

or

$$N_I = 2.3 \times 10^{15} \text{ cm}^{-3}$$

4.35

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^5} \Rightarrow$$

$$n_o = 1.125 \times 10^{15} \text{ cm}^{-3}$$

$$n_o > p_o \Rightarrow \text{n-type}$$

4.36

$$kT = (0.0259) \left(\frac{200}{300} \right) = 0.01727 \text{ eV}$$

$$n_i^2 = (4.7 \times 10^{17}) (7 \times 10^{18}) \left(\frac{200}{300} \right)^3 \times \exp \left[\frac{-1.42}{0.01727} \right]$$

or

$$n_i = 1.38 \text{ cm}^{-3}$$

Now

$$n_o p_o = n_i^2 \Rightarrow 5 p_o^2 = n_i^2$$

or

$$p_o = \frac{n_i}{\sqrt{5}} \Rightarrow p_o = 0.617 \text{ cm}^{-3}$$

And

$$n_o = 5 p_o \Rightarrow n_o = 3.09 \text{ cm}^{-3}$$

4.37

Computer Plot

4.38

Computer Plot

4.39

Computer Plot

4.40

n-type, so majority carrier = electrons

$$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2} \right)^2 + n_i^2} = 10^{13} + \sqrt{(10^{13})^2 + (2 \times 10^{13})^2}$$

or

$$n_o = 3.24 \times 10^{13} \text{ cm}^{-3}$$

Then

$$p_o = \frac{n_i^2}{n_o} = \frac{(2 \times 10^{13})^2}{3.24 \times 10^{13}} \Rightarrow p_o = 1.23 \times 10^{13} \text{ cm}^{-3}$$

4.41

(a) $N_a > N_d \Rightarrow$ n-type

$$n_o = N_d - N_a = 2 \times 10^{16} - 1 \times 10^{16}$$

or

$$n_o = 1 \times 10^{16} \text{ cm}^{-3}$$

Then

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{10^{16}} \Rightarrow$$

$$p_o = 2.25 \times 10^4 \text{ cm}^{-3}$$

(b)

$N_a > N_d \Rightarrow$ p-type

$$p_o = N_a - N_d = 3 \times 10^{16} - 2 \times 10^{15}$$

or

$$p_o = 2.8 \times 10^{16} \text{ cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{2.8 \times 10^{16}} \Rightarrow$$

$$n_o = 8.04 \times 10^3 \text{ cm}^{-3}$$

4.42

(a) $n_o < n_i \Rightarrow$ p-type

(b) $n_o = 4.5 \times 10^4 \text{ cm}^{-3} \Rightarrow$ electrons: minority carrier

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{4.5 \times 10^4} \Rightarrow$$

$$p_o = 5 \times 10^{15} \text{ cm}^{-3} \Rightarrow \text{holes: majority carrier}$$

(c)

$$p_o = N_a - N_d$$

so

$$5 \times 10^{15} = N_a - 5 \times 10^{15} \Rightarrow N_a = 10^{16} \text{ cm}^{-3}$$

Acceptor impurity concentration,

$$N_d = 5 \times 10^{15} \text{ cm}^{-3} \text{ Donor impurity concentration}$$

4.43

$$E_{Fi} - E_F = kT \ln \left(\frac{p_o}{n_i} \right)$$

For Germanium:

$T(^{\circ}K)$	$kT(\text{eV})$	$n_i(\text{cm}^{-3})$
200	0.01727	2.16×10^{10}
400	0.03454	8.6×10^{14}
600	0.0518	3.82×10^{16}

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2} \text{ and } N_a = 10^{15} \text{ cm}^{-3}$$

$T(^{\circ}K)$	$p_o(\text{cm}^{-3})$	$E_{Fi} - E_F (\text{eV})$
200	1.0×10^{15}	0.1855
400	1.49×10^{15}	0.01898
600	3.87×10^{16}	0.000674

4.44

$$E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$$

For Germanium,

$$T = 300 \text{ K} \Rightarrow n_i = 2.4 \times 10^{13} \text{ cm}^{-3}$$

$$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$N_d(\text{cm}^{-3})$	$n_o(\text{cm}^{-3})$	$E_F - E_{Fi} (\text{eV})$
10^{14}	1.05×10^{14}	0.0382
10^{16}	10^{16}	0.156
10^{18}	10^{18}	0.2755

4.45

$$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

Now

$$n_i = 0.05 n_o$$

so

$$n_o = 1.5 \times 10^{15} + \sqrt{(1.5 \times 10^{15})^2 + [(0.05)n_o]^2}$$

which yields

$$n_o = 3.0075 \times 10^{15} \text{ cm}^{-3}$$

Then

$$n_i = 1.504 \times 10^{14} \text{ cm}^{-3}$$

We have

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

so

$$(1.504 \times 10^{14})^2 = (4.7 \times 10^{17})(7 \times 10^{18}) \left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.42}{(0.0259)(T/300)}\right]$$

By trial and error

$$T \approx 762 \text{ K}$$

4.46

Computer Plot

4.47

Computer Plot

4.48

$$(a) \quad E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4} (0.0259) \ln(10) \Rightarrow$$

$$E_{Fi} - E_{midgap} = +0.0447 \text{ eV}$$

(b)

Impurity atoms to be added so

$$E_{midgap} - E_F = 0.45 \text{ eV}$$

(i) p-type, so add acceptor impurities

$$(ii) \quad E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947 \text{ eV}$$

$$p_o = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) = 10^5 \exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_o = N_a = 1.97 \times 10^{13} \text{ cm}^{-3}$$

4.49

$$n_o = N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

so

$$N_d = 5 \times 10^{15} + 2.8 \times 10^{19} \exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5 \times 10^{15} + 6.95 \times 10^{15}$$

so

$$N_d = 1.2 \times 10^{16} \text{ cm}^{-3}$$

4.50

$$(a) \quad p_o = N_a = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

or

$$\exp\left[\frac{+(E_F - E_V)}{kT}\right] = \frac{N_V}{N_a} = \frac{1.04 \times 10^{19}}{7 \times 10^{15}} = 1.49 \times 10^3$$

Then

$$E_F - E_V = (0.0259) \ln(1.49 \times 10^3)$$

or

$$\underline{E_F - E_V = 0.189 \text{ eV}}$$

(b)

$$\text{If } E_F - E_V = 0.1892 - 0.0259 = 0.1633 \text{ eV}$$

Then

$$N_a = 1.04 \times 10^{19} \exp\left(\frac{-0.1633}{0.0259}\right) = 1.90 \times 10^{16} \text{ cm}^{-3}$$

so that

$$\Delta N_a = 1.90 \times 10^{16} - 7 \times 10^{15} \Rightarrow$$

$$\underline{\Delta N_a = 1.2 \times 10^{16} \text{ cm}^{-3}}$$

Acceptor impurities to be added

4.51

$$(a) \quad E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = (0.0259) \ln\left(\frac{10^{15}}{1.5 \times 10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.2877 \text{ eV}}$$

(b)

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = 0.2877 \text{ eV}$$

(c)

$$\text{For (a), } \underline{n_o = N_d = 10^{15} \text{ cm}^{-3}}$$

For (b)

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{10^{15}} \Rightarrow \underline{n_o = 2.25 \times 10^5 \text{ cm}^{-3}}$$

4.52

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right) = (0.0259) \ln\left(\frac{p_o}{n_i}\right) = 0.45 \text{ eV}$$

Then

$$p_o = (1.8 \times 10^6) \exp\left(\frac{0.45}{0.0259}\right) \Rightarrow$$

$$\underline{p_o = 6.32 \times 10^{13} \text{ cm}^{-3}}$$

Now

$$p_o < N_a, \text{ Donors must be added}$$

$$p_o = N_a - N_d \Rightarrow N_d = N_a - p_o$$

so

$$N_d = 10^{15} - 6.32 \times 10^{13} \Rightarrow$$

$$\underline{N_d = 9.368 \times 10^{14} \text{ cm}^{-3}}$$

4.53

$$(a) \quad E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = (0.0259) \ln\left(\frac{2 \times 10^{15}}{1.5 \times 10^{10}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.3056 \text{ eV}}$$

(b)

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = (0.0259) \ln\left(\frac{10^{16}}{1.5 \times 10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3473 \text{ eV}}$$

(c)

$$\underline{E_F = E_{Fi}}$$

(d)

$$kT = 0.03453 \text{ eV}, n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right) = (0.03453) \ln\left(\frac{10^{14}}{2.38 \times 10^{12}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.1291 \text{ eV}}$$

(e)

$$kT = 0.04317 \text{ eV}, n_i = 8.54 \times 10^{13} \text{ cm}^{-3}$$

$$E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right) = (0.04317) \ln\left(\frac{1.49 \times 10^{14}}{8.54 \times 10^{13}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.0024 \text{ eV}}$$

4.54

$$\begin{aligned} \text{(a)} \quad E_F - E_{Fi} &= kT \ln \left(\frac{N_d}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{2 \times 10^{15}}{1.8 \times 10^6} \right) \Rightarrow \\ \underline{E_F - E_{Fi} &= 0.5395 \text{ eV}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad E_{Fi} - E_F &= kT \ln \left(\frac{N_a}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{10^{16}}{1.8 \times 10^6} \right) \Rightarrow \\ \underline{E_{Fi} - E_F &= 0.5811 \text{ eV}} \end{aligned}$$

$$\text{(c)} \quad \underline{E_F = E_{Fi} \quad E_{Fi} -}$$

$$\begin{aligned} \text{(d)} \quad kT &= 0.03453 \text{ eV}, n_i = 3.28 \times 10^9 \text{ cm}^{-3} \\ E_{Fi} - E_F &= (0.03453) \ln \left(\frac{10^{14}}{3.28 \times 10^9} \right) \Rightarrow \\ \underline{E_{Fi} - E_F &= 0.3565 \text{ eV}} \end{aligned}$$

(e)

$$\begin{aligned} kT &= 0.04317 \text{ eV}, n_i = 2.81 \times 10^{11} \text{ cm}^{-3} \\ E_F - E_{Fi} &= kT \ln \left(\frac{n_o}{n_i} \right) \\ &= (0.04317) \ln \left(\frac{10^{14}}{2.81 \times 10^{11}} \right) \Rightarrow \\ \underline{E_F - E_{Fi} &= 0.2536 \text{ eV}} \end{aligned}$$

4.55

p-type

$$\begin{aligned} E_{Fi} - E_F &= kT \ln \left(\frac{p_o}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}} \right) \Rightarrow \\ \underline{E_{Fi} - E_F &= 0.3294 \text{ eV}} \end{aligned}$$

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Chapter 5

Problem Solutions

5.1

(a) $n_o = 10^{16} \text{ cm}^{-3}$

and

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.8 \times 10^6)^2}{10^{16}} \Rightarrow$$

$$p_o = 3.24 \times 10^{-4} \text{ cm}^{-3}$$

(b)

$$J = e\mu_n n_o E$$

For GaAs doped at $N_d = 10^{16} \text{ cm}^{-3}$,

$$\mu_n \approx 7500 \text{ cm}^2 / V - s$$

Then

$$J = (1.6 \times 10^{-19})(7500)(10^{16})(10)$$

or

$$J = 120 \text{ A / cm}^2$$

(b) (i) $p_o = 10^{16} \text{ cm}^{-3}$, $n_o = 3.24 \times 10^{-4} \text{ cm}^{-3}$

(ii) For GaAs doped at $N_a = 10^{16} \text{ cm}^{-3}$,

$$\mu_p \approx 310 \text{ cm}^2 / V - s$$

$$J = e\mu_p p_o E$$

$$= (1.6 \times 10^{-19})(310)(10^{16})(10) \Rightarrow$$

$$J = 4.96 \text{ A / cm}^2$$

5.2

(a) $V = IR \Rightarrow 10 = (0.1R) \Rightarrow$

$$R = 100 \Omega$$

(b)

$$R = \frac{L}{\sigma A} \Rightarrow \sigma = \frac{L}{RA} \Rightarrow$$

$$\sigma = \frac{10^{-3}}{(100)(10^{-3})} \Rightarrow$$

$$\sigma = 0.01 (\Omega - \text{cm})^{-1}$$

(c)

$$\sigma \approx e\mu_n N_d$$

or

$$0.01 = (1.6 \times 10^{-19})(1350)N_d$$

or

$$N_d = 4.63 \times 10^{13} \text{ cm}^{-3}$$

(d)

$$\sigma \approx e\mu_p p_o \Rightarrow$$

$$0.01 = (1.6 \times 10^{-19})(480)p_o$$

or

$$p_o = 1.30 \times 10^{14} \text{ cm}^{-3} = N_a - N_d = N_a - 10^{15}$$

or

$$N_a = 1.13 \times 10^{15} \text{ cm}^{-3}$$

Note: For the doping concentrations obtained, the assumed mobility values are valid.

5.3

(a) $R = \frac{\rho L}{A} = \frac{L}{\sigma A}$ and $\sigma \approx e\mu_n N_d$

For $N_d = 5 \times 10^{16} \text{ cm}^{-3}$, $\mu_n \approx 1100 \text{ cm}^2 / V - s$

Then

$$R = \frac{0.1}{(1.6 \times 10^{-19})(1100)(5 \times 10^{16})(100)(10^{-4})^2}$$

or

$$R = 1.136 \times 10^4 \Omega$$

Then

$$I = \frac{V}{R} = \frac{5}{1.136 \times 10^4} \Rightarrow I = 0.44 \text{ mA}$$

(b)

In this case

$$R = 1.136 \times 10^3 \Omega$$

Then

$$I = \frac{V}{R} = \frac{5}{1.136 \times 10^3} \Rightarrow I = 4.4 \text{ mA}$$

(c)

$$E = \frac{V}{L}$$

For (a), $E = \frac{5}{0.10} = 50 \text{ V / cm}$

And

$$v_d = \mu_n E = (1100)(50) \text{ or } v_d = 5.5 \times 10^4 \text{ cm / s}$$

For (b), $E = \frac{V}{L} = \frac{5}{0.01} = 500 \text{ V / cm}$

And

$$v_d = (1100)(500) \Rightarrow v_d = 5.5 \times 10^5 \text{ cm / s}$$

5.4

(a) GaAs:

$$R = \frac{\rho L}{A} = \frac{V}{I} = \frac{10}{20} = 0.5 \text{ k}\Omega = \frac{L}{\sigma A}$$

Now

$$\sigma \approx e\mu_p N_a$$

For $N_a = 10^{17} \text{ cm}^{-3}$, $\mu_p \approx 210 \text{ cm}^2 / V - s$

Then

$$\sigma = (1.6 \times 10^{-19})(210)(10^{17}) = 3.36 (\Omega - \text{cm})^{-1}$$

So

$$L = R\sigma A = (500)(3.36)(85 \times 10^{-8})$$

or

$$L = 14.3 \text{ }\mu\text{m}$$

(b) Silicon

For $N_a = 10^{17} \text{ cm}^{-3}$, $\mu_p \approx 310 \text{ cm}^2 / V - s$

Then

$$\sigma = (1.6 \times 10^{-19})(310)(10^{17}) = 4.96 (\Omega - \text{cm})^{-1}$$

So

$$L = R\sigma A = (500)(4.96)(85 \times 10^{-8})$$

or

$$L = 21.1 \text{ }\mu\text{m}$$

5.5

(a) $E = \frac{V}{L} = \frac{3}{1} = 3 \text{ V} / \text{cm}$

$$v_d = \mu_n E \Rightarrow \mu_n = \frac{v_d}{E} = \frac{10^4}{3}$$

or

$$\mu_n = 3333 \text{ cm}^2 / V - s$$

(b)

$$v_d = \mu_n E = (800)(3)$$

or

$$v_d = 2.4 \times 10^3 \text{ cm} / s$$

5.6

(a) Silicon: For $E = 1 \text{ kV} / \text{cm}$,

$$v_d = 1.2 \times 10^6 \text{ cm} / s$$

Then

$$t_i = \frac{d}{v_d} = \frac{10^{-4}}{1.2 \times 10^6} \Rightarrow t_i = 8.33 \times 10^{-11} \text{ s}$$

For GaAs, $v_d = 7.5 \times 10^6 \text{ cm} / s$

Then

$$t_i = \frac{d}{v_d} = \frac{10^{-4}}{7.5 \times 10^6} \Rightarrow t_i = 1.33 \times 10^{-11} \text{ s}$$

(b)

Silicon: For $E = 50 \text{ kV} / \text{cm}$,

$$v_d = 9.5 \times 10^6 \text{ cm} / s$$

Then

$$t_i = \frac{d}{v_d} = \frac{10^{-4}}{9.5 \times 10^6} \Rightarrow t_i = 1.05 \times 10^{-11} \text{ s}$$

GaAs, $v_d = 7 \times 10^6 \text{ cm} / s$

Then

$$t_i = \frac{d}{v_d} = \frac{10^{-4}}{7 \times 10^6} \Rightarrow t_i = 1.43 \times 10^{-11} \text{ s}$$

5.7

For an intrinsic semiconductor,

$$\sigma_i = en_i(\mu_n + \mu_p)$$

(a)

For $N_d = N_a = 10^{14} \text{ cm}^{-3}$,

$$\mu_n = 1350 \text{ cm}^2 / V - s, \mu_p = 480 \text{ cm}^2 / V - s$$

Then

$$\sigma_i = (1.6 \times 10^{-19})(1.5 \times 10^{10})(1350 + 480)$$

or

$$\sigma_i = 4.39 \times 10^{-6} (\Omega - \text{cm})^{-1}$$

(b)

For $N_d = N_a = 10^{18} \text{ cm}^{-3}$,

$$\mu_n \approx 300 \text{ cm}^2 / V - s, \mu_p \approx 130 \text{ cm}^2 / V - s$$

Then

$$\sigma_i = (1.6 \times 10^{-19})(1.5 \times 10^{10})(300 + 130)$$

or

$$\sigma_i = 1.03 \times 10^{-6} (\Omega - \text{cm})^{-1}$$

5.8

(a) GaAs

$$\sigma \approx e\mu_p p_o \Rightarrow 5 = (1.6 \times 10^{-19})\mu_p p_o$$

From Figure 5.3, and using trial and error, we find

$$p_o \approx 1.3 \times 10^{17} \text{ cm}^{-3}, \mu_p \approx 240 \text{ cm}^2 / V - s$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{1.3 \times 10^{17}} \quad \text{or} \quad n_o = 2.49 \times 10^{-5} \text{ cm}^{-3}$$

(b) Silicon:

$$\sigma = \frac{1}{\rho} \approx e\mu_n n_o$$

or

$$n_o = \frac{1}{\rho e \mu_n} = \frac{1}{(8)(1.6 \times 10^{-19})(1350)}$$

or

$$n_o = 5.79 \times 10^{14} \text{ cm}^{-3}$$

and

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5.79 \times 10^{14}} \Rightarrow p_o = 3.89 \times 10^5 \text{ cm}^{-3}$$

Note: For the doping concentrations obtained in part (b), the assumed mobility values are valid.

5.9

$$\sigma_i = en_i(\mu_n + \mu_p)$$

Then

$$10^{-6} = (1.6 \times 10^{-19})(1000 + 600)n_i$$

or

$$n_i(300K) = 3.91 \times 10^9 \text{ cm}^{-3}$$

Now

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

or

$$E_g = kT \ln\left(\frac{N_c N_v}{n_i^2}\right) = (0.0259) \ln\left[\frac{(10^{19})^2}{(3.91 \times 10^9)^2}\right]$$

or

$$E_g = 1.122 \text{ eV}$$

Now

$$n_i^2(500K) = (10^{19})^2 \exp\left[\frac{-1.122}{(0.0259)(500/300)}\right]$$

$$= 5.15 \times 10^{26}$$

or

$$n_i(500K) = 2.27 \times 10^{13} \text{ cm}^{-3}$$

Then

$$\sigma_i = (1.6 \times 10^{-19})(2.27 \times 10^{13})(1000 + 600)$$

so

$$\sigma_i(500K) = 5.81 \times 10^{-3} (\Omega - \text{cm})^{-1}$$

5.10

(a) (i) Silicon: $\sigma_i = en_i(\mu_n + \mu_p)$

$$\sigma_i = (1.6 \times 10^{-19})(1.5 \times 10^{10})(1350 + 480)$$

or

$$\sigma_i = 4.39 \times 10^{-6} (\Omega - \text{cm})^{-1}$$

(ii) Ge:

$$\sigma_i = (1.6 \times 10^{-19})(2.4 \times 10^{13})(3900 + 1900)$$

or

$$\sigma_i = 2.23 \times 10^{-2} (\Omega - \text{cm})^{-1}$$

(iii) GaAs:

$$\sigma_i = (1.6 \times 10^{-19})(1.8 \times 10^6)(8500 + 400)$$

or

$$\sigma_i = 2.56 \times 10^{-9} (\Omega - \text{cm})^{-1}$$

$$(b) \quad R = \frac{L}{\sigma A}$$

$$(i) \quad R = \frac{200 \times 10^{-4}}{(4.39 \times 10^{-6})(85 \times 10^{-8})} \Rightarrow$$

$$R = 5.36 \times 10^9 \Omega$$

$$(ii) \quad R = \frac{200 \times 10^{-4}}{(2.23 \times 10^{-2})(85 \times 10^{-8})} \Rightarrow$$

$$R = 1.06 \times 10^6 \Omega$$

$$(iii) \quad R = \frac{200 \times 10^{-4}}{(2.56 \times 10^{-9})(85 \times 10^{-8})} \Rightarrow$$

$$R = 9.19 \times 10^{12} \Omega$$

5.11

$$(a) \quad \rho = 5 = \frac{1}{e\mu_n N_d}$$

Assume $\mu_n = 1350 \text{ cm}^2 / \text{V} - \text{s}$

Then

$$N_d = \frac{1}{(1.6 \times 10^{-19})(1350)(5)} \Rightarrow$$

$$N_d = 9.26 \times 10^{14} \text{ cm}^{-3}$$

(b)

$$T = 200K \rightarrow T = -75C$$

$$T = 400K \rightarrow T = 125C$$

From Figure 5.2,

$$T = -75C, N_d = 10^{15} \text{ cm}^{-3} \Rightarrow$$

$$\mu_n \approx 2500 \text{ cm}^2 / V - s$$

$$T = 125^\circ\text{C}, N_d = 10^{15} \text{ cm}^{-3} \Rightarrow$$

$$\mu_n \approx 700 \text{ cm}^2 / V - s$$

Assuming $n_o = N_d = 9.26 \times 10^{14} \text{ cm}^{-3}$ over the temperature range,
For $T = 200\text{K}$,

$$\rho = \frac{1}{(1.6 \times 10^{-19})(2500)(9.26 \times 10^{14})} \Rightarrow$$

$$\rho = 2.7 \text{ } \Omega - \text{cm}$$

For $T = 400\text{K}$,

$$\rho = \frac{1}{(1.6 \times 10^{-19})(700)(9.26 \times 10^{14})} \Rightarrow$$

$$\rho = 9.64 \text{ } \Omega - \text{cm}$$

5.12

Computer plot

5.13

(a) $E = 10 \text{ V} / \text{cm} \Rightarrow |v_d| = \mu_n E$

$$v_d = (1350)(10) \Rightarrow v_d = 1.35 \times 10^4 \text{ cm} / \text{s}$$

so

$$T = \frac{1}{2} m_n^* v_d^2 = \frac{1}{2} (1.08)(9.11 \times 10^{-31})(1.35 \times 10^4)^2$$

or

$$T = 8.97 \times 10^{-27} \text{ J} \Rightarrow 5.6 \times 10^{-8} \text{ eV}$$

(b)

$$E = 1 \text{ kV} / \text{cm},$$

$$v_d = (1350)(1000) = 1.35 \times 10^6 \text{ cm} / \text{s}$$

Then

$$T = \frac{1}{2} (1.08)(9.11 \times 10^{-31})(1.35 \times 10^6)^2$$

or

$$T = 8.97 \times 10^{-23} \text{ J} \Rightarrow 5.6 \times 10^{-4} \text{ eV}$$

5.14

(a) $n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$

$$= (2 \times 10^{19})(1 \times 10^{19}) \exp\left(\frac{-1.10}{0.0259}\right)$$

$$= 7.18 \times 10^{19} \Rightarrow n_i = 8.47 \times 10^9 \text{ cm}^{-3}$$

For $N_d = 10^{14} \text{ cm}^{-3} \gg n_i \Rightarrow n_o = 10^{14} \text{ cm}^{-3}$

Then

$$J = \sigma E = e \mu_n n_o E$$

$$= (1.6 \times 10^{-19})(1000)(10^{14})(100)$$

or

$$J = 1.60 \text{ A} / \text{cm}^2$$

(b)

A 5% increase is due to a 5% increase in electron concentration. So

$$n_o = 1.05 \times 10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

We can write

$$(1.05 \times 10^{14} - 5 \times 10^{13})^2 = (5 \times 10^{13})^2 + n_i^2$$

so

$$n_i^2 = 5.25 \times 10^{26}$$

$$= (2 \times 10^{19})(1 \times 10^{19}) \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

which yields

$$2.625 \times 10^{-12} = \left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.10}{kT}\right)$$

By trial and error, we find

$$T = 456\text{K}$$

5.15

(a) $\sigma = e \mu_n n_o + e \mu_p p_o$ and $n_o = \frac{n_i^2}{p_o}$

Then

$$\sigma = \frac{e \mu_n n_i^2}{p_o} + e \mu_p p_o$$

To find the minimum conductivity,

$$\frac{d\sigma}{dp_o} = 0 = \frac{(-1)e \mu_n n_i^2}{p_o^2} + e \mu_p \Rightarrow$$

which yields

$$p_o = n_i \left(\frac{\mu_n}{\mu_p}\right)^{1/2} \quad (\text{Answer to part (b)})$$

Substituting into the conductivity expression

$$\sigma = \sigma_{\min} = \frac{e \mu_n n_i^2}{\left[n_i \left(\frac{\mu_n}{\mu_p}\right)^{1/2}\right]} + e \mu_p \left[n_i \left(\frac{\mu_n}{\mu_p}\right)^{1/2}\right]$$

which simplifies to

$$\sigma_{\min} = 2en_i \sqrt{\mu_n \mu_p}$$

The intrinsic conductivity is defined as

$$\sigma_i = en_i(\mu_n + \mu_p) \Rightarrow en_i = \frac{\sigma_i}{\mu_n + \mu_p}$$

The minimum conductivity can then be written as

$$\sigma_{\min} = \frac{2\sigma_i \sqrt{\mu_n \mu_p}}{\mu_n + \mu_p}$$

5.16

$$\sigma = e\mu n_i = \frac{1}{\rho}$$

Now

$$\frac{1/\rho_1}{1/\rho_2} = \frac{1/50}{1/5} = \frac{5}{50} = 0.10 = \frac{\exp\left(\frac{-E_g}{2kT_1}\right)}{\exp\left(\frac{-E_g}{2kT_2}\right)}$$

or

$$0.10 = \exp\left[-E_g\left(\frac{1}{2kT_1} - \frac{1}{2kT_2}\right)\right]$$

$$kT_1 = 0.0259$$

$$kT_2 = (0.0259)\left(\frac{330}{300}\right) = 0.02849$$

$$\frac{1}{2kT_1} = 19.305, \quad \frac{1}{2kT_2} = 17.550$$

Then

$$E_g(19.305 - 17.550) = \ln(10)$$

or

$$E_g = 1.312 \text{ eV}$$

5.17

$$\begin{aligned} \frac{1}{\mu} &= \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3} \\ &= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{500} \\ &= 0.00050 + 0.000667 + 0.0020 \end{aligned}$$

or

$$\mu = 316 \text{ cm}^2 / \text{V} \cdot \text{s}$$

5.18

$$\mu_n = (1300)\left(\frac{T}{300}\right)^{-3/2} = (1300)\left(\frac{300}{T}\right)^{+3/2}$$

(a)

$$\text{At } T = 200\text{K}, \mu_n = (1300)(1.837) \Rightarrow$$

$$\mu_n = 2388 \text{ cm}^2 / \text{V} \cdot \text{s}$$

(b)

$$\text{At } T = 400\text{K}, \mu_n = (1300)(0.65) \Rightarrow$$

$$\mu_n = 844 \text{ cm}^2 / \text{V} \cdot \text{s}$$

5.19

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{250} + \frac{1}{500} = 0.006$$

Then

$$\mu = 167 \text{ cm}^2 / \text{V} \cdot \text{s}$$

5.20

Computer plot

5.21

Computer plot

5.22

$$J_n = eD_n \frac{dn}{dx} = eD_n \left(\frac{5 \times 10^{14} - n(0)}{0.01 - 0} \right)$$

$$0.19 = (1.6 \times 10^{-19})(25) \left(\frac{5 \times 10^{14} - n(0)}{0.010} \right)$$

Then

$$\frac{(0.19)(0.010)}{(1.6 \times 10^{-19})(25)} = 5 \times 10^{14} - n(0)$$

which yields

$$n(0) = 0.25 \times 10^{14} \text{ cm}^{-3}$$

5.23

$$\begin{aligned} J &= eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x} \\ &= (1.6 \times 10^{-19})(25) \left(\frac{10^{16} - 10^{15}}{0 - 0.10} \right) \end{aligned}$$

or

$$|J| = 0.36 \text{ A} / \text{cm}^2$$

For $A = 0.05 \text{ cm}^2$

$$I = AJ = (0.05)(0.36) \Rightarrow I = 18 \text{ mA}$$

5.24

$$J_n = eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x}$$

so

$$-400 = (1.6 \times 10^{-19}) D_n \left(\frac{10^{17} - 6 \times 10^{16}}{0 - 4 \times 10^{-4}} \right)$$

or

$$-400 = D_n (-16)$$

Then

$$D_n = 25 \text{ cm}^2 / \text{s}$$

5.25

$$\begin{aligned} J &= -eD_p \frac{dp}{dx} \\ &= -eD_p \frac{d}{dx} \left[10^{16} \left(1 - \frac{x}{L} \right) \right] = -eD_p \left(\frac{-10^{16}}{L} \right) \\ &= \frac{(1.6 \times 10^{-19})(10)(10^{16})}{10 \times 10^{-4}} \end{aligned}$$

or

$$J = 16 \text{ A / cm}^2 = \text{constant at all three points}$$

5.26

$$\begin{aligned} J_p(x=0) &= -eD_p \frac{dp}{dx} \Big|_{x=0} \\ &= -eD_p \frac{10^{15}}{(-L_p)} = \frac{(1.6 \times 10^{-19})(10)(10^{15})}{5 \times 10^{-4}} \end{aligned}$$

or

$$J_p(x=0) = 3.2 \text{ A / cm}^2$$

Now

$$\begin{aligned} J_n(x=0) &= eD_n \frac{dn}{dx} \Big|_{x=0} \\ &= eD_n \left(\frac{5 \times 10^{14}}{L_n} \right) = \frac{(1.6 \times 10^{-19})(25)(5 \times 10^{14})}{10^{-3}} \end{aligned}$$

or

$$J_n(x=0) = 2 \text{ A / cm}^2$$

Then

$$J = J_p(x=0) + J_n(x=0) = 3.2 + 2$$

or

$$J = 5.2 \text{ A / cm}^2$$

5.27

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dp} \left[10^{15} \exp\left(\frac{-x}{22.5}\right) \right]$$

Distance x is in μm , so $22.5 \rightarrow 22.5 \times 10^{-4} \text{ cm}$.

Then

$$\begin{aligned} J_p &= -eD_p (10^{15}) \left(\frac{-1}{22.5 \times 10^{-4}} \right) \exp\left(\frac{-x}{22.5}\right) \\ &= \frac{+(1.6 \times 10^{-19})(48)(10^{15})}{22.5 \times 10^{-4}} \exp\left(\frac{-x}{22.5}\right) \end{aligned}$$

or

$$J_p = 3.41 \exp\left(\frac{-x}{22.5}\right) \text{ A / cm}^2$$

5.28

$$J_n = e\mu_n n E + eD_n \frac{dn}{dx}$$

or

$$\begin{aligned} -40 &= (1.6 \times 10^{-19})(960) \left[10^{16} \exp\left(\frac{-x}{18}\right) \right] E \\ &+ (1.6 \times 10^{-19})(25)(10^{16}) \left(\frac{-1}{18 \times 10^{-4}} \right) \exp\left(\frac{-x}{18}\right) \end{aligned}$$

Then

$$-40 = 1.536 \left[\exp\left(\frac{-x}{18}\right) \right] E - 22.2 \exp\left(\frac{-x}{18}\right)$$

Then

$$E = \frac{22.2 \exp\left(\frac{-x}{18}\right) - 40}{1.536 \exp\left(\frac{-x}{18}\right)} \Rightarrow$$

$$E = 14.5 - 26 \exp\left(\frac{+x}{18}\right)$$

5.29

$$J_T = J_{n,dif} + J_{p,dif}$$

(a) $J_{p,dif} = -eD_p \frac{dp}{dx}$ and

$$p(x) = 10^{15} \exp\left(\frac{-x}{L}\right) \text{ where } L = 12 \mu\text{m}$$

so

$$J_{p,dif} = -eD_p (10^{15}) \left(\frac{-1}{L} \right) \exp\left(\frac{-x}{L}\right)$$

or

$$J_{p,dif} = \frac{(1.6 \times 10^{-19})(12)(10^{15})}{12 \times 10^{-4}} \exp\left(\frac{-x}{12}\right)$$

or

$$J_{p,dif} = +1.6 \exp\left(\frac{-x}{L}\right) \text{ A / cm}^2$$

(b)

$$J_{n,drf} = J_T - J_{p,dif}$$

or

$$J_{n,drf} = 4.8 - 1.6 \exp\left(\frac{-x}{L}\right)$$

(c)

$$J_{n,drf} = e\mu_n n_o E$$

Then

$$\begin{aligned} (1.6 \times 10^{-19})(1000)(10^{16})E \\ = 4.8 - 1.6 \exp\left(\frac{-x}{L}\right) \end{aligned}$$

which yields

$$E = \left[3 - 1 \times \exp\left(\frac{-x}{L}\right) \right] \text{ V / cm}$$

5.30

$$(a) \quad J = e\mu_n n(x)E + eD_n \frac{dn(x)}{dx}$$

Now $\mu_n = 8000 \text{ cm}^2 / \text{V} \cdot \text{s}$ so that

$$D_n = (0.0259)(8000) = 207 \text{ cm}^2 / \text{s}$$

Then

$$\begin{aligned} 100 = (1.6 \times 10^{-19})(8000)(12)n(x) \\ + (1.6 \times 10^{-19})(207) \frac{dn(x)}{dx} \end{aligned}$$

which yields

$$100 = 1.54 \times 10^{-14} n(x) + 3.31 \times 10^{-17} \frac{dn(x)}{dx}$$

Solution is of the form

$$n(x) = A + B \exp\left(\frac{-x}{d}\right)$$

so that

$$\frac{dn(x)}{dx} = \frac{-B}{d} \exp\left(\frac{-x}{d}\right)$$

Substituting into the differential equation, we have

$$\begin{aligned} 100 = (1.54 \times 10^{-14}) \left[A + B \exp\left(\frac{-x}{d}\right) \right] \\ - \frac{(3.31 \times 10^{-17})}{d} B \exp\left(\frac{-x}{d}\right) \end{aligned}$$

This equation is valid for all x , so

$$100 = 1.54 \times 10^{-14} A$$

or

$$A = 6.5 \times 10^{15}$$

Also

$$\begin{aligned} 1.54 \times 10^{-14} B \exp\left(\frac{-x}{d}\right) \\ - \frac{(3.31 \times 10^{-17})}{d} B \exp\left(\frac{-x}{d}\right) = 0 \end{aligned}$$

which yields

$$d = 2.15 \times 10^{-3} \text{ cm}$$

At $x = 0$, $e\mu_n n(0)E = 50$

so that

$$50 = (1.6 \times 10^{-19})(8000)(12)(A + B)$$

which yields $B = -3.24 \times 10^{15}$

Then

$$n(x) = 6.5 \times 10^{15} - 3.24 \times 10^{15} \exp\left(\frac{-x}{d}\right) \text{ cm}^{-3}$$

(b)

$$\text{At } x = 0, n(0) = 6.5 \times 10^{15} - 3.24 \times 10^{15}$$

Or

$$n(0) = 3.26 \times 10^{15} \text{ cm}^{-3}$$

At $x = 50 \text{ } \mu\text{m}$,

$$n(50) = 6.5 \times 10^{15} - 3.24 \times 10^{15} \exp\left(\frac{-50}{21.5}\right)$$

or

$$n(50) = 6.18 \times 10^{15} \text{ cm}^{-3}$$

(c)

$$\text{At } x = 50 \text{ } \mu\text{m}, J_{drf} = e\mu_n n(50)E$$

$$= (1.6 \times 10^{-19})(8000)(6.18 \times 10^{15})(12)$$

or

$$J_{drf}(x = 50) = 94.9 \text{ A / cm}^2$$

Then

$$J_{dif}(x = 50) = 100 - 94.9 \Rightarrow$$

$$J_{dif}(x = 50) = 5.1 \text{ A / cm}^2$$

5.31

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

(a) $E_F - E_{Fi} = ax + b$, $b = 0.4$

$$0.15 = a(10^{-3}) + 0.4 \text{ so that } a = -2.5 \times 10^2$$

Then

$$E_F - E_{Fi} = 0.4 - 2.5 \times 10^2 x$$

So

$$n = n_i \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{kT}\right)$$

(b)

$$\begin{aligned} J_n &= eD_n \frac{dn}{dx} \\ &= eD_n n_i \left(\frac{-2.5 \times 10^2}{kT}\right) \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{kT}\right) \end{aligned}$$

Assume $T = 300K$, $kT = 0.0259 \text{ eV}$, and

$$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

Then

$$\begin{aligned} J_n &= \frac{-(1.6 \times 10^{-19})(25)(1.5 \times 10^{10})(2.5 \times 10^2)}{(0.0259)} \\ &\quad \times \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{0.0259}\right) \end{aligned}$$

or

$$J_n = -5.79 \times 10^{-4} \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{0.0259}\right)$$

(i) At $x = 0$, $J_n = -2.95 \times 10^3 \text{ A/cm}^2$

(ii) At $x = 5 \mu\text{m}$, $J_n = -23.7 \text{ A/cm}^2$

5.32

(a) $J_n = e\mu_n nE + eD_n \frac{dn}{dx}$

$$\begin{aligned} -80 &= (1.6 \times 10^{-19})(1000)(10^{16})\left(1 - \frac{x}{L}\right)E \\ &\quad + (1.6 \times 10^{-19})(25.9)\left(\frac{-10^{16}}{L}\right) \end{aligned}$$

where $L = 10 \times 10^{-4} = 10^{-3} \text{ cm}$

We find

$$-80 = 1.6E - 1.6\left(\frac{x}{10^{-3}}\right)E - 41.44$$

or

$$80 = 1.6\left(\frac{x}{L} - 1\right)E + 41.44$$

Solving for the electric field, we find

$$E = \frac{38.56}{\left(\frac{x}{L} - 1\right)}$$

(b)

For $J_n = -20 \text{ A/cm}^2$

$$20 = 1.6\left(\frac{x}{L} - 1\right)E + 41.44$$

Then

$$E = \frac{21.44}{\left(1 - \frac{x}{L}\right)}$$

5.33

(a) $J = e\mu_n nE + eD_n \frac{dn}{dx}$

Let $n = N_d = N_{do} \exp(-\alpha x)$, $J = 0$

Then

$$0 = \mu_n N_{do} [\exp(-\alpha x)]E + D_n N_{do} (-\alpha) \exp(-\alpha x)$$

or

$$0 = E + \frac{D_n}{\mu_n} (-\alpha)$$

Since $\frac{D_n}{\mu_n} = \frac{kT}{e}$

So

$$E = \alpha \left(\frac{kT}{e}\right)$$

(b)

$$\begin{aligned} V &= - \int_0^{1/\alpha} E dx = -\alpha \left(\frac{kT}{e}\right) \int_0^{1/\alpha} dx \\ &= - \left[\alpha \left(\frac{kT}{e}\right) \right] \cdot \left(\frac{1}{\alpha}\right) \text{ so that } V = - \left(\frac{kT}{e}\right) \end{aligned}$$

5.34

From Example 5.5

$$E_x = \frac{(0.0259)(10^{19})}{(10^{16} - 10^{19}x)} = \frac{(0.0259)(10^3)}{(1 - 10^3x)}$$

$$V = - \int_0^{10^{-4}} E_x dx = -(0.0259)(10^3) \int_0^{10^{-4}} \frac{dx}{(1 - 10^3x)}$$

$$= -(0.0259)(10^3) \left(\frac{-1}{10^3} \right) \ln[1 - 10^3 x]_{0}^{10^{-4}}$$

$$= (0.0259) [\ln(1 - 0.1) - \ln(1)]$$

or

$$\underline{V = -2.73 \text{ mV}}$$

5.35

From Equation [5.40]

$$E_x = - \left(\frac{kT}{e} \right) \left(\frac{1}{N_d(x)} \right) \cdot \frac{dN_d(x)}{dx}$$

Now

$$1000 = -(0.0259) \left(\frac{1}{N_d(x)} \right) \cdot \frac{dN_d(x)}{dx}$$

or

$$\frac{dN_d(x)}{dx} + 3.86 \times 10^4 N_d(x) = 0$$

Solution is of the form

$$N_d(x) = A \exp(-\alpha x)$$

and

$$\frac{dN_d(x)}{dx} = -A\alpha \exp(-\alpha x)$$

Substituting into the differential equation

$$-A\alpha \exp(-\alpha x) + 3.86 \times 10^4 A \exp(-\alpha x) = 0$$

which yields

$$\underline{\alpha = 3.86 \times 10^4 \text{ cm}^{-1}}$$

At $x = 0$, the actual value of $N_d(0)$ is arbitrary.

5.36

$$(a) \quad J_n = J_{drf} + J_{dif} = 0$$

$$J_{dif} = eD_n \frac{dn}{dx} = eD_n \frac{dN_d(x)}{dx}$$

$$= \frac{eD_n}{(-L)} \cdot N_{do} \exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n \left(\frac{kT}{e} \right) = (6000)(0.0259) = 155.4 \text{ cm}^2 / s$$

Then

$$J_{dif} = \frac{-(1.6 \times 10^{-19})(155.4)(5 \times 10^{16})}{(0.1 \times 10^{-4})} \exp\left(\frac{-x}{L}\right)$$

or

$$\underline{J_{dif} = -1.24 \times 10^5 \exp\left(\frac{-x}{L}\right) \text{ A / cm}^2}$$

(b)

$$0 = J_{drf} + J_{dif}$$

Now

$$J_{drf} = e\mu_n nE$$

$$= (1.6 \times 10^{-19})(6000)(5 \times 10^{16}) \left[\exp\left(\frac{-x}{L}\right) \right] E$$

$$= 48E \exp\left(\frac{-x}{L}\right)$$

We have

$$J_{drf} = -J_{dif}$$

so

$$48E \exp\left(\frac{-x}{L}\right) = 1.24 \times 10^5 \exp\left(\frac{-x}{L}\right)$$

which yields

$$\underline{E = 2.58 \times 10^3 \text{ V / cm}}$$

5.37

Computer Plot

5.38

$$(a) \quad D = \mu \left(\frac{kT}{e} \right) = (925)(0.0259)$$

so

$$\underline{D = 23.96 \text{ cm}^2 / s}$$

(b)

$$\text{For } D = 28.3 \text{ cm}^2 / s$$

$$\mu = \frac{28.3}{0.0259} \Rightarrow \underline{\mu = 1093 \text{ cm}^2 / \text{V} \cdot s}$$

5.39

We have $L = 10^{-1} \text{ cm} = 10^{-3} \text{ m}$,

$$W = 10^{-2} \text{ cm} = 10^{-4} \text{ m}, \quad d = 10^{-3} \text{ cm} = 10^{-5} \text{ m}$$

(a)

We have

$$p = 10^{16} \text{ cm}^{-3} = 10^{22} \text{ m}^{-3}, \quad I_x = 1 \text{ mA} = 10^{-3} \text{ A}$$

Then

$$V_H = \frac{I_x B_z}{epd} = \frac{(10^{-3})(3.5 \times 10^{-2})}{(1.6 \times 10^{-19})(10^{22})(10^{-5})}$$

or

$$\underline{V_H = 2.19 \text{ mV}}$$

(b)

$$E_H = \frac{V_H}{W} = \frac{2.19 \times 10^{-3}}{10^{-2}}$$

or

$$E_H = 0.219 \text{ V / cm}$$

5.40

(a) $V_H = \frac{-I_x B_z}{ned} = \frac{-(250 \times 10^{-6})(5 \times 10^{-2})}{(5 \times 10^{21})(1.6 \times 10^{-19})(5 \times 10^{-5})}$

or

$$V_H = -0.3125 \text{ mV}$$

(b)

$$E_H = \frac{V_H}{W} = \frac{-0.3125 \times 10^{-3}}{2 \times 10^{-2}} \Rightarrow$$

$$E_H = -1.56 \times 10^{-2} \text{ V / cm}$$

(c)

$$\mu_n = \frac{I_x L}{enV_x W d}$$

$$= \frac{(250 \times 10^{-6})(10^{-3})}{(1.6 \times 10^{-19})(5 \times 10^{21})(0.1)(2 \times 10^{-4})(5 \times 10^{-5})}$$

or

$$\mu_n = 0.3125 \text{ m}^2 / \text{V} - \text{s} = 3125 \text{ cm}^2 / \text{V} - \text{s}$$

5.41

(a) $V_H = \text{positive} \Rightarrow \text{p-type}$

(b)

$$V_H = \frac{I_x B_z}{epd} \Rightarrow p = \frac{I_x B_z}{eV_H d}$$

$$= \frac{(0.75 \times 10^{-3})(10^{-1})}{(1.6 \times 10^{-19})(5.8 \times 10^{-3})(10^{-5})}$$

or

$$p = 8.08 \times 10^{21} \text{ m}^{-3} = 8.08 \times 10^{15} \text{ cm}^{-3}$$

(c)

$$\mu_p = \frac{I_x L}{epV_x W d}$$

$$= \frac{(0.75 \times 10^{-3})(10^{-3})}{(1.6 \times 10^{-19})(8.08 \times 10^{21})(15)(10^{-4})(10^{-5})}$$

or

$$\mu_p = 3.87 \times 10^{-2} \text{ m}^2 / \text{V} - \text{s} = 387 \text{ cm}^2 / \text{V} - \text{s}$$

5.42

(a) $V_H = E_H W = -(16.5 \times 10^{-3})(5 \times 10^{-2})$

or

$$V_H = -0.825 \text{ mV}$$

(b)

$V_H = \text{negative} \Rightarrow \text{n-type}$

(c)

$$n = \frac{-I_x B_z}{edV_H}$$

$$= \frac{-(0.5 \times 10^{-3})(6.5 \times 10^{-2})}{(1.6 \times 10^{-19})(5 \times 10^{-5})(-0.825 \times 10^{-3})}$$

or

$$n = 4.92 \times 10^{21} \text{ m}^{-3} = 4.92 \times 10^{15} \text{ cm}^{-3}$$

(d)

$$\mu_n = \frac{I_x L}{enV_x W d}$$

$$= \frac{(0.5 \times 10^{-3})(0.5 \times 10^{-2})}{(1.6 \times 10^{-19})(4.92 \times 10^{21})(1.25)(5 \times 10^{-4})(5 \times 10^{-5})}$$

or

$$\mu_n = 0.102 \text{ m}^2 / \text{V} - \text{s} = 1020 \text{ cm}^2 / \text{V} - \text{s}$$

5.43

(a) $V_H = \text{negative} \Rightarrow \text{n-type}$

(b) $n = \frac{-I_x B_z}{edV_H} \Rightarrow n = 8.68 \times 10^{14} \text{ cm}^{-3}$

(c) $\mu_n = \frac{I_x L}{enV_x W d} \Rightarrow \mu_n = 8182 \text{ cm}^2 / \text{V} - \text{s}$

(d) $\sigma = \frac{1}{\rho} = e\mu_n n = (1.6 \times 10^{-19})(8182)(8.68 \times 10^{14})$
or $\rho = 0.88 (\Omega - \text{cm})$

Chapter 6

Problem Solutions

6.1

n-type semiconductor, low-injection so that

$$R' = \frac{\delta p}{\tau_{pO}} = \frac{5 \times 10^{13}}{10^{-6}}$$

or

$$R' = 5 \times 10^{19} \text{ cm}^{-3} \text{ s}^{-1}$$

6.2

(a) $R_{nO} = \frac{n_o}{\tau_{nO}}$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{(10^{10})^2}{10^{16}} = 10^4 \text{ cm}^{-3}$$

Then

$$R_{nO} = \frac{10^4}{2 \times 10^{-7}} \Rightarrow R_{nO} = 5 \times 10^{10} \text{ cm}^{-3} \text{ s}^{-1}$$

(b)

$$R_n = \frac{\delta n}{\tau_{nO}} = \frac{10^{12}}{2 \times 10^{-7}} \text{ or } R_n = 5 \times 10^{18} \text{ cm}^{-3} \text{ s}^{-1}$$

so

$$\Delta R_n = R_n - R_{nO} = 5 \times 10^{18} - 5 \times 10^{10} \Rightarrow$$

$$\Delta R_n \approx 5 \times 10^{18} \text{ cm}^{-3} \text{ s}^{-1}$$

6.3

(a) Recombination rates are equal

$$\frac{n_o}{\tau_{nO}} = \frac{p_o}{\tau_{pO}}$$

$$n_o = N_d = 10^{16} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

So

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25 \times 10^4}{20 \times 10^{-6}}$$

or

$$\tau_{nO} = 8.89 \times 10^{-6} \text{ s}$$

(b) Generation Rate = Recombination Rate

So

$$G = \frac{2.25 \times 10^4}{20 \times 10^{-6}} \Rightarrow G = 1.125 \times 10^9 \text{ cm}^{-3} \text{ s}^{-1}$$

(c)

$$R = G = 1.125 \times 10^9 \text{ cm}^{-3} \text{ s}^{-1}$$

6.4

(a) $E = h\nu = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34})(3 \times 10^8)}{6300 \times 10^{-10}}$

or

$$E = 3.15 \times 10^{-19} \text{ J} \quad \text{This is the energy of 1 photon.}$$

Now

$$1 \text{ W} = 1 \text{ J/s} \Rightarrow 3.17 \times 10^{18} \text{ photons/s}$$

$$\text{Volume} = (1)(0.1) = 0.1 \text{ cm}^3$$

Then

$$g = \frac{3.17 \times 10^{18}}{0.1} \Rightarrow$$

$$g = 3.17 \times 10^{19} \text{ e-h pairs / cm}^3 \text{ s}$$

(b)

$$\delta n = \delta p = g\tau = (3.17 \times 10^{19})(10 \times 10^{-6})$$

or

$$\delta n = \delta p = 3.17 \times 10^{14} \text{ cm}^{-3}$$

6.5

We have

$$\frac{\partial p}{\partial t} = -\nabla \cdot F_p^+ + g_p - \frac{p}{\tau_p}$$

and

$$J_p = e\mu_p pE - eD_p \nabla p$$

The hole particle current density is

$$F_p^+ = \frac{J_p}{(+e)} = \mu_p pE - D_p \nabla p$$

Now

$$\nabla \cdot F_p^+ = \mu_p \nabla \cdot (pE) - D_p \nabla \cdot \nabla p$$

We can write

$$\nabla \cdot (pE) = E \cdot \nabla p + p \nabla \cdot E$$

and

$$\nabla \cdot \nabla p = \nabla^2 p$$

so

$$\nabla \cdot F_p^+ = \mu_p (E \cdot \nabla p + p \nabla \cdot E) - D_p \nabla^2 p$$

Then

$$\frac{\partial p}{\partial t} = -\mu_p (E \cdot \nabla p + p \nabla \cdot E) + D_p \nabla^2 p + g_p - \frac{p}{\tau_p}$$

We can then write

$$D_p \nabla^2 p - \mu_p (E \cdot \nabla p + p \nabla \cdot E) + g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

6.6

From Equation [6.18]

$$\frac{\partial p}{\partial t} = -\nabla \cdot F_p^+ + g_p - \frac{p}{\tau_p}$$

For steady-state, $\frac{\partial p}{\partial t} = 0$

Then

$$0 = -\nabla \cdot F_p^+ + g_p - R_p$$

and for a one-dimensional case,

$$\begin{aligned} \frac{dF_p^+}{dx} &= g_p - R_p = 10^{20} - 2 \times 10^{19} \Rightarrow \\ \frac{dF_p^+}{dx} &= 8 \times 10^{19} \text{ cm}^{-3} \text{ s}^{-1} \end{aligned}$$

6.7

From Equation [6.18],

$$0 = -\frac{dF_p^+}{dx} + 0 - 2 \times 10^{19}$$

or

$$\frac{dF_p^+}{dx} = -2 \times 10^{19} \text{ cm}^{-3} \text{ s}^{-1}$$

6.8

We have the continuity equations

$$\begin{aligned} (1) \quad D_p \nabla^2 (\delta p) - \mu_p [E \cdot \nabla (\delta p) + p \nabla \cdot E] \\ + g_p - \frac{p}{\tau_p} = \frac{\partial (\delta p)}{\partial t} \end{aligned}$$

and

$$\begin{aligned} (2) \quad D_n \nabla^2 (\delta n) + \mu_n [E \cdot \nabla (\delta n) + n \nabla \cdot E] \\ + g_n - \frac{n}{\tau_n} = \frac{\partial (\delta n)}{\partial t} \end{aligned}$$

By charge neutrality

$$\delta n = \delta p \equiv \delta n \Rightarrow \nabla (\delta n) = \nabla (\delta p)$$

$$\text{and } \nabla^2 (\delta n) = \nabla^2 (\delta p) \quad \text{and} \quad \frac{\partial (\delta n)}{\partial t} = \frac{\partial (\delta p)}{\partial t}$$

Also

$$g_n = g_p \equiv g, \quad \frac{p}{\tau_p} = \frac{n}{\tau_n} \equiv R$$

Then we can write

$$\begin{aligned} (1) \quad D_p \nabla^2 (\delta n) - \mu_p [E \cdot \nabla (\delta n) + p \nabla \cdot E] \\ + g - R = \frac{\partial (\delta n)}{\partial t} \end{aligned}$$

and

$$\begin{aligned} (2) \quad D_n \nabla^2 (\delta n) + \mu_n [E \cdot \nabla (\delta n) + n \nabla \cdot E] \\ + g - R = \frac{\partial (\delta n)}{\partial t} \end{aligned}$$

Multiply Equation (1) by $\mu_n n$ and Equation (2) by $\mu_p p$, and then add the two equations.

We find

$$\begin{aligned} (\mu_n n D_p + \mu_p p D_n) \nabla^2 (\delta n) \\ + \mu_n \mu_p (p - n) E \cdot \nabla (\delta n) \\ + (\mu_n n + \mu_p p)(g - R) = (\mu_n n + \mu_p p) \frac{\partial (\delta n)}{\partial t} \end{aligned}$$

Divide by $(\mu_n n + \mu_p p)$, then

$$\begin{aligned} \left(\frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \right) \nabla^2 (\delta n) \\ + \left[\frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} \right] E \cdot \nabla (\delta n) \\ + (g - R) = \frac{\partial (\delta n)}{\partial t} \end{aligned}$$

Define

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

$$\text{and } \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Then we have

$$\begin{aligned} D' \nabla^2 (\delta n) + \mu' E \cdot \nabla (\delta n) + (g - R) = \frac{\partial (\delta n)}{\partial t} \\ \text{Q.E.D.} \end{aligned}$$

6.9

For Ge: $T = 300K$, $n_i = 2.4 \times 10^{13} \text{ cm}^{-3}$

$$n = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{(10^{13})^2 + (2.4 \times 10^{13})^2}$$

or

$$n = 3.6 \times 10^{13} \text{ cm}^{-3}$$

Also

$$p = \frac{n_i^2}{n} = \frac{(2.4 \times 10^{13})^2}{3.6 \times 10^{13}} = 1.6 \times 10^{13} \text{ cm}^{-3}$$

We have

$$\mu_n = 3900, \mu_p = 1900$$

$$D_n = 101, D_p = 49.2$$

Now

$$D' = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

$$= \frac{(101)(49.2)(3.6 \times 10^{13} + 1.6 \times 10^{13})}{(101)(3.6 \times 10^{13}) + (49.2)(1.6 \times 10^{13})}$$

or

$$D' = 58.4 \text{ cm}^2 / \text{s}$$

Also

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

$$= \frac{(3900)(1900)(1.6 \times 10^{13} - 3.6 \times 10^{13})}{(3900)(3.6 \times 10^{13}) + (1900)(1.6 \times 10^{13})}$$

or

$$\mu' = -868 \text{ cm}^2 / \text{V} \cdot \text{s}$$

Now

$$\frac{n}{\tau_n} = \frac{p}{\tau_p} \Rightarrow \frac{3.6 \times 10^{13}}{\tau_n} = \frac{1.6 \times 10^{13}}{24 \mu\text{s}}$$

which yields

$$\tau_n = 54 \mu\text{s}$$

6.10

$$\sigma = e\mu_n n + e\mu_p p$$

With excess carriers present

$$n = n_o + \delta n \text{ and } p = p_o + \delta p$$

For an n-type semiconductor, we can write

$$\delta n = \delta p \equiv \delta p$$

Then

$$\sigma = e\mu_n (n_o + \delta p) + e\mu_p (p_o + \delta p)$$

or

$$\sigma = e\mu_n n_o + e\mu_p p_o + e(\mu_n + \mu_p)(\delta p)$$

so

$$\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$$

In steady-state, $\delta p = g' \tau$

So that

$$\Delta\sigma = e(\mu_n + \mu_p)(g' \tau_{pO})$$

6.11

n-type, so that minority carriers are holes.
Uniform generation throughout the sample
means we have

$$g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

Homogeneous solution is of the form

$$(\delta p)_H = A \exp\left(\frac{-t}{\tau_{pO}}\right)$$

and the particular solution is

$$(\delta p)_P = g' \tau_{pO}$$

so that the total solution is

$$(\delta p) = g' \tau_{pO} + A \exp\left(\frac{-t}{\tau_{pO}}\right)$$

At $t = 0$, $\delta p = 0$ so that

$$0 = g' \tau_{pO} + A \Rightarrow A = -g' \tau_{pO}$$

Then

$$\delta p = g' \tau_{pO} \left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right) \right]$$

The conductivity is

$$\sigma = e\mu_n n_o + e\mu_p p_o + e(\mu_n + \mu_p)(\delta p)$$

$$\approx e\mu_n n_o + e(\mu_n + \mu_p)(\delta p)$$

so

$$\sigma = (1.6 \times 10^{-19})(1000)(5 \times 10^{16})$$

$$+ (1.6 \times 10^{-19})(1000 + 420)(5 \times 10^{21})(10^{-7})$$

$$\times \left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right) \right]$$

Then

$$\sigma = 8 + 0.114 \left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right) \right]$$

where $\tau_{pO} = 10^{-7} \text{ s}$

6.12

n-type GaAs:

$$\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$$

In steady-state, $\delta p = g'\tau_{pO}$. Then

$$\Delta\sigma = (1.6 \times 10^{-19})(8500 + 400)(2 \times 10^{21})(2 \times 10^{-7})$$

or

$$\Delta\sigma = 0.57 (\Omega \cdot \text{cm})^{-1}$$

The steady-state excess carrier recombination rate

$$R' = g' = 2 \times 10^{21} \text{ cm}^{-3} \text{ s}^{-1}$$

6.13

For $t < 0$, steady-state, so

$$\delta p(0) = g'\tau_{pO} = (5 \times 10^{21})(3 \times 10^{-7}) \Rightarrow$$

$$\delta p(0) = 1.5 \times 10^{15} \text{ cm}^{-3}$$

Now

$$\sigma = e\mu_n n_O + e(\mu_n + \mu_p)(\delta p)$$

For $t \geq 0$, $\delta p = \delta p(0) \exp(-t/\tau_{pO})$

Then

$$\sigma = (1.6 \times 10^{-19})(1350)(5 \times 10^{16})$$

$$+ (1.6 \times 10^{-19})(1350 + 480)(1.5 \times 10^{15}) \exp(-t/\tau_{pO})$$

or

$$\sigma = 10.8 + 0.439 \exp(-t/\tau_{pO})$$

We have that

$$I = AJ = A\sigma E = \frac{A\sigma V}{L}$$

so

$$I = \frac{(10^{-4})(5)}{(0.10)} [10.8 + 0.439 \exp(-t/\tau_{pO})]$$

or

$$I = [54 + 2.20 \exp(-t/\tau_{pO})] \text{ mA}$$

where

$$\tau_{pO} = 3 \times 10^{-7} \text{ s}$$

6.14

(a) p-type GaAs,

$$D_n \nabla^2(\delta n) + \mu_n E \cdot \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform generation rate, so that

$$\nabla(\delta n) = \nabla^2(\delta n) = 0, \text{ then}$$

$$g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

The solution is of the form

$$\delta n = g'\tau_{nO}[1 - \exp(-t/\tau_{nO})]$$

Now

$$R'_n = \frac{\delta n}{\tau_{nO}} = g'[1 - \exp(-t/\tau_{nO})]$$

(b)

Maximum value at steady-state, $n_O = 10^{14} \text{ cm}^{-3}$

So

$$(\delta n)_O = g'\tau_{nO} \Rightarrow \tau_{nO} = \frac{(\delta n)_O}{g'} = \frac{10^{14}}{10^{20}}$$

or

$$\tau_{nO} = 10^{-6} \text{ s}$$

(c)

Determine t at which

$$(i) \quad \delta n = (0.75) \times 10^{14} \text{ cm}^{-3}$$

We have

$$0.75 \times 10^{14} = 10^{14} [1 - \exp(-t/\tau_{nO})]$$

which yields

$$t = \tau_{nO} \ln\left(\frac{1}{1-0.75}\right) \Rightarrow t = 1.39 \mu\text{s}$$

$$(ii) \quad \delta n = 0.5 \times 10^{14} \text{ cm}^{-3}$$

We find

$$t = \tau_{nO} \ln\left(\frac{1}{1-0.5}\right) \Rightarrow t = 0.693 \mu\text{s}$$

$$(iii) \quad \delta n = 0.25 \times 10^{14} \text{ cm}^{-3}$$

We find

$$t = \tau_{nO} \ln\left(\frac{1}{1-0.25}\right) \Rightarrow t = 0.288 \mu\text{s}$$

6.15

(a)

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Then

$$R_{pO} = \frac{P_O}{\tau_{pO}} \Rightarrow \tau_{pO} = \frac{P_O}{R_{pO}} = \frac{2.25 \times 10^4}{10^{11}}$$

or

$$\tau_{pO} = 2.25 \times 10^{-7} \text{ s}$$

Now

$$R'_p = \frac{\delta p}{\tau_{pO}} = \frac{10^{14}}{2.25 \times 10^{-7}} \Rightarrow$$

or

$$R'_p = 4.44 \times 10^{20} \text{ cm}^{-3} \text{ s}^{-1}$$

Recombination rate increases by the factor

$$\frac{R'_p}{R_{pO}} = \frac{4.44 \times 10^{20}}{10^{11}} \Rightarrow \frac{R'_p}{R_{pO}} = 4.44 \times 10^9$$

(b)

From part (a), $\tau_{pO} = 2.25 \times 10^{-7} \text{ s}$

6.16

Silicon, n-type. For $0 \leq t \leq 10^{-7} \text{ s}$

$$\begin{aligned} \delta p &= g' \tau_{pO} [1 - \exp(-t/\tau_{pO})] \\ &= (2 \times 10^{20})(10^{-7}) [1 - \exp(-t/\tau_{pO})] \end{aligned}$$

or

$$\delta p = 2 \times 10^{13} [1 - \exp(-t/\tau_{pO})]$$

At $t = 10^{-7} \text{ s}$,

$$\delta p(10^{-7}) = 2 \times 10^{13} [1 - \exp(-1)]$$

or

$$\delta p(10^{-7}) = 1.26 \times 10^{13} \text{ cm}^{-3}$$

For $t > 10^{-7} \text{ s}$,

$$\delta p = (1.26 \times 10^{13}) \exp\left[\frac{-(t - 10^{-7})}{\tau_{pO}}\right]$$

where

$$\tau_{pO} = 10^{-7} \text{ s}$$

6.17

(a) For $0 < t < 2 \times 10^{-6} \text{ s}$

$$\begin{aligned} \delta n &= g' \tau_{nO} [1 - \exp(-t/\tau_{nO})] \\ &= (10^{20})(10^{-6}) [1 - \exp(-t/\tau_{nO})] \end{aligned}$$

or

$$\delta n = 10^{14} [1 - \exp(-t/\tau_{nO})]$$

where $\tau_{nO} = 10^{-6} \text{ s}$

At $t = 2 \times 10^{-6} \text{ s}$

$$\delta n(2 \mu\text{s}) = (10^{14}) [1 - \exp(-2/1)]$$

or

$$\delta n(2 \mu\text{s}) = 0.865 \times 10^{14} \text{ cm}^{-3}$$

For $t > 2 \times 10^{-6} \text{ s}$

$$\delta n = 0.865 \times 10^{14} \exp\left[\frac{-(t - 2 \times 10^{-6})}{\tau_{nO}}\right]$$

(b) (i) At $t = 0$, $\delta n = 0$

(ii) At $t = 2 \times 10^{-6} \text{ s}$, $\delta n = 0.865 \times 10^{14} \text{ cm}^{-3}$

(iii) At $t \rightarrow \infty$, $\delta n = 0$

6.18

p-type, minority carriers are electrons

In steady-state, $\frac{\partial(\delta n)}{\partial t} = 0$, then

(a)

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A \exp(-x/L_n) + B \exp(+x/L_n)$$

But $\delta n = 0$ as $x \rightarrow \infty$ so that $B \equiv 0$.

At $x = 0$, $\delta n = 10^{13} \text{ cm}^{-3}$

Then

$$\delta n = 10^{13} \exp(-x/L_n)$$

Now

$$L_n = \sqrt{D_n \tau_{nO}}, \text{ where } D_n = \mu_n \left(\frac{kT}{e} \right)$$

or

$$D_n = (0.0259)(1200) = 31.1 \text{ cm}^2 / \text{s}$$

Then

$$L_n = \sqrt{(31.1)(5 \times 10^{-7})} \Rightarrow$$

or

$$L_n = 39.4 \mu\text{m}$$

(b)

$$J_n = eD_n \frac{d(\delta n)}{dx} = \frac{eD_n (10^{13})}{(-L_n)} \exp(-x/L_n)$$

$$= \frac{-(1.6 \times 10^{-19})(31.1)(10^{13})}{39.4 \times 10^{-4}} \exp(-x/L_n)$$

or

$$J_n = -12.6 \exp(-x/L_n) \text{ mA/cm}^2$$

6.19(a) p-type silicon, $p_{p0} = 10^{14} \text{ cm}^{-3}$ and

$$n_{p0} = \frac{n_i^2}{p_{p0}} = \frac{(1.5 \times 10^{10})^2}{10^{14}} = 2.25 \times 10^6 \text{ cm}^{-3}$$

(b) Excess minority carrier concentration

$$\delta n = n_p - n_{p0}$$

At $x = 0$, $n_p = 0$ so that

$$\delta n(0) = 0 - n_{p0} = -2.25 \times 10^6 \text{ cm}^{-3}$$

(c) For the one-dimensional case,

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{n0}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \text{ where } L_n^2 = D_n \tau_{n0}$$

The general solution is of the form

$$\delta n = A \exp(-x/L_n) + B \exp(+x/L_n)$$

For $x \rightarrow \infty$, δn remains finite, so that $B = 0$.

Then the solution is

$$\delta n = -n_{p0} \exp(-x/L_n)$$

6.20

p-type so electrons are the minority carriers

$$D_n \nabla^2(\delta n) + \mu_n E \cdot \nabla(\delta n) + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$

For steady state, $\frac{\partial(\delta n)}{\partial t} = 0$ and for $x > 0$, $g' = 0$, $E = 0$, so we have

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{n0}} = 0 \text{ or } \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

where $L_n^2 = D_n \tau_{n0}$

The solution is of the form

$$\delta n = A \exp(-x/L_n) + B \exp(+x/L_n)$$

The excess concentration δn must remain finite, so that $B = 0$. At $x = 0$, $\delta n(0) = 10^{15} \text{ cm}^{-3}$, so the solution is

$$\delta n = 10^{15} \exp(-x/L_n)$$

We have that $\mu_n = 1050 \text{ cm}^2 / V - s$, then

$$D_n = \mu_n \left(\frac{kT}{e} \right) = (1050)(0.0259) = 27.2 \text{ cm}^2 / s$$

Then

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{(27.2)(8 \times 10^{-7})} \Rightarrow$$

$$L_n = 46.6 \text{ } \mu\text{m}$$

(a)

Electron diffusion current density at $x = 0$

$$J_n = eD_n \frac{d(\delta n)}{dx} \Big|_{x=0}$$

$$= eD_n \frac{d}{dx} [10^{15} \exp(-x/L_n)] \Big|_{x=0}$$

$$= \frac{-eD_n (10^{15})}{L_n} = \frac{-(1.6 \times 10^{-19})(27.2)(10^{15})}{46.6 \times 10^{-4}}$$

or

$$J_n = -0.934 \text{ A/cm}^2$$

Since $\delta p = \delta n$, excess holes diffuse at the same rate as excess electrons, then

$$J_p(x=0) = +0.934 \text{ A/cm}^2$$

(b)

At $x = L_n$,

$$J_n = eD_n \frac{d(\delta n)}{dx} \Big|_{x=L_n} = \frac{eD_n (10^{15})}{(-L_n)} \exp(-1)$$

$$= \frac{-(1.6 \times 10^{-19})(27.2)(10^{15})}{46.6 \times 10^{-4}} \exp(-1)$$

or

$$J_n = -0.344 \text{ A/cm}^2$$

Then

$$J_p = +0.344 \text{ A/cm}^2$$

6.21

n-type, so we have

$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p E_o \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{p0}} = 0$$

Assume the solution is of the form

$$\delta p = A \exp(sx)$$

Then

$$\frac{d(\delta p)}{dx} = A s \exp(sx), \quad \frac{d^2(\delta p)}{dx^2} = A s^2 \exp(sx)$$

Substituting into the differential equation

$$D_p A s^2 \exp(sx) - \mu_p E_o A s \exp(sx) - \frac{A \exp(sx)}{\tau_{pO}} = 0$$

or

$$D_p s^2 - \mu_p E_o s - \frac{1}{\tau_{pO}} = 0$$

Dividing by D_p

$$s^2 - \frac{\mu_p}{D_p} E_o s - \frac{1}{L_p^2} = 0$$

The solution for s is

$$s = \frac{1}{2} \left[\frac{\mu_p}{D_p} E_o \pm \sqrt{\left(\frac{\mu_p}{D_p} E_o \right)^2 + \frac{4}{L_p^2}} \right]$$

This can be rewritten as

$$s = \frac{1}{L_p} \left[\frac{\mu_p L_p E_o}{2 D_p} \pm \sqrt{\left(\frac{\mu_p L_p E_o}{2 D_p} \right)^2 + 1} \right]$$

We may define

$$\beta \equiv \frac{\mu_p L_p E_o}{2 D_p}$$

Then

$$s = \frac{1}{L_p} \left[\beta \pm \sqrt{1 + \beta^2} \right]$$

In order that $\delta p = 0$ for $x > 0$, use the minus sign for $x > 0$ and the plus sign for $x < 0$.

Then the solution is

$$\delta p(x) = A \exp(s_- x) \text{ for } x > 0$$

$$\delta p(x) = A \exp(s_+ x) \text{ for } x < 0$$

where

$$s_{\pm} = \frac{1}{L_p} \left[\beta \pm \sqrt{1 + \beta^2} \right]$$

6.22

Computer Plot

6.23

(a) From Equation [6.55],

$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n E_o \frac{d(\delta n)}{dx} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} + \frac{\mu_n}{D_n} E_o \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

We have that

$$\frac{D_n}{\mu_n} = \left(\frac{kT}{e} \right) \text{ so we can define}$$

$$\frac{\mu_n}{D_n} E_o = \frac{E_o}{(kT/e)} \equiv \frac{1}{L'}$$

Then we can write

$$\frac{d^2(\delta n)}{dx^2} + \frac{1}{L'} \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

Solution will be of the form

$$\delta n = \delta n(0) \exp(-\alpha x) \text{ where } \alpha > 0$$

Then

$$\frac{d(\delta n)}{dx} = -\alpha(\delta n) \text{ and } \frac{d^2(\delta n)}{dx^2} = \alpha^2(\delta n)$$

Substituting into the differential equation, we have

$$\alpha^2(\delta n) + \frac{1}{L'} [-\alpha(\delta n)] - \frac{\delta n}{L_n^2} = 0$$

or

$$\alpha^2 - \frac{\alpha}{L'} - \frac{1}{L_n^2} = 0$$

which yields

$$\alpha = \frac{1}{L_n} \left\{ \frac{L_n}{2L'} + \sqrt{\left(\frac{L_n}{2L'} \right)^2 + 1} \right\}$$

Note that if $E_o = 0$, $L' \rightarrow \infty$, then $\alpha = \frac{1}{L_n}$

(b)

$$L_n = \sqrt{D_n \tau_{nO}} \text{ where } D_n = \mu_n \left(\frac{dT}{e} \right)$$

or

$$D_n = (1200)(0.0259) = 31.1 \text{ cm}^2 / \text{s}$$

Then

$$L_n = \sqrt{(31.1)(5 \times 10^{-7})} = 39.4 \text{ } \mu\text{m}$$

For $E_o = 12 \text{ V} / \text{cm}$, then

$$L' = \frac{(kT/e)}{E_o} = \frac{0.0259}{12} = 21.6 \times 10^{-4} \text{ cm}$$

Then

$$\alpha = 5.75 \times 10^2 \text{ cm}^{-1}$$

(c)

Force on the electrons due to the electric field is in the negative x-direction. Therefore, the effective diffusion of the electrons is reduced and the concentration drops off faster with the applied electric field.

6.24

p-type so the minority carriers are electrons, then

$$D_n \nabla^2(\delta n) + \mu_n E \cdot \nabla(\delta n) + g' - \frac{\delta n}{\tau_{no}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform illumination means that

$\nabla(\delta n) = \nabla^2(\delta n) = 0$. For $\tau_{no} = \infty$, we are left with

$$\frac{d(\delta n)}{dt} = g' \text{ which gives } \delta n = g't + C_1$$

For $t < 0$, $\delta n = 0$ which means that $C_1 = 0$.

Then

$$\delta n = G'_o t \text{ for } 0 \leq t \leq T$$

For $t > T$, $g' = 0$ so we have $\frac{d(\delta n)}{dt} = 0$

Or

$$\delta n = G'_o T \text{ (No recombination)}$$

6.25

n-type so minority carriers are holes, then

$$D_p \nabla^2(\delta p) - \mu_p E \cdot \nabla(\delta p) + g' - \frac{\delta p}{\tau_{po}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{po} = \infty$, $E = 0$, $\frac{\partial(\delta p)}{\partial t} = 0$ (steady state). Then we have

$$D_p \frac{d^2(\delta p)}{dx^2} + g' = 0 \text{ or } \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G'_o = \text{constant}$. Then

$$\frac{d(\delta p)}{dx} = -\frac{G'_o}{D_p} x + C_1 \text{ and}$$

$$\delta p = -\frac{G'_o}{2D_p} x^2 + C_1 x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_3 \text{ and}$$

$$\delta p = C_3 x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0, \frac{d(\delta p)}{dx} = C_5, \text{ and}$$

$$\delta p = C_5 x + C_6$$

The boundary conditions are

(1) $\delta p = 0$ at $x = +3L$; (2) $\delta p = 0$ at $x = -3L$;

(3) δp continuous at $x = +L$; (4) δp continuous at $x = -L$; The flux must be continuous so that

(5) $\frac{d(\delta p)}{dx}$ continuous at $x = +L$; (6) $\frac{d(\delta p)}{dx}$ continuous at $x = -L$.

Applying these boundary conditions, we find

$$\delta p = \frac{G'_o}{2D_p} (5L^2 - x^2) \text{ for } -L < x < +L$$

$$\delta p = \frac{G'_o L}{D_p} (3L - x) \text{ for } L < x < 3L$$

$$\delta p = \frac{G'_o L}{D_p} (3L + x) \text{ for } -3L < x < -L$$

6.26

$$\mu_p = \frac{d}{E_o t} = \frac{0.75}{\left(\frac{2.5}{1}\right)(160 \times 10^{-6})} = 1875 \text{ cm}^2 / \text{V} \cdot \text{s}$$

Then

$$D_p = \frac{(\mu_p E_o)^2 (\Delta t)^2}{16 t_o} = \frac{\left[(1875) \left(\frac{2.5}{1} \right) \right]^2 (75.5 \times 10^{-6})^2}{16 (160 \times 10^{-6})}$$

which gives

$$D_p = 48.9 \text{ cm}^2 / \text{s}$$

From the Einstein relation,

$$\frac{D_p}{\mu_p} = \frac{kT}{e} = \frac{48.9}{1875} = 0.02608 \text{ V}$$

6.27

Assume that $f(x, t) = (4\pi Dt)^{-1/2} \exp\left(\frac{-x^2}{4Dt}\right)$

is the solution to the differential equation

$$D\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f}{\partial t}$$

To prove: we can write

$$\frac{\partial f}{\partial x} = (4\pi Dt)^{-1/2} \left(\frac{-2x}{4Dt}\right) \exp\left(\frac{-x^2}{4Dt}\right)$$

and

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= (4\pi Dt)^{-1/2} \left(\frac{-2x}{4Dt}\right)^2 \exp\left(\frac{-x^2}{4Dt}\right) \\ &\quad + (4\pi Dt)^{-1/2} \left(\frac{-2}{4Dt}\right) \exp\left(\frac{-x^2}{4Dt}\right) \end{aligned}$$

Also

$$\begin{aligned} \frac{\partial f}{\partial t} &= (4\pi Dt)^{-1/2} \left(\frac{-x^2}{4D}\right) \left(\frac{-1}{t^2}\right) \exp\left(\frac{-x^2}{4Dt}\right) \\ &\quad + (4\pi D)^{-1/2} \left(\frac{-1}{2}\right) t^{-3/2} \exp\left(\frac{-x^2}{4Dt}\right) \end{aligned}$$

Substituting the expressions for $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial f}{\partial t}$ into the differential equation, we find $0 = 0$, Q.E.D.

6.28

Computer Plot

6.29

n-type

$$\delta n = \delta p = g' \tau_{pO} = (10^{21})(10^{-6}) = 10^{15} \text{ cm}^{-3}$$

We have $n_O = 10^{16} \text{ cm}^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Now

$$\begin{aligned} E_{Fn} - E_{Fi} &= kT \ln\left(\frac{n_O + \delta n}{n_i}\right) \\ &= (0.0259) \ln\left(\frac{10^{16} + 10^{15}}{1.5 \times 10^{10}}\right) \end{aligned}$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.3498 \text{ eV}}$$

and

$$\begin{aligned} E_{Fi} - E_{Fp} &= kT \ln\left(\frac{p_O + \delta p}{n_i}\right) \\ &= (0.0259) \ln\left(\frac{2.25 \times 10^4 + 10^{15}}{1.5 \times 10^{10}}\right) \end{aligned}$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.2877 \text{ eV}}$$

6.30

(a) p-type

$$\begin{aligned} E_{Fi} - E_F &= kT \ln\left(\frac{p_O}{n_i}\right) \\ &= (0.0259) \ln\left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}}\right) \end{aligned}$$

or

$$\underline{E_{Fi} - E_F = 0.3294 \text{ eV}}$$

(b)

$$\delta n = \delta p = 5 \times 10^{14} \text{ cm}^{-3}$$

and

$$n_O = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}$$

Then

$$\begin{aligned} E_{Fn} - E_{Fi} &= kT \ln\left(\frac{n_O + \delta n}{n_i}\right) \\ &= (0.0259) \ln\left(\frac{4.5 \times 10^4 + 5 \times 10^{14}}{1.5 \times 10^{10}}\right) \end{aligned}$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.2697 \text{ eV}}$$

and

$$\begin{aligned} E_{Fi} - E_{Fp} &= kT \ln\left(\frac{p_O + \delta p}{n_i}\right) \\ &= (0.0259) \ln\left(\frac{5 \times 10^{15} + 5 \times 10^{14}}{1.5 \times 10^{10}}\right) \end{aligned}$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.3318 \text{ eV}}$$

6.31

n-type GaAs; $n_o = 5 \times 10^{16} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.8 \times 10^6)^2}{5 \times 10^{16}} = 6.48 \times 10^{-5} \text{ cm}^{-3}$$

We have

$$\delta n = \delta p = (0.1)N_d = 5 \times 10^{15} \text{ cm}^{-3}$$

(a)

$$\begin{aligned} E_{Fn} - E_{Fi} &= kT \ln \left(\frac{n_o + \delta n}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{5 \times 10^{16} + 5 \times 10^{15}}{1.8 \times 10^6} \right) \end{aligned}$$

or

$$E_{Fn} - E_{Fi} = 0.6253 \text{ eV}$$

We have

$$\begin{aligned} E_F - E_{Fi} &= kT \ln \left(\frac{N_d}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{5 \times 10^{16}}{1.8 \times 10^6} \right) \end{aligned}$$

or

$$E_F - E_{Fi} = 0.6228 \text{ eV}$$

Now

$$\begin{aligned} E_{Fn} - E_F &= (E_{Fn} - E_{Fi}) - (E_F - E_{Fi}) \\ &= 0.6253 - 0.6228 \end{aligned}$$

so

$$E_{Fn} - E_F = 0.0025 \text{ eV}$$

(b)

$$\begin{aligned} E_{Fi} - E_{Fp} &= kT \ln \left(\frac{p_o + \delta p}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.8 \times 10^6} \right) \end{aligned}$$

or

$$E_{Fi} - E_{Fp} = 0.5632 \text{ eV}$$

6.32

Quasi-Fermi level for minority carrier electrons

$$E_{Fn} - E_{Fi} = kT \ln \left(\frac{n_o + \delta n}{n_i} \right)$$

We have

$$\delta n = (10^{14}) \left(\frac{x}{50 \mu\text{m}} \right)$$

Neglecting the minority carrier electron concentration

$$E_{Fn} - E_{Fi} = kT \ln \left[\frac{(10^{14})(x)}{(50 \mu\text{m})(1.8 \times 10^6)} \right]$$

We find

$x(\mu\text{m})$	$E_{Fn} - E_{Fi} \text{ (eV)}$
0	-0.581
1	+0.361
2	+0.379
10	+0.420
20	+0.438
50	+0.462

Quasi-Fermi level for holes: we have

$$E_{Fi} - E_{Fp} = kT \ln \left(\frac{p_o + \delta p}{n_i} \right)$$

We have $p_o = 10^{16} \text{ cm}^{-3}$, $\delta p = \delta n$

We find

$x(\mu\text{m})$	$E_{Fi} - E_{Fp} \text{ (eV)}$
0	+0.58115
50	+0.58140

6.33

(a) We can write

$$E_{Fi} - E_F = kT \ln \left(\frac{p_o}{n_i} \right)$$

and

$$E_{Fi} - E_{Fp} = kT \ln \left(\frac{p_o + \delta p}{n_i} \right)$$

so that

$$\begin{aligned} (E_{Fi} - E_{Fp}) - (E_{Fi} - E_F) &= E_F - E_{Fp} \\ &= kT \ln \left(\frac{p_o + \delta p}{n_i} \right) - kT \ln \left(\frac{p_o}{n_i} \right) \end{aligned}$$

or

$$E_F - E_{Fp} = kT \ln \left(\frac{p_o + \delta p}{p_o} \right) = (0.01)kT$$

Then

$$\frac{p_o + \delta p}{p_o} = \exp(0.01) = 1.010 \Rightarrow$$

$$\frac{\delta p}{p_o} = 0.010 \Rightarrow \text{low-injection, so that}$$

$$\delta p = 5 \times 10^{12} \text{ cm}^{-3}$$

(b)

$$E_{Fn} - E_{Fi} \approx kT \ln \left(\frac{\delta p}{n_i} \right)$$

$$= (0.0259) \ln \left(\frac{5 \times 10^{12}}{1.5 \times 10^{10}} \right)$$

or

$$E_{Fn} - E_{Fi} = 0.1505 \text{ eV}$$

6.34

Computer Plot

6.35

Computer Plot

6.36

(a)

$$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n (n + n') + C_p (p + p')}$$

$$= \frac{(np - n_i^2)}{\tau_{pO} (n + n') + \tau_{nO} (p + p')}$$

For $n = p = 0$

$$R = \frac{-n_i^2}{\tau_{pO} n_i + \tau_{nO} n_i} \Rightarrow R = \frac{-n_i}{\tau_{pO} + \tau_{nO}}$$

(b)

We had defined the net generation rate as

$g - R = g_o + g' - (R_o + R')$ where

$g_o = R_o$ since these are the thermal equilibrium generation and recombination rates. If $g' = 0$,

then $g - R = -R'$ and $R' = \frac{-n_i}{\tau_{pO} + \tau_{nO}}$ so that

$g - R = + \frac{n_i}{\tau_{pO} + \tau_{nO}}$. Thus a negative

recombination rate implies a net positive generation rate.

6.37

We have that

$$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n (n + n') + C_p (p + p')}$$

$$= \frac{(np - n_i^2)}{\tau_{pO} (n + n_i) + \tau_{nO} (p + n_i)}$$

If $n = n_o + \delta n$ and $p = p_o + \delta n$, then

$$R = \frac{(n_o + \delta n)(p_o + \delta n) - n_i^2}{\tau_{pO} (n_o + \delta n + n_i) + \tau_{nO} (p_o + \delta n + n_i)}$$

$$= \frac{n_o p_o + \delta n (n_o + p_o) + (\delta n)^2 - n_i^2}{\tau_{pO} (n_o + \delta n + n_i) + \tau_{nO} (p_o + \delta n + n_i)}$$

If $\delta n \ll n_i$, we can neglect the $(\delta n)^2$; also

$$n_o p_o = n_i^2$$

Then

$$R = \frac{\delta n (n_o + p_o)}{\tau_{pO} (n_o + n_i) + \tau_{nO} (p_o + n_i)}$$

(a)

For n-type, $n_o \gg p_o$, $n_o \gg n_i$

Then

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO}} = 10^{+7} \text{ s}^{-1}$$

(b)

Intrinsic, $n_o = p_o = n_i$

Then

$$\frac{R}{\delta n} = \frac{2n_i}{\tau_{pO} (2n_i) + \tau_{nO} (2n_i)}$$

or

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO} + \tau_{nO}} = \frac{1}{10^{-7} + 5 \times 10^{-7}} \Rightarrow$$

$$\frac{R}{\delta n} = 1.67 \times 10^{+6} \text{ s}^{-1}$$

(c)

p-type, $p_o \gg n_o$, $p_o \gg n_i$

Then

$$\frac{R}{\delta n} = \frac{1}{\tau_{nO}} = \frac{1}{5 \times 10^{-7}} = 2 \times 10^{+6} \text{ s}^{-1}$$

6.38

(a) From Equation [6.56],

$$D_p \frac{d^2(\delta p)}{dx^2} + g' - \frac{\delta p}{\tau_{pO}} = 0$$

Solution is of the form

$$\delta p = g' \tau_{pO} + A \exp(-x/L_p) + B \exp(+x/L_p)$$

At $x = \infty$, $\delta p = g' \tau_{pO}$, so that $B \equiv 0$,

Then

$$\delta p = g' \tau_{pO} + A \exp(-x/L_p)$$

We have

$$D_p \frac{d(\delta p)}{dx} \Big|_{x=0} = s(\delta p) \Big|_{x=0}$$

We can write

$$\frac{d(\delta p)}{dx} \Big|_{x=0} = \frac{-A}{L_p} \quad \text{and} \quad (\delta p) \Big|_{x=0} = g' \tau_{pO} + A$$

Then

$$\frac{-AD_p}{L_p} = s(g' \tau_{pO} + A)$$

Solving for A we find

$$A = \frac{-sg' \tau_{pO}}{\frac{D_p}{L_p} + s}$$

The excess concentration is then

$$\delta p = g' \tau_{pO} \left[1 - \frac{s}{(D_p/L_p) + s} \cdot \exp\left(\frac{-x}{L_p}\right) \right]$$

where

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(10)(10^{-7})} = 10^{-3} \text{ cm}$$

Now

$$\delta p = (10^{21})(10^{-7}) \left[1 - \frac{s}{(10/10^{-3}) + s} \exp\left(\frac{-x}{L_p}\right) \right]$$

or

$$\delta p = 10^{14} \left[1 - \frac{s}{10^4 + s} \exp\left(\frac{-x}{L_p}\right) \right]$$

(i) $s = 0$, $\delta p = 10^{14} \text{ cm}^{-3}$

(ii) $s = 2000 \text{ cm} / \text{s}$,

$$\delta p = 10^{14} \left[1 - 0.167 \exp\left(\frac{-x}{L_p}\right) \right]$$

(iii) $s = \infty$, $\delta p = 10^{14} \left[1 - \exp\left(\frac{-x}{L_p}\right) \right]$

(b) (i) $s = 0$, $\delta p(0) = 10^{14} \text{ cm}^{-3}$

(ii) $s = 2000 \text{ cm} / \text{s}$, $\delta p(0) = 0.833 \times 10^{14} \text{ cm}^{-3}$

(iii) $s = \infty$, $\delta p(0) = 0$

6.39

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(25)(5 \times 10^{-7})} = 35.4 \times 10^{-4} \text{ cm}$$

(a)

$$\text{At } x = 0, g' \tau_{nO} = (2 \times 10^{21})(5 \times 10^{-7}) = 10^{15} \text{ cm}^{-3}$$

$$\text{Or } \delta n_O = g' \tau_{nO} = 10^{15} \text{ cm}^{-3}$$

For $x > 0$

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \Rightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A \exp(-x/L_n) + B \exp(+x/L_n)$$

$$\text{At } x = 0, \delta n = \delta n_O = A + B$$

$$\text{At } x = W,$$

$$\delta n = 0 = A \exp(-W/L_n) + B \exp(+W/L_n)$$

Solving these two equations, we find

$$A = \frac{-\delta n_O \exp(+2W/L_n)}{1 - \exp(2W/L_n)}$$

$$B = \frac{\delta n_O}{1 - \exp(2W/L_n)}$$

Substituting into the general solution, we find

$$\delta n = \frac{\delta n_O}{[\exp(+W/L_n) - \exp(-W/L_n)]} \times \{ \exp[+(W-x)/L_n] - \exp[-(W-x)/L_n] \}$$

or

$$\delta n = \frac{\delta n_O \sinh[(W-x)/L_n]}{\sinh[W/L_n]}$$

where

$$\delta n_O = 10^{15} \text{ cm}^{-3} \quad \text{and} \quad L_n = 35.4 \mu\text{m}$$

(b)

If $\tau_{nO} = \infty$, we have

$$\frac{d^2(\delta n)}{dx^2} = 0$$

so the solution is of the form

$$\delta n = Cx + D$$

Applying the boundary conditions, we find

$$\delta n = \delta n_o \left(1 - \frac{x}{W} \right)$$

6.40

For $\tau_{po} = \infty$, we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \quad \text{so that} \quad \frac{d(\delta p)}{dx} = A \quad \text{and}$$

$$\delta p = Ax + B$$

At $x = W$

$$-D_p \frac{d(\delta p)}{dx} \Big|_{x=W} = s \cdot (\delta p) \Big|_{x=W}$$

or

$$-D_p A = s(AW + B)$$

which yields

$$B = \frac{-A}{s} (D_p + sW)$$

At $x = 0$, the flux of excess holes is

$$10^{19} = -D_p \frac{d(\delta p)}{dx} \Big|_{x=0} = -D_p A$$

so that

$$A = \frac{-10^{19}}{10} = -10^{18} \text{ cm}^{-4}$$

and

$$B = \frac{10^{18}}{s} (10 + sW) = 10^{18} \left(\frac{10}{s} + W \right)$$

The solution is now

$$\delta p = 10^{18} \left(W - x + \frac{10}{s} \right)$$

(a)

For $s = \infty$,

$$\delta p = 10^{18} (20 \times 10^{-4} - x) \text{ cm}^{-3}$$

(b)

For $s = 2 \times 10^3 \text{ cm/s}$

$$\delta p = 10^{18} (70 \times 10^{-4} - x) \text{ cm}^{-3}$$

6.41

For $-W < x < 0$,

$$D_n \frac{d^2(\delta n)}{dx^2} + G'_o = 0$$

so that

$$\frac{d(\delta n)}{dx} = -\frac{G'_o}{D_p} x + C_1$$

and

$$\delta n = -\frac{G'_o}{2D_n} x^2 + C_1 x + C_2$$

For $0 < x < W$,

$$\frac{d^2(\delta n)}{dx^2} = 0, \text{ so } \frac{d(\delta n)}{dx} = C_3, \delta n = C_3 x + C_4$$

The boundary conditions are:

$$(1) \quad s = 0 \text{ at } x = -W, \text{ so that } \frac{d(\delta n)}{dx} \Big|_{x=-W} = 0$$

$$(2) \quad s = \infty \text{ at } x = +W, \text{ so that } \delta n(W) = 0$$

$$(3) \quad \delta n \text{ continuous at } x = 0$$

$$(4) \quad \frac{d(\delta n)}{dx} \text{ continuous at } x = 0$$

Applying the boundary conditions, we find

$$C_1 = C_3 = -\frac{G'_o W}{D_n}, \quad C_2 = C_4 = +\frac{G'_o W^2}{D_n}$$

Then, for $-W < x < 0$

$$\delta n = \frac{G'_o}{2D_n} (-x^2 - 2Wx + 2W^2)$$

and for $0 < x < +W$

$$\delta n = \frac{G'_o W}{D_n} (W - x)$$

6.42

Computer Plot

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Chapter 7

Problem Solutions

7.1

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

where $V_t = 0.0259 \text{ V}$ and $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$

We find

(a)

For $N_d = 10^{15} \text{ cm}^{-3}$	$V_{bi} \text{ (V)}$
(i) $N_a = 10^{15} \text{ cm}^{-3}$	0.575 V
(ii) $N_a = 10^{16} \text{ cm}^{-3}$	0.635
(iii) $N_a = 10^{17} \text{ cm}^{-3}$	0.695
(iv) $N_a = 10^{18} \text{ cm}^{-3}$	0.754

(b)

For $N_d = 10^{18} \text{ cm}^{-3}$	$V_{bi} \text{ (V)}$
(i) $N_a = 10^{15} \text{ cm}^{-3}$	0.754 V
(ii) $N_a = 10^{16} \text{ cm}^{-3}$	0.814
(iii) $N_a = 10^{17} \text{ cm}^{-3}$	0.874
(iv) $N_a = 10^{18} \text{ cm}^{-3}$	0.933

7.2

Si: $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$

Ge: $n_i = 2.4 \times 10^{13} \text{ cm}^{-3}$

GaAs: $n_i = 1.8 \times 10^6 \text{ cm}^{-3}$

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \text{ and } V_t = 0.0259 \text{ V}$$

(a)

$N_d = 10^{14} \text{ cm}^{-3}$, $N_a = 10^{17} \text{ cm}^{-3}$

Then

$$\text{Si: } V_{bi} = 0.635 \text{ V}, \text{ Ge: } V_{bi} = 0.253 \text{ V},$$

$$\text{GaAs: } V_{bi} = 1.10 \text{ V}$$

(b)

$N_d = 5 \times 10^{16} \text{ cm}^{-3}$, $N_a = 5 \times 10^{16} \text{ cm}^{-3}$

Then

$$\text{Si: } V_{bi} = 0.778 \text{ V}, \text{ Ge: } V_{bi} = 0.396 \text{ V},$$

$$\text{GaAs: } V_{bi} = 1.25 \text{ V}$$

(c)

$$N_d = 10^{17} \text{ cm}^{-3}, N_a = 10^{17} \text{ cm}^{-3}$$

Then

$$\text{Si: } V_{bi} = 0.814 \text{ V}, \text{ Ge: } V_{bi} = 0.432 \text{ V},$$

$$\text{GaAs: } V_{bi} = 1.28 \text{ V}$$

7.3

Computer Plot

7.4

Computer Plot

7.5

(a) n-side:

$$\begin{aligned} E_F - E_{Fi} &= kT \ln \left(\frac{N_d}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}} \right) \end{aligned}$$

or

$$E_F - E_{Fi} = 0.3294 \text{ eV}$$

p-side:

$$\begin{aligned} E_{Fi} - E_F &= kT \ln \left(\frac{N_a}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{10^{17}}{1.5 \times 10^{10}} \right) \end{aligned}$$

or

$$E_{Fi} - E_F = 0.4070 \text{ eV}$$

(b)

$$V_{bi} = 0.3294 + 0.4070$$

or

$$V_{bi} = 0.7364 \text{ V}$$

(c)

$$\begin{aligned} V_{bi} &= V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ &= (0.0259) \ln \left[\frac{(10^{17})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] \end{aligned}$$

or

$$V_{bi} = 0.7363 \text{ V}$$

(d)

$$x_n = \left[\frac{2 \epsilon V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.736)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{10^{17}}{5 \times 10^{15}} \right) \left(\frac{1}{10^{17} + 5 \times 10^{15}} \right) \right]^{1/2}$$

or

$$x_n = 0.426 \mu m$$

Now

$$x_p = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.736)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{5 \times 10^{15}}{10^{17}} \right) \left(\frac{1}{10^{17} + 5 \times 10^{15}} \right) \right]^{1/2}$$

or

$$x_p = 0.0213 \mu m$$

We have

$$|E_{\max}| = \frac{eN_d x_n}{\epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(5 \times 10^{15})(0.426 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 3.29 \times 10^4 \text{ V / cm}$$

7.6

(a) n-side

$$E_F - E_{Fi} = (0.0259) \ln \left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}} \right) \Rightarrow$$

$$E_F - E_{Fi} = 0.3653 \text{ eV}$$

p-side

$$E_{Fi} - E_F = (0.0259) \ln \left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}} \right) \Rightarrow$$

$$E_{Fi} - E_F = 0.3653 \text{ eV}$$

(b)

$$V_{bi} = 0.3653 + 0.3653 \Rightarrow$$

$$V_{bi} = 0.7306 \text{ V}$$

(c)

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$= (0.0259) \ln \left[\frac{(2 \times 10^{16})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.7305 \text{ V}$$

(d)

$$x_n = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.7305)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{2 \times 10^{16}}{2 \times 10^{16}} \right) \left(\frac{1}{2 \times 10^{16} + 2 \times 10^{16}} \right) \right]^{1/2}$$

or

$$x_n = 0.154 \mu m$$

By symmetry

$$x_p = 0.154 \mu m$$

Now

$$|E_{\max}| = \frac{eN_d x_n}{\epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(2 \times 10^{16})(0.154 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 4.76 \times 10^4 \text{ V / cm}$$

7.7

$$(b) \quad n_o = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right]$$

$$= 2.8 \times 10^{19} \exp \left(\frac{-0.21}{0.0259} \right)$$

or

$$n_o = N_d = 8.43 \times 10^{15} \text{ cm}^{-3} \quad (\text{n-region})$$

$$p_o = N_v \exp \left[\frac{-(E_F - E_v)}{kT} \right]$$

$$= 1.04 \times 10^{19} \exp \left(\frac{-0.18}{0.0259} \right)$$

or

$$p_o = N_a = 9.97 \times 10^{15} \text{ cm}^{-3} \quad (\text{p-region})$$

(c)

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$= (0.0259) \ln \left[\frac{(9.97 \times 10^{15})(8.43 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$$

or

$$\underline{V_{bi} = 0.690 \text{ V}}$$

7.8

(a) GaAs: $V_{bi} = 1.20 \text{ V}$, $n_i = 1.8 \times 10^6 \text{ cm}^{-3}$

$$x_p = 0.2W = 0.2(x_n + x_p)$$

or

$$\frac{x_p}{x_n} = 0.25$$

Also

$$N_d x_n = N_a x_p \Rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 0.25$$

Now

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

or

$$1.20 = (0.0259) \ln \left(\frac{0.25 N_a^2}{n_i^2} \right)$$

Then

$$\frac{0.25 N_a^2}{n_i^2} = \exp \left(\frac{1.20}{0.0259} \right)$$

or

$$N_a = 2n_i \exp \left[\frac{1.20}{2(0.0259)} \right]$$

or

$$\underline{N_a = 4.14 \times 10^{16} \text{ cm}^{-3}}$$

(b)

$$N_d = 0.25 N_a \Rightarrow \underline{N_d = 1.04 \times 10^{16} \text{ cm}^{-3}}$$

(c)

$$\begin{aligned} x_n &= \left[\frac{2 \epsilon V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.20)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{4}{1} \right) \left(\frac{1}{4.14 \times 10^{16} + 1.04 \times 10^{16}} \right) \right]^{1/2} \end{aligned}$$

or

$$\underline{x_n = 0.366 \text{ } \mu\text{m}}$$

(d)

$$x_p = 0.25 x_n \Rightarrow \underline{x_p = 0.0916 \text{ } \mu\text{m}}$$

(e)

$$\begin{aligned} E_{\max} &= \frac{e N_d x_n}{\epsilon} = \frac{e N_a x_p}{\epsilon} \\ &= \frac{(1.6 \times 10^{-19})(1.04 \times 10^{16})(0.366 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$\underline{E_{\max} = 5.25 \times 10^4 \text{ V/cm}}$$

7.9

$$(a) \quad V_{bi} = (0.0259) \ln \left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2} \right]$$

or

$$\underline{V_{bi} = 0.635 \text{ V}}$$

(b)

$$\begin{aligned} x_n &= \left[\frac{2 \epsilon V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{10^{16}}{10^{15}} \right) \left(\frac{1}{10^{16} + 10^{15}} \right) \right]^{1/2} \end{aligned}$$

$$\text{or } \underline{x_n = 0.864 \text{ } \mu\text{m}}$$

Now

$$\begin{aligned} x_p &= \left[\frac{2 \epsilon V_{bi}}{e} \left(\frac{N_d}{N_a} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{10^{15}}{10^{16}} \right) \left(\frac{1}{10^{16} + 10^{15}} \right) \right]^{1/2} \end{aligned}$$

$$\text{or } \underline{x_p = 0.0864 \text{ } \mu\text{m}}$$

(c)

$$\begin{aligned} E_{\max} &= \frac{e N_d x_n}{\epsilon} \\ &= \frac{(1.6 \times 10^{-19})(10^{15})(0.864 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$\underline{E_{\max} = 1.34 \times 10^4 \text{ V/cm}}$$

7.10

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \text{ and}$$

$$n_i^2 = N_c N_v \exp \left(\frac{-E_g}{kT} \right)$$

We can write

$$N_c N_v = N_{co} N_{vo} \left(\frac{T}{300} \right)^3$$

Now

$$\begin{aligned} V_{bi} &= V_t [\ln(N_a N_d) - \ln(n_i^2)] \\ &= V_t [\ln(N_a N_d) - \ln(N_{co} N_{vo}) \\ &\quad - \ln \left(\frac{T}{300} \right)^3 + \frac{E_g}{kT}] \end{aligned}$$

or

$$V_{bi} = V_t \left[\ln \left(\frac{N_a N_d}{N_{co} N_{vo}} \right) - 3 \ln \left(\frac{T}{300} \right) + \frac{E_g}{kT} \right]$$

or

$$\begin{aligned} 0.40 &= (0.0250) \left(\frac{T}{300} \right) \\ &\times \left[\ln \left[\frac{(5 \times 10^{15})(10^{16})}{(2.8 \times 10^{19})(1.04 \times 10^{19})} \right] - 3 \ln \left(\frac{T}{300} \right) \right. \\ &\quad \left. + \frac{1.12}{(0.0259)(T/300)} \right] \end{aligned}$$

Then

$$15.44 = \left(\frac{T}{300} \right) \left[-15.58 - 3 \ln \left(\frac{T}{300} \right) + \frac{43.24}{(T/300)} \right]$$

By trial and error

$$\underline{T = 490K}$$

7.11

$$\begin{aligned} \text{(a)} \quad V_{bi} &= V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ &= (0.0259) \ln \left[\frac{(5 \times 10^{17})(10^{17})}{(1.5 \times 10^{10})^2} \right] \end{aligned}$$

or

$$\underline{V_{bi} = 0.8556 V}$$

(b)

For a 1% change in V_{bi} , assume that the change is due to n_i^2 , where the major dependence on temperature is given by

$$n_i^2 \propto \exp \left(\frac{-E_g}{kT} \right)$$

Now

$$\begin{aligned} \frac{V_{bi}(T_2)}{V_{bi}(T_1)} &= \frac{\ln \left[\frac{N_a N_d}{n_i^2(T_2)} \right]}{\ln \left[\frac{N_a N_d}{n_i^2(T_1)} \right]} \\ &= \frac{\ln(N_a N_d) - \ln[n_i^2(T_2)]}{\ln(N_a N_d) - \ln[n_i^2(T_1)]} \\ &= \frac{\ln(N_a N_d) - \ln(N_c N_v) - \left(\frac{-E_g}{kT_2} \right)}{\ln(N_a N_d) - \ln(N_c N_v) - \left(\frac{-E_g}{kT_1} \right)} \\ &= \left\{ \ln[(5 \times 10^{17})(10^{17})] \right. \\ &\quad \left. - \ln[(2.8 \times 10^{19})(1.04 \times 10^{19})] + \frac{E_g}{kT_2} \right\} \\ &\quad / \left\{ \ln[(5 \times 10^{17})(10^{17})] \right. \\ &\quad \left. - \ln[(2.8 \times 10^{19})(1.04 \times 10^{19})] + \frac{E_g}{kT_1} \right\} \end{aligned}$$

or

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{79.897 - 88.567 + \frac{E_g}{kT_2}}{79.897 - 88.567 + \frac{E_g}{kT_1}}$$

We can write

$$0.990 = \frac{-8.67 + \frac{E_g}{kT_2}}{-8.67 + \frac{1.12}{0.0259}} = \frac{-8.67 + \frac{E_g}{kT_2}}{34.57}$$

so that

$$\frac{E_g}{kT_2} = 42.90 = \frac{1.12}{(0.0259) \left(\frac{T_2}{300} \right)}$$

We then find

$$\underline{T_2 = 302.4K}$$

7.12

(b) For $N_d = 10^{16} \text{ cm}^{-3}$,

$$\begin{aligned} E_F - E_{Fi} &= kT \ln \left(\frac{N_d}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right) \end{aligned}$$

or

$$\underline{E_F - E_{Fi} = 0.3473 \text{ eV}}$$

For $N_d = 10^{15} \text{ cm}^{-3}$,

$$E_F - E_{Fi} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right)$$

or

$$\underline{E_F - E_{Fi} = 0.2877 \text{ eV}}$$

Then

$$V_{bi} = 0.3473 - 0.2877$$

or

$$\underline{V_{bi} = 0.0596 \text{ V}}$$

7.13

$$(a) \quad V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$= (0.0259) \ln \left[\frac{(10^{16})(10^{12})}{(1.5 \times 10^{10})^2} \right]$$

or

$$\underline{V_{bi} = 0.456 \text{ V}}$$

(b)

$$\begin{aligned} x_n &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.456)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{10^{12}}{10^{16}} \right) \left(\frac{1}{10^{16} + 10^{12}} \right) \right]^{1/2} \end{aligned}$$

or

$$\underline{x_n = 2.43 \times 10^{-7} \text{ cm}}$$

(c)

$$\begin{aligned} x_p &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.456)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{10^{16}}{10^{12}} \right) \left(\frac{1}{10^{16} + 10^{12}} \right) \right]^{1/2} \end{aligned}$$

or

$$\underline{x_p = 2.43 \times 10^{-3} \text{ cm}}$$

(d)

$$\begin{aligned} |E_{\max}| &= \frac{e N_d x_n}{\epsilon} \\ &= \frac{(1.6 \times 10^{-19})(10^{16})(2.43 \times 10^{-7})}{(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$\underline{|E_{\max}| = 3.75 \times 10^2 \text{ V/cm}}$$

7.14

Assume Silicon, so

$$\begin{aligned} L_D &= \left(\frac{\epsilon kT}{e^2 N_d} \right)^{1/2} \\ &= \left[\frac{(11.7)(8.85 \times 10^{-14})(0.0259)(1.6 \times 10^{-19})}{(1.6 \times 10^{-19})^2 N_d} \right]^{1/2} \end{aligned}$$

or

$$L_D = \left(\frac{1.676 \times 10^5}{N_d} \right)^{1/2}$$

$$(a) \quad N_d = 8 \times 10^{14} \text{ cm}^{-3}, \quad \underline{L_D = 0.1447 \text{ } \mu\text{m}}$$

$$(b) \quad N_d = 2.2 \times 10^{16} \text{ cm}^{-3}, \quad \underline{L_D = 0.02760 \text{ } \mu\text{m}}$$

$$(c) \quad N_d = 8 \times 10^{17} \text{ cm}^{-3}, \quad \underline{L_D = 0.004577 \text{ } \mu\text{m}}$$

Now

$$(a) \quad V_{bi} = 0.7427 \text{ V}$$

$$(b) \quad V_{bi} = 0.8286 \text{ V}$$

$$(c) \quad V_{bi} = 0.9216 \text{ V}$$

Also

$$\begin{aligned} x_n &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi})}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{8 \times 10^{17}}{N_d} \right) \left(\frac{1}{8 \times 10^{17} + N_d} \right) \right]^{1/2} \end{aligned}$$

Then

$$(a) \quad \underline{x_n = 1.096 \text{ } \mu\text{m}}$$

$$(b) \quad \underline{x_n = 0.2178 \text{ } \mu\text{m}}$$

$$(c) \quad \underline{x_n = 0.02731 \text{ } \mu\text{m}}$$

Now

$$(a) \quad \underline{\frac{L_D}{x_n} = 0.1320}$$

$$(b) \quad \frac{L_D}{x_n} = 0.1267$$

$$(c) \quad \frac{L_D}{x_n} = 0.1677$$

7.15 Computer Plot

7.16

$$(a) \quad V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ = (0.0259) \ln \left[\frac{(2 \times 10^{16})(2 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$$

or

$$(b) \quad \underline{V_{bi} = 0.671 \text{ V}} \\ W = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \\ = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_R)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left[\frac{2 \times 10^{16} + 2 \times 10^{15}}{(2 \times 10^{16})(2 \times 10^{15})} \right] \right\}^{1/2}$$

or

$$W = [7.12 \times 10^{-9} (V_{bi} + V_R)]^{1/2}$$

$$\text{For } V_R = 0, \quad \underline{W = 0.691 \times 10^{-4} \text{ cm}}$$

$$\text{For } V_R = 8 \text{ V}, \quad \underline{W = 2.48 \times 10^{-4} \text{ cm}}$$

(c)

$$E_{\max} = \frac{2(V_{bi} + V_R)}{W}$$

$$\text{For } V_R = 0, \quad \underline{E_{\max} = 1.94 \times 10^4 \text{ V / cm}}$$

$$\text{For } V_R = 8 \text{ V}, \quad \underline{E_{\max} = 7.0 \times 10^4 \text{ V / cm}}$$

7.17

$$(a) \quad V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ = (0.0259) \ln \left[\frac{(5 \times 10^{17})(10^{17})}{(1.5 \times 10^{10})^2} \right]$$

or

$$\underline{V_{bi} = 0.856 \text{ V}}$$

(b)

$$x_n = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ = \left[\frac{2(11.7)(8.85 \times 10^{-14})(5.856)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{5 \times 10^{17}}{1 \times 10^{17}} \right) \left(\frac{1}{5 \times 10^{17} + 1 \times 10^{17}} \right) \right]^{1/2}$$

or

$$\underline{x_n = 0.251 \text{ } \mu\text{m}}$$

Also

$$x_p = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e} \left(\frac{N_d}{N_a} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ = \left[\frac{2(11.7)(8.85 \times 10^{-14})(5.856)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{1 \times 10^{17}}{5 \times 10^{17}} \right) \left(\frac{1}{5 \times 10^{17} + 1 \times 10^{17}} \right) \right]^{1/2}$$

or

$$\underline{x_p = 0.0503 \text{ } \mu\text{m}}$$

Also

$$W = x_n + x_p$$

or

$$\underline{W = 0.301 \text{ } \mu\text{m}}$$

(c)

$$E_{\max} = \frac{2(V_{bi} + V_R)}{W} = \frac{2(5.856)}{0.301 \times 10^{-4}}$$

or

$$\underline{E_{\max} = 3.89 \times 10^5 \text{ V / cm}}$$

(d)

$$C_T = \frac{\epsilon A}{W} = \frac{(11.7)(8.85 \times 10^{-14})(10^{-4})}{0.301 \times 10^{-4}}$$

or

$$\underline{C_T = 3.44 \text{ pF}}$$

7.18

$$(a) \quad V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ = (0.0259) \ln \left[\frac{50 N_a^2}{(1.5 \times 10^{10})^2} \right]$$

We can write

$$\exp\left(\frac{0.752}{0.0259}\right) = \frac{50N_a^2}{(1.5 \times 10^{10})^2}$$

or

$$N_a = \frac{1.5 \times 10^{10}}{\sqrt{50}} \exp\left[\frac{0.752}{2(0.0259)}\right]$$

and

$$N_a = 4.28 \times 10^{15} \text{ cm}^{-3}$$

Then

$$N_d = 2.14 \times 10^{17} \text{ cm}^{-3}$$

(b)

$$x_p \approx W \approx \left[\frac{2 \in (V_{bi} + V_R)}{e} \cdot \left(\frac{1}{N_a} \right) \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(10.752)}{(1.6 \times 10^{-19})(4.28 \times 10^{15})} \right]^{1/2}$$

or

$$x_p = 1.80 \text{ } \mu\text{m}$$

(c)

$$C' \approx \left[\frac{e \in N_a}{2(V_{bi} + V_R)} \right]^{1/2}$$

$$= \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(4.28 \times 10^{15})}{2(10.752)} \right]^{1/2}$$

or

$$C' = 5.74 \times 10^{-9} \text{ F / cm}^2$$

7.19

(a) Neglecting change in V_{bi}

$$\frac{C'(2N_a)}{C'(N_a)} = \left\{ \frac{\left[\frac{2}{(2N_a + N_d)} \right]}{\left(\frac{1}{N_a + N_d} \right)} \right\}^{1/2}$$

For a $n^+p \Rightarrow N_d \gg N_a$

Then

$$\frac{C'(2N_a)}{C'(N_a)} = \sqrt{2} = 1.414$$

so a 41.4% change.

(b)

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT \ln\left(\frac{2N_a N_d}{n_i^2}\right)}{kT \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

$$= \frac{kT \ln 2 + kT \ln\left(\frac{N_a N_d}{n_i^2}\right)}{kT \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

So we can write this as

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT \ln 2 + V_{bi}(N_a)}{V_{bi}(N_a)}$$

so

$$\Delta V_{bi} = kT \ln 2 = (0.0259) \ln 2$$

or

$$\Delta V_{bi} = 17.95 \text{ mV}$$

17.20

(a)

$$\frac{W(A)}{W(B)} = \frac{\left[\frac{2 \in (V_{biA} + V_R)}{e} \left(\frac{N_a + N_{dA}}{N_a N_{dA}} \right) \right]^{1/2}}{\left[\frac{2 \in (V_{biB} + V_R)}{e} \left(\frac{N_a + N_{dB}}{N_a N_{dB}} \right) \right]^{1/2}}$$

or

$$\frac{W(A)}{W(B)} = \left[\frac{(V_{biA} + V_R)}{(V_{biB} + V_R)} \cdot \frac{(N_a + N_{dA})}{(N_a + N_{dB})} \cdot \left(\frac{N_{dB}}{N_{dA}} \right) \right]^{1/2}$$

We find

$$V_{biA} = (0.0259) \ln \left[\frac{(10^{18})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.7543 \text{ V}$$

$$V_{biB} = (0.0259) \ln \left[\frac{(10^{18})(10^{16})}{(1.5 \times 10^{10})^2} \right] = 0.8139 \text{ V}$$

So we find

$$\frac{W(A)}{W(B)} = \left[\left(\frac{0.7543}{0.8139} \right) \left(\frac{10^{18} + 10^{15}}{10^{18} + 10^{16}} \right) \left(\frac{10^{16}}{10^{15}} \right) \right]^{1/2}$$

or

$$\frac{W(A)}{W(B)} = 3.13$$

(b)

$$\frac{E(A)}{E(B)} = \frac{\frac{2(V_{biA} + V_R)}{W(A)}}{\frac{2(V_{biB} + V_R)}{W(B)}} = \frac{W(B)}{W(A)} \cdot \frac{(V_{biA} + V_R)}{(V_{biB} + V_R)}$$

$$= \left(\frac{1}{3.13} \right) \left(\frac{5.7543}{5.8139} \right)$$

or

$$\frac{E(A)}{E(B)} = 0.316$$

(c)

$$\frac{C'_j(A)}{C'_j(B)} = \frac{\left[\frac{\epsilon N_a N_{dA}}{2(V_{biA} + V_R)(N_a + N_{dA})} \right]^{1/2}}{\left[\frac{\epsilon N_a N_{dB}}{2(V_{biB} + V_R)(N_a + N_{dB})} \right]^{1/2}}$$

$$= \left[\left(\frac{N_{dA}}{N_{dB}} \right) \left(\frac{V_{biB} + V_R}{V_{biA} + V_R} \right) \left(\frac{N_a + N_{dB}}{N_a + N_{dA}} \right) \right]^{1/2}$$

$$= \left[\left(\frac{10^{15}}{10^{16}} \right) \left(\frac{5.8139}{5.7543} \right) \left(\frac{10^{18} + 10^{16}}{10^{18} + 10^{15}} \right) \right]^{1/2}$$

or

$$\frac{C'_j(A)}{C'_j(B)} = 0.319$$

17.21

(a) $V_{bi} = (0.0259) \ln \left[\frac{(4 \times 10^{15})(4 \times 10^{17})}{(1.5 \times 10^{10})^2} \right] \Rightarrow$

$$V_{bi} = 0.766 \text{ V}$$

Now

$$|E_{\max}| = \left[\frac{2e(V_{bi} + V_R)}{\epsilon} \left(\frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2}$$

so

$$(3 \times 10^5)^2 = \left[\frac{2(1.6 \times 10^{-19})}{(11.7)(8.85 \times 10^{-14})} \right] (V_{bi} + V_R)$$

$$\times \left[\frac{(4 \times 10^{15})(4 \times 10^{17})}{4 \times 10^{15} + 4 \times 10^{17}} \right]$$

or

$$9 \times 10^{10} = 1.22 \times 10^9 (V_{bi} + V_R) \Rightarrow$$

$$V_{bi} + V_R = 73.77 \text{ V}$$

and

$$V_R = 73 \text{ V}$$

(b)

$$V_{bi} = (0.0259) \ln \left[\frac{(4 \times 10^{16})(4 \times 10^{17})}{(1.5 \times 10^{10})^2} \right] \Rightarrow$$

$$V_{bi} = 0.826 \text{ V}$$

$$(3 \times 10^5)^2 = \left[\frac{2(1.6 \times 10^{-19})}{(11.7)(8.85 \times 10^{-14})} \right] (V_{bi} + V_R)$$

$$\times \left[\frac{(4 \times 10^{16})(4 \times 10^{17})}{4 \times 10^{16} + 4 \times 10^{17}} \right]$$

which yields

$$V_{bi} + V_R = 8.007 \text{ V}$$

and

$$V_R = 7.18 \text{ V}$$

(c)

$$V_{bi} = (0.0259) \ln \left[\frac{(4 \times 10^{17})(4 \times 10^{17})}{(1.5 \times 10^{10})^2} \right] \Rightarrow$$

$$V_{bi} = 0.886 \text{ V}$$

$$(3 \times 10^5)^2 = \left[\frac{2(1.6 \times 10^{-19})}{(11.7)(8.85 \times 10^{-14})} \right] (V_{bi} + V_R)$$

$$\times \left[\frac{(4 \times 10^{17})(4 \times 10^{17})}{4 \times 10^{17} + 4 \times 10^{17}} \right]$$

which yields

$$V_{bi} + V_R = 1.456 \text{ V}$$

and

$$V_R = 0.570 \text{ V}$$

17.22

(a) We have

$$\frac{C_j(0)}{C_j(10)} = \frac{\left[\frac{\epsilon N_a N_d}{2V_{bi}(N_a + N_d)} \right]^{1/2}}{\left[\frac{\epsilon N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{1/2}}$$

or

$$\frac{C_j(0)}{C_j(10)} = 3.13 = \left(\frac{V_{bi} + V_R}{V_{bi}} \right)^{1/2}$$

For $V_R = 10\text{ V}$, we find

$$(3.13)^2 V_{bi} = V_{bi} + 10$$

or

$$\underline{V_{bi} = 1.14\text{ V}}$$

(b)

$$x_p = 0.2W = 0.2(x_p + x_n)$$

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \Rightarrow$$

so

$$1.14 = (0.0259) \ln \left[\frac{0.25 N_a^2}{(1.8 \times 10^6)^2} \right]$$

We can then write

$$N_a = \frac{1.8 \times 10^6}{\sqrt{0.25}} \exp \left[\frac{1.14}{2(0.0259)} \right]$$

or

$$\underline{N_a = 1.3 \times 10^{16}\text{ cm}^{-3}}$$

and

$$\underline{N_d = 3.25 \times 10^{15}\text{ cm}^{-3}}$$

7.23

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{16})(5 \times 10^{16})}{(1.8 \times 10^6)^2} \right]$$

or

$$\underline{V_{bi} = 1.20\text{ V}}$$

Now

$$\frac{C'_j(V_{R1})}{C'_j(V_{R2})} = \frac{\left[\frac{1}{V_{bi} + V_{R1}} \right]^{1/2}}{\left[\frac{1}{V_{bi} + V_{R2}} \right]^{1/2}} = \left[\frac{V_{bi} + V_{R2}}{V_{bi} + V_{R1}} \right]^{1/2}$$

So

$$(3)^2 = \frac{1.20 + V_{R2}}{1.20 + 1} \Rightarrow$$

$$\underline{V_{R2} = 18.6\text{ V}}$$

7.24

$$C' = \left[\frac{e \in N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{1/2}$$

and

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{18})(10^{15})}{(1.5 \times 10^{10})^2} \right]$$

or

$$\underline{V_{bi} = 0.754\text{ V}}$$

For $N_a \gg N_d$, we have

$$C' = \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})}{2(V_{bi} + V_R)} \right]^{1/2}$$

or

$$C' = \left[\frac{8.28 \times 10^{-17}}{V_{bi} + V_R} \right]^{1/2}$$

For $V_R = 1\text{ V}$, $C' = 6.87 \times 10^{-9}\text{ F/cm}^2$

For $V_R = 10\text{ V}$, $C' = 2.77 \times 10^{-9}\text{ F/cm}^2$

If $A = 6 \times 10^{-4}\text{ cm}^2$, then

For $V_R = 1\text{ V}$, $C = 4.12\text{ pF}$

For $V_R = 10\text{ V}$, $C = 1.66\text{ pF}$

The resonant frequency is given by

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

so that

For $V_R = 1\text{ V}$, $f_o = 1.67\text{ MHz}$

For $V_R = 10\text{ V}$, $f_o = 2.63\text{ MHz}$

7.25

$$|E_{\max}| = \frac{eN_d x_n}{\epsilon}$$

For a p^+n junction,

$$x_n \approx \left[\frac{2\epsilon(V_{bi} + V_R)}{eN_d} \right]^{1/2}$$

so that

$$|E_{\max}| = \left[\frac{2eN_d}{\epsilon} (V_{bi} + V_R) \right]^{1/2}$$

Assuming that $V_{bi} \ll V_R$, then

$$N_d = \frac{\epsilon E_{\max}^2}{2eV_R} = \frac{(11.7)(8.85 \times 10^{-14})(10^6)^2}{2(1.6 \times 10^{-19})(10)}$$

or

$$N_d = 3.24 \times 10^{17} \text{ cm}^{-3}$$

7.26

$$x_n = 0.1W = 0.1(x_n + x_p)$$

which yields

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} = 9$$

We can write

$$\begin{aligned} V_{bi} &= V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ &= (0.0259) \ln \left[\frac{9 N_a^2}{(1.5 \times 10^{10})^2} \right] \end{aligned}$$

We also have

$$C'_j = \frac{C_T}{A} = \frac{3.5 \times 10^{-12}}{5.5 \times 10^{-4}} = 6.36 \times 10^{-9} \text{ F / cm}^2$$

so

$$6.36 \times 10^{-9} = \left[\frac{e \epsilon N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{1/2}$$

Which becomes

$$\begin{aligned} 4.05 \times 10^{-17} &= \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})N_a(9N_a)}{2(V_{bi} + V_R)(N_a + 9N_a)} \end{aligned}$$

or

$$4.05 \times 10^{-17} = \frac{7.46 \times 10^{-32} N_a}{(V_{bi} + V_R)}$$

If $V_R = 1.2 \text{ V}$, then by iteration we find

$$N_a = 9.92 \times 10^{14} \text{ cm}^{-3}$$

$$V_{bi} = 0.632 \text{ V}$$

$$N_d = 8.93 \times 10^{15} \text{ cm}^{-3}$$

7.27

$$\begin{aligned} \text{(a)} \quad V_{bi} &= V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ &= (0.0259) \ln \left[\frac{(5 \times 10^{15})(10^{14})}{(1.5 \times 10^{10})^2} \right] \end{aligned}$$

or

$$V_{bi} = 0.557 \text{ V}$$

(b)

$$\begin{aligned} x_p &= \left[\frac{2 \epsilon V_{bi}}{e} \left(\frac{N_d}{N_a} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.557)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{10^{14}}{5 \times 10^{15}} \right) \left(\frac{1}{10^{14} + 5 \times 10^{15}} \right) \right]^{1/2} \end{aligned}$$

or

$$x_p = 5.32 \times 10^{-6} \text{ cm}$$

Also

$$\begin{aligned} x_n &= \left[\frac{2 \epsilon V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.557)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{5 \times 10^{15}}{10^{14}} \right) \left(\frac{1}{10^{14} + 5 \times 10^{15}} \right) \right]^{1/2} \end{aligned}$$

or

$$x_n = 2.66 \times 10^{-4} \text{ cm}$$

(c)

For $x_n = 30 \text{ } \mu\text{m}$, we have

$$\begin{aligned} 30 \times 10^{-4} &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_R)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{5 \times 10^{15}}{10^{14}} \right) \left(\frac{1}{10^{14} + 5 \times 10^{15}} \right) \right]^{1/2} \end{aligned}$$

which becomes

$$9 \times 10^{-6} = 1.27 \times 10^{-7} (V_{bi} + V_R)$$

We find

$$V_R = 70.3 \text{ V}$$

7.28

An n^+p junction with $N_a = 10^{14} \text{ cm}^{-3}$,

(a)

A one-sided junction and assume $V_R \gg V_{bi}$, then

$$x_p = \left[\frac{2 \epsilon V_R}{e N_a} \right]^{1/2}$$

so

$$(50 \times 10^{-4})^2 = \frac{2(11.7)(8.85 \times 10^{-14})V_R}{(1.6 \times 10^{-19})(10^{14})}$$

which yields

$$\underline{V_R = 193 \text{ V}}$$

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p \left(\frac{N_a}{N_d} \right)$$

so

$$x_n = (50 \times 10^{-4}) \left(\frac{10^{14}}{10^{16}} \right) \Rightarrow$$

or

$$\underline{x_n = 0.5 \text{ } \mu\text{m}}$$

(c)

$$E_{\max} = \frac{2(V_{bi} + V_R)}{W} = \frac{2(193)}{50.5 \times 10^{-4}}$$

or

$$\underline{E_{\max} = 7.72 \times 10^4 \text{ V/cm}}$$

7.29

$$(a) \quad V_{bi} = (0.0259) \ln \left[\frac{(10^{18})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.796 \text{ V}$$

$$\begin{aligned} C = AC' &= A \left[\frac{e \in N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{1/2} \\ &= (5 \times 10^{-5}) \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(V_{bi} + V_R)} \right. \\ &\quad \left. \times \frac{(10^{18})(5 \times 10^{15})}{(10^{18} + 5 \times 10^{15})} \right]^{1/2} \end{aligned}$$

or

$$C = (5 \times 10^{-5}) \left[\frac{4.121 \times 10^{-16}}{(V_{bi} + V_R)} \right]^{1/2}$$

For $V_R = 0$, $C = 1.14 \text{ pF}$

For $V_R = 3 \text{ V}$, $C = 0.521 \text{ pF}$

For $V_R = 6 \text{ V}$, $C = 0.389 \text{ pF}$

We can write

$$\left(\frac{1}{C} \right)^2 = \frac{1}{A^2} \left[\frac{2(V_{bi} + V_R)(N_a + N_d)}{e \in N_a N_d} \right]$$

For the p^+n junction

$$\left(\frac{1}{C} \right)^2 \approx \frac{1}{A^2} \left[\frac{2(V_{bi} + V_R)}{e \in N_d} \right]$$

so that

$$\frac{\Delta(1/C)^2}{\Delta V_R} = \frac{1}{A^2} \cdot \frac{2}{e \in N_d}$$

We have

$$\text{For } V_R = 0, \left(\frac{1}{C} \right)^2 = 7.69 \times 10^{23}$$

$$\text{For } V_R = 6 \text{ V}, \left(\frac{1}{C} \right)^2 = 6.61 \times 10^{24}$$

Then, for $\Delta V_R = 6 \text{ V}$,

$$\Delta(1/C)^2 = 5.84 \times 10^{24}$$

We find

$$\begin{aligned} N_d &= \frac{2}{A^2 e \in} \cdot \frac{1}{\left(\frac{\Delta(1/C)^2}{\Delta V_R} \right)} \\ &= \frac{2}{(5 \times 10^{-5})^2 (1.6 \times 10^{-19}) (11.7) (8.85 \times 10^{-14})} \\ &\quad \times \frac{1}{\left(\frac{5.84 \times 10^{24}}{6} \right)} \end{aligned}$$

so that

$$\underline{N_d = 4.96 \times 10^{15} \approx 5 \times 10^{15} \text{ cm}^{-3}}$$

Now, for a straight line

$$y = mx + b$$

$$m = \frac{\Delta(1/C)^2}{\Delta V_R} = \frac{5.84 \times 10^{24}}{6}$$

$$\text{At } V_R = 0, \left(\frac{1}{C} \right)^2 = 7.69 \times 10^{23} = b$$

Then

$$\left(\frac{1}{C} \right)^2 = \left(\frac{5.84 \times 10^{24}}{6} \right) \cdot V_R + 7.69 \times 10^{23}$$

$$\text{Now, at } \left(\frac{1}{C} \right)^2 = 0,$$

$$0 = \left(\frac{5.84 \times 10^{24}}{6} \right) \cdot V_R + 7.69 \times 10^{23}$$

which yields

$$\underline{V_R = -V_{bi} = -0.790 \text{ V}}$$

or

$$V_{bi} \approx 0.796 \text{ V}$$

(b)

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{18})(6 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.860 \text{ V}$$

$$C = (5 \times 10^{-5}) \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(V_{bi} + V_R)} \times \frac{(10^{18})(6 \times 10^{16})}{(10^{18} + 6 \times 10^{16})} \right]^{1/2}$$

or

$$C = (5 \times 10^{-5}) \left[\frac{4.689 \times 10^{-15}}{V_{bi} + V_R} \right]^{1/2}$$

Then

$$\text{For } V_R = 0, \quad C = 3.69 \text{ pF}$$

$$\text{For } V_R = 3 \text{ V}, \quad C = 1.74 \text{ pF}$$

$$\text{For } V_R = 6 \text{ V}, \quad C = 1.31 \text{ pF}$$

7.30

$$C' = \frac{C}{A} = \frac{1.3 \times 10^{-12}}{10^{-5}} = 1.3 \times 10^{-7} \text{ F / cm}^2$$

(a) For a one-sided junction

$$C' = \left[\frac{e \epsilon N_L}{2(V_{bi} + V_R)} \right]^{1/2}$$

where N_L is the doping concentration in the low-doped region.

$$\text{We have } V_{bi} + V_R = 0.95 + 0.05 = 1.00 \text{ V}$$

Then

$$\begin{aligned} & (1.3 \times 10^{-7})^2 \\ &= \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})N_L}{2(1)} \end{aligned}$$

which yields

$$N_L = 2.04 \times 10^{17} \text{ cm}^{-3}$$

(b)

$$V_{bi} = V_t \ln \left(\frac{N_L N_H}{n_i^2} \right)$$

where N_H is the doping concentration in the high-doped region.

So

$$0.95 = (0.0259) \ln \left[\frac{(2.04 \times 10^{17})N_H}{(1.5 \times 10^{10})^2} \right]$$

which yields

$$N_H = 9.38 \times 10^{18} \text{ cm}^{-3}$$

7.31

Computer Plot

7.32

$$(a) \quad V_{bi} = V_t \ln \left(\frac{N_{aO} N_{dO}}{n_i^2} \right)$$

(c) p-region

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon} = \frac{-eN_{aO}}{\epsilon}$$

or

$$E = \frac{-eN_{aO}x}{\epsilon} + C_1$$

We have

$$E = 0 \text{ at } x = -x_p \Rightarrow C_1 = \frac{-eN_{aO}x_p}{\epsilon}$$

Then for $-x_p < x < 0$

$$E = \frac{-eN_{aO}}{\epsilon} (x + x_p)$$

n-region, $0 < x < x_o$

$$\frac{dE_1}{dx} = \frac{\rho(x)}{\epsilon} = \frac{eN_{dO}}{2\epsilon}$$

or

$$E_1 = \frac{eN_{dO}x}{2\epsilon} + C_2$$

n-region, $x_o < x < x_n$

$$\frac{dE_2}{dx} = \frac{\rho(x)}{\epsilon} = \frac{eN_{dO}}{\epsilon}$$

or

$$E_2 = \frac{eN_{dO}x}{\epsilon} + C_3$$

We have $E_2 = 0$ at $x = x_n$, then

$$C_3 = \frac{-eN_{dO}x_n}{\epsilon}$$

so that for $x_o < x < x_n$

$$E_2 = \frac{-eN_{dO}}{\epsilon} (x_n - x)$$

We also have

$$E_2 = E_1 \text{ at } x = x_o$$

Then

$$\frac{eN_{d0}x_o}{2\epsilon} + C_2 = \frac{-eN_{d0}}{\epsilon}(x_n - x_o)$$

or

$$C_2 = \frac{-eN_{d0}}{\epsilon}\left(x_n - \frac{x_o}{2}\right)$$

Then, for $0 < x < x_{o2}$

$$E_1 = \frac{eN_{d0}x}{2\epsilon} - \frac{eN_{d0}}{\epsilon}\left(x_n - \frac{x_o}{2}\right)$$

7.33

$$(a) \frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon} = \frac{-dE(x)}{dx}$$

For $-2 < x < -1 \mu m$, $\rho(x) = +eN_d$

So

$$\frac{dE}{dx} = \frac{eN_d}{\epsilon} \Rightarrow E = \frac{eN_dx}{\epsilon} + C_1$$

At $x = -2 \mu m \equiv -x_o$, $E = 0$

So

$$0 = \frac{-eN_dx_o}{\epsilon} + C_1 \Rightarrow C_1 = \frac{eN_dx_o}{\epsilon}$$

Then

$$E = \frac{eN_d}{\epsilon}(x + x_o)$$

At $x = 0$, $E(0) = E(x = -1 \mu m)$, so

$$\begin{aligned} E(0) &= \frac{eN_d}{\epsilon}(-1+2)x10^{-4} \\ &= \frac{(1.6 \times 10^{-19})(5 \times 10^{15})}{(11.7)(8.85 \times 10^{-14})}(1 \times 10^{-4}) \end{aligned}$$

which yields

$$E(0) = 7.73 \times 10^4 \text{ V/cm}$$

(c)

Magnitude of potential difference is

$$\begin{aligned} |\phi| &= \int E dx = \frac{eN_d}{\epsilon} \int (x + x_o) dx \\ &= \frac{eN_d}{\epsilon} \left(\frac{x^2}{2} + x_o \cdot x \right) + C_2 \end{aligned}$$

Let $\phi = 0$ at $x = -x_o$, then

$$0 = \frac{eN_d}{\epsilon} \left(\frac{x_o^2}{2} - x_o^2 \right) + C_2 \Rightarrow C_2 = \frac{eN_dx_o^2}{2\epsilon}$$

Then we can write

$$|\phi| = \frac{eN_d}{2\epsilon}(x + x_o)^2$$

At $x = -1 \mu m$

$$|\phi_1| = \frac{(1.6 \times 10^{-19})(5 \times 10^{15})}{2(11.7)(8.85 \times 10^{-14})} [(-1+2)x10^{-4}]$$

or

$$|\phi_1| = 3.86 \text{ V}$$

Potential difference across the intrinsic region

$$|\phi_i| = E(0) \cdot d = (7.73 \times 10^4)(2 \times 10^{-4})$$

or

$$|\phi_i| = 15.5 \text{ V}$$

By symmetry, potential difference across the p-region space charge region is also 3.86 V . The total reverse-bias voltage is then

$$V_R = 2(3.86) + 15.5 \Rightarrow V_R = 23.2 \text{ V}$$

7.34

(a) For the linearly graded junction,

$$\rho(x) = eax,$$

Then

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon} = \frac{eax}{\epsilon}$$

Now

$$E = \int \frac{eax}{\epsilon} dx = \frac{ea}{\epsilon} \cdot \frac{x^2}{2} + C_1$$

At $x = +x_o$ and $x = -x_o$, $E = 0$

So

$$0 = \frac{ea}{\epsilon} \left(\frac{x_o^2}{2} \right) + C_1 \Rightarrow C_1 = \frac{-ea}{\epsilon} \left(\frac{x_o^2}{2} \right)$$

Then

$$E = \frac{ea}{2\epsilon}(x^2 - x_o^2)$$

(b)

$$\phi(x) = -\int E dx = \frac{-ea}{2\epsilon} \left[\frac{x^3}{3} - x_o^2 \cdot x \right] + C_2$$

Set $\phi = 0$ at $x = -x_o$, then

$$0 = \frac{-ea}{2\epsilon} \left[\frac{-x_o^3}{3} + x_o^3 \right] + C_2 \Rightarrow C_2 = \frac{eax_o^3}{3\epsilon}$$

Then

$$\phi(x) = \frac{-ea}{2\epsilon} \left(\frac{x^3}{3} - x_o^2 \cdot x \right) + \frac{eax_o^3}{3\epsilon}$$

7.35

We have that

$$C' = \left[\frac{ea \epsilon^2}{12(V_{bi} + V_R)} \right]^{1/3}$$

then

$$\begin{aligned} & (7.2 \times 10^{-9})^3 \\ &= \left[\frac{a(1.6 \times 10^{-19})[(11.7)(8.85 \times 10^{-14})]^2}{12(0.7 + 3.5)} \right] \end{aligned}$$

which yields

$$a = 1.1 \times 10^{20} \text{ cm}^{-4}$$

Chapter 8

Problem Solutions

8.1

In the forward bias

$$I_f \approx I_s \exp\left(\frac{eV}{kT}\right)$$

Then

$$\frac{I_{f1}}{I_{f2}} = \frac{I_s}{I_s} \cdot \frac{\exp\left(\frac{eV_1}{kT}\right)}{\exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\frac{e}{kT}(V_1 - V_2)\right]$$

or

$$V_1 - V_2 = \left(\frac{kT}{e}\right) \ln\left(\frac{I_{f1}}{I_{f2}}\right)$$

(a)

$$\text{For } \frac{I_{f1}}{I_{f2}} = 10 \Rightarrow \underline{V_1 - V_2 = 59.9 \text{ mV} \approx 60 \text{ mV}}$$

(b)

$$\text{For } \frac{I_{f1}}{I_{f2}} = 100 \Rightarrow V_1 - V_2 = 119.3 \text{ mV} \approx 120 \text{ mV}$$

8.2

$$I = I_s \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

or we can write this as

$$\frac{I}{I_s} + 1 = \exp\left(\frac{eV}{kT}\right)$$

so that

$$V = \left(\frac{kT}{e}\right) \ln\left(\frac{I}{I_s} + 1\right)$$

In reverse bias, I is negative, so at

$$\frac{I}{I_s} = -0.90, \text{ we have}$$

$$V = (0.0259) \ln(1 - 0.90) \Rightarrow$$

or

$$\underline{V = -59.6 \text{ mV}}$$

8.3

Computer Plot

8.4

The cross-sectional area is

$$A = \frac{I}{J} = \frac{10 \times 10^{-3}}{20} = 5 \times 10^{-4} \text{ cm}^2$$

We have

$$J \approx J_s \exp\left(\frac{V_D}{V_t}\right) \Rightarrow 20 = J_s \exp\left(\frac{0.65}{0.0259}\right)$$

so that

$$J_s = 2.52 \times 10^{-10} \text{ A / cm}^2$$

We can write

$$J_s = en_i^2 \left[\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

We want

$$\frac{\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}}}{\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_{pO}}}} = 0.10$$

or

$$\begin{aligned} & \frac{\frac{1}{N_a} \cdot \sqrt{\frac{25}{5 \times 10^{-7}}}}{\frac{1}{N_a} \cdot \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5 \times 10^{-7}}}} \\ &= \frac{7.07 \times 10^3}{7.07 \times 10^3 + \frac{N_a}{N_d} (4.47 \times 10^3)} = 0.10 \end{aligned}$$

which yields

$$\frac{N_a}{N_d} = 14.24$$

Now

$$\begin{aligned} J_s &= 2.52 \times 10^{-10} = (1.6 \times 10^{-19}) (1.5 \times 10^{10})^2 \\ &\times \left[\frac{1}{(14.24)N_d} \cdot \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5 \times 10^{-7}}} \right] \end{aligned}$$

We find

$$N_d = 7.1 \times 10^{14} \text{ cm}^{-3}$$

and

$$\underline{N_a = 1.01 \times 10^{16} \text{ cm}^{-3}}$$

8.5

(a)

$$\begin{aligned}\frac{J_n}{J_n + J_p} &= \frac{\frac{eD_n n_{pO}}{L_n}}{\frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}} \\ &= \frac{\sqrt{\frac{D_n}{\tau_{nO}} \cdot \frac{n_i^2}{N_a}}}{\sqrt{\frac{D_n}{\tau_{nO}} \cdot \frac{n_i^2}{N_a}} + \sqrt{\frac{D_p}{\tau_{pO}} \cdot \frac{n_i^2}{N_d}}} \\ &= \frac{1}{1 + \sqrt{\frac{D_p \tau_{nO}}{D_n \tau_{pO}} \cdot \left(\frac{N_a}{N_d}\right)}}\end{aligned}$$

We have

$$\frac{D_p}{D_n} = \frac{\mu_p}{\mu_n} = \frac{1}{2.4} \quad \text{and} \quad \frac{\tau_{nO}}{\tau_{pO}} = \frac{1}{0.1}$$

so

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + \sqrt{\frac{1}{2.4} \cdot \frac{1}{0.1} \left(\frac{N_a}{N_d}\right)}}$$

or

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + (2.04) \left(\frac{N_a}{N_d}\right)}$$

(b)

Using Einstein's relation, we can write

$$\begin{aligned}\frac{J_n}{J_n + J_p} &= \frac{\frac{e\mu_n}{L_n} \cdot \frac{n_i^2}{N_a}}{\frac{e\mu_n}{L_n} \cdot \frac{n_i^2}{N_a} + \frac{e\mu_p}{L_p} \cdot \frac{n_i^2}{N_d}} \\ &= \frac{e\mu_n N_d}{e\mu_n N_d + \frac{L_n}{L_p} \cdot e\mu_p N_a}\end{aligned}$$

We have

$$\sigma_n = e\mu_n N_d \quad \text{and} \quad \sigma_p = e\mu_p N_a$$

Also

$$\frac{L_n}{L_p} = \sqrt{\frac{D_n \tau_{nO}}{D_p \tau_{pO}}} = \sqrt{\frac{2.4}{0.1}} = 4.90$$

Then

$$\frac{J_n}{J_n + J_p} = \frac{(\sigma_n / \sigma_p)}{(\sigma_n / \sigma_p) + 4.90}$$

8.6

For a silicon p^+n junction,

$$\begin{aligned}I_s &= Aen_i^2 \cdot \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \\ &= (10^{-4})(1.6 \times 10^{-19})(1.5 \times 10^{10})^2 \cdot \frac{1}{10^{16}} \sqrt{\frac{12}{10^{-7}}}\end{aligned}$$

or

$$I_s = 3.94 \times 10^{-15} \text{ A}$$

Then

$$I_D = I_s \exp\left(\frac{V_D}{V_t}\right) = (3.94 \times 10^{-15}) \exp\left(\frac{0.50}{0.0259}\right)$$

or

$$I_D = 9.54 \times 10^{-7} \text{ A}$$

8.7

We want

$$\frac{J_n}{J_n + J_p} = 0.95$$

$$\begin{aligned}&= \frac{\frac{eD_n n_{pO}}{L_n}}{\frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}} = \frac{\frac{D_n}{L_n N_a}}{\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}} \\ &= \frac{\frac{D_n}{L_n}}{\frac{D_n}{L_n} + \frac{D_p}{L_p} \cdot \frac{N_a}{N_d}}\end{aligned}$$

We obtain

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(25)(0.1 \times 10^{-6})} \Rightarrow$$

$$L_n = 15.8 \mu\text{m}$$

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(10)(0.1 \times 10^{-6})} \Rightarrow$$

$$L_p = 10 \mu\text{m}$$

Then

$$0.95 = \frac{\frac{25}{15.8}}{\frac{25}{15.8} + \frac{10}{10} \cdot \left(\frac{N_a}{N_d} \right)}$$

which yields

$$\frac{N_a}{N_d} = 0.083$$

8.8

(a) p-side: $E_{Fi} - E_F = kT \ln \left(\frac{N_a}{n_i} \right)$

$$= (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}} \right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.329 \text{ eV}}$$

Also

n-side: $E_F - E_{Fi} = kT \ln \left(\frac{N_d}{n_i} \right)$

$$= (0.0259) \ln \left(\frac{10^{17}}{1.5 \times 10^{10}} \right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.407 \text{ eV}}$$

(b)

We can find

$$D_n = (1250)(0.0259) = 32.4 \text{ cm}^2 / \text{s}$$

$$D_p = (420)(0.0259) = 10.9 \text{ cm}^2 / \text{s}$$

Now

$$J_s = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

$$= (1.6 \times 10^{-19}) (1.5 \times 10^{10})^2$$

$$\times \left[\frac{1}{5 \times 10^{15}} \sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}} \sqrt{\frac{10.9}{10^{-7}}} \right]$$

or

$$J_s = 4.48 \times 10^{-11} \text{ A} / \text{cm}^2$$

Then

$$I_s = AJ_s = (10^{-4}) (4.48 \times 10^{-11})$$

or

$$\underline{I_s = 4.48 \times 10^{-15} \text{ A}}$$

We find

$$I = I_s \exp \left(\frac{V_D}{V_i} \right)$$

$$= (4.48 \times 10^{-15}) \exp \left(\frac{0.5}{0.0259} \right)$$

or

$$\underline{I = 1.08 \text{ } \mu\text{A}}$$

(c)

The hole current is proportional to

$$I_p \propto en_i^2 \cdot A \cdot \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= (1.6 \times 10^{-19}) (1.5 \times 10^{10})^2 (10^{-4}) \left(\frac{1}{10^{17}} \right) \sqrt{\frac{10.9}{10^{-7}}}$$

or

$$I_p \propto 3.76 \times 10^{-16} \text{ A}$$

Then

$$\frac{I_p}{I} = \frac{3.76 \times 10^{-16}}{4.48 \times 10^{-15}} \Rightarrow \underline{\frac{I_p}{I} = 0.0839}$$

8.9

$$I = I_s \left[\exp \left(\frac{V_a}{V_i} \right) - 1 \right]$$

For a p^+n diode,

$$I_s = A \left(\frac{eD_p p_{nO}}{L_p} \right) = A \left(e \sqrt{\frac{D_p}{\tau_{pO}}} \cdot \frac{n_i^2}{N_d} \right)$$

$$= (10^{-4}) \left[(1.6 \times 10^{-19}) \sqrt{\frac{10}{10^{-6}}} \cdot \frac{(2.4 \times 10^{13})^2}{10^{16}} \right]$$

or

$$\underline{I_s = 2.91 \times 10^{-9} \text{ A}}$$

(a)

For $V_a = +0.2 \text{ V}$,

$$I = (2.91 \times 10^{-9}) \left[\exp \left(\frac{0.2}{0.0259} \right) - 1 \right]$$

or

$$\underline{I = 6.55 \text{ } \mu\text{A}}$$

(b)

For $V_a = -0.2 \text{ V}$,

$$I = (2.91 \times 10^{-9}) \left[\exp \left(\frac{-0.2}{0.0259} \right) - 1 \right]$$

$$\approx -2.91 \times 10^{-9} \text{ A}$$

$$\text{or } \underline{I = -I_s = -2.91 \text{ nA}}$$

8.10

For an n^+p silicon diode

$$I_s = Aen_i^2 \cdot \frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} \\ = \frac{(10^{-4})(1.6 \times 10^{-19})(1.5 \times 10^{10})^2}{10^{16}} \sqrt{\frac{25}{10^{-6}}}$$

or

$$I_s = 1.8 \times 10^{-15} \text{ A}$$

(a)

For $V_a = 0.5 \text{ V}$

$$I_D = I_s \exp\left(\frac{V_a}{V_t}\right) = (1.8 \times 10^{-15}) \exp\left(\frac{0.5}{0.0259}\right)$$

or

$$I_D = 4.36 \times 10^{-7} \text{ A}$$

(b)

For $V_a = -0.5 \text{ V}$

$$I_D = -I_s = -1.8 \times 10^{-15} \text{ A}$$

8.11

(a) We find

$$D_p = \mu_p \left(\frac{kT}{e} \right) = (480)(0.0259) = 12.4 \text{ cm}^2 / \text{s}$$

and

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(12.4)(0.1 \times 10^{-6})} \Rightarrow \\ L_p = 11.1 \text{ } \mu\text{m}$$

Also

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

Then

$$J_{pO} = \frac{eD_p p_{nO}}{L_p} = \frac{(1.6 \times 10^{-19})(12.4)(2.25 \times 10^5)}{(11.1 \times 10^{-4})}$$

or

$$J_{pO} = 4.02 \times 10^{-10} \text{ A / cm}^2$$

For $A = 10^{-4} \text{ cm}^2$, then

$$I_{pO} = 4.02 \times 10^{-14} \text{ A}$$

(b)

We have

$$D_n = \mu_n \left(\frac{kT}{e} \right) = (1350)(0.0259) = 35 \text{ cm}^2 / \text{s}$$

and

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(35)(0.4 \times 10^{-6})} \Rightarrow \\ L_n = 37.4 \text{ } \mu\text{m}$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}$$

Then

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} = \frac{(1.6 \times 10^{-19})(35)(4.5 \times 10^4)}{(37.4 \times 10^{-4})}$$

or

$$J_{nO} = 6.74 \times 10^{-11} \text{ A / cm}^2$$

For $A = 10^{-4} \text{ cm}^2$, then

$$I_{nO} = 6.74 \times 10^{-15} \text{ A}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \\ = (0.0259) \ln\left[\frac{(5 \times 10^{15})(10^{15})}{(1.5 \times 10^{10})^2}\right]$$

or

$$V_{bi} = 0.617 \text{ V}$$

Then for

$$V_a = \frac{1}{2} V_{bi} = 0.309 \text{ V}$$

We find

$$p_n = p_{nO} \exp\left(\frac{eV_a}{kT}\right) \\ = (2.25 \times 10^5) \exp\left(\frac{0.309}{0.0259}\right)$$

or

$$p_n = 3.42 \times 10^{10} \text{ cm}^{-3}$$

(d)

The total current is

$$I = (I_{pO} + I_{nO}) \exp\left(\frac{eV_a}{kT}\right) \\ = (4.02 \times 10^{-14} + 6.74 \times 10^{-15}) \exp\left(\frac{0.309}{0.0259}\right)$$

or

$$I = 7.13 \times 10^{-9} \text{ A}$$

The hole current is

$$I_p = I_{pO} \exp\left(\frac{eV_a}{kT}\right) \exp\left[\frac{-(x-x_n)}{L_p}\right]$$

The electron current is given by

$$\begin{aligned} I_n &= I - I_p \\ &= 7.13 \times 10^{-9} - (4.02 \times 10^{-14}) \\ &\quad \times \exp\left(\frac{0.309}{0.0259}\right) \exp\left[\frac{-(x-x_n)}{L_p}\right] \end{aligned}$$

At $x = x_n + \frac{1}{2} L_p$

$$I_n = 7.13 \times 10^{-9} - (6.10 \times 10^{-9}) \exp\left(\frac{-1}{2}\right)$$

or

$$I_n = 3.43 \times 10^{-9} \text{ A}$$

8.12

(a) The excess hole concentration is given by

$$\begin{aligned} \delta p_n &= p_n - p_{nO} \\ &= p_{nO} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{-x}{L_p}\right) \end{aligned}$$

We find

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

and

$$\begin{aligned} L_p &= \sqrt{D_p \tau_{pO}} = \sqrt{(8)(0.01 \times 10^{-6})} \Rightarrow \\ L_p &= 2.83 \text{ } \mu\text{m} \end{aligned}$$

Then

$$\begin{aligned} \delta p_n &= (2.25 \times 10^4) \\ &\quad \times \left[\exp\left(\frac{0.610}{0.0259}\right) - 1 \right] \exp\left(\frac{-x}{2.83 \times 10^{-4}}\right) \end{aligned}$$

or

$$\delta p_n = 3.81 \times 10^{14} \exp\left(\frac{-x}{2.83 \times 10^{-4}}\right) \text{ cm}^{-3}$$

(b)

We have

$$\begin{aligned} J_p &= -eD_p \frac{d(\delta p_n)}{dx} \\ &= \frac{eD_p (3.81 \times 10^{14})}{(2.83 \times 10^{-4})} \exp\left(\frac{-x}{2.83 \times 10^{-4}}\right) \end{aligned}$$

At $x = 3 \times 10^{-4} \text{ cm}$,

$$J_p = \frac{(1.6 \times 10^{-19})(8)(3.81 \times 10^{14})}{2.83 \times 10^{-4}} \exp\left(\frac{-3}{2.83}\right)$$

or

$$J_p = 0.597 \text{ A / cm}^2$$

(c)

We have

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} \exp\left(\frac{eV_a}{kT}\right)$$

We can determine that

$$n_{pO} = 4.5 \times 10^3 \text{ cm}^{-3} \text{ and } L_n = 10.7 \text{ } \mu\text{m}$$

Then

$$J_{nO} = \frac{(1.6 \times 10^{-19})(23)(4.5 \times 10^3)}{10.7 \times 10^{-4}} \exp\left(\frac{0.610}{0.0259}\right)$$

or

$$J_{nO} = 0.262 \text{ A / cm}^2$$

We can also find

$$J_{pO} = 1.72 \text{ A / cm}^2$$

Then, at $x = 3 \text{ } \mu\text{m}$,

$$\begin{aligned} J_n(3 \text{ } \mu\text{m}) &= J_{nO} + J_{pO} - J_p(3 \text{ } \mu\text{m}) \\ &= 0.262 + 1.72 - 0.597 \end{aligned}$$

or

$$J_n(3 \text{ } \mu\text{m}) = 1.39 \text{ A / cm}^2$$

8.13

(a) From Problem 8.9 (Ge diode)

Low injection means

$$p_n(0) = (0.1)N_d = 10^{15} \text{ cm}^{-3}$$

Now

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(2.4 \times 10^{13})^2}{10^{16}} = 5.76 \times 10^{10} \text{ cm}^{-3}$$

We have

$$p_n(0) = p_{nO} \exp\left(\frac{V_a}{V_t}\right)$$

or

$$\begin{aligned} V_a &= V_t \ln\left[\frac{p_n(0)}{p_{nO}}\right] \\ &= (0.0259) \ln\left(\frac{10^{15}}{5.76 \times 10^{10}}\right) \end{aligned}$$

or

$$V_a = 0.253 \text{ V}$$

(b)

For Problem 8.10 (Si diode)

$$n_p(0) = (0.1)N_a = 10^{15} \text{ cm}^{-3}$$

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Then

$$\begin{aligned} V_a &= V_t \ln \left[\frac{n_p(0)}{n_{pO}} \right] \\ &= (0.0259) \ln \left(\frac{10^{15}}{2.25 \times 10^4} \right) \end{aligned}$$

or

$$\underline{V_a = 0.635 \text{ V}}$$

8.14

The excess electron concentration is given by

$$\begin{aligned} \delta n_p &= n_p - n_{pO} \\ &= n_{pO} \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right] \exp \left(\frac{-x}{L_n} \right) \end{aligned}$$

The total number of excess electrons is

$$N_p = A \int_0^{\infty} \delta n_p dx$$

We may note that

$$\int_0^{\infty} \exp \left(\frac{-x}{L_n} \right) dx = -L_n \exp \left(\frac{-x}{L_n} \right) \Big|_0^{\infty} = L_n$$

Then

$$N_p = AL_n n_{pO} \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right]$$

We can find

$$D_n = 35 \text{ cm}^2 / \text{s} \quad \text{and} \quad L_n = 59.2 \text{ } \mu\text{m}$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{8 \times 10^{15}} = 2.81 \times 10^4 \text{ cm}^{-3}$$

Then

$$\begin{aligned} N_p &= (10^{-3})(59.2 \times 10^{-4})(2.81 \times 10^4) \\ &\quad \times \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right] \end{aligned}$$

or

$$N_p = 0.166 \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right]$$

Then we find the total number of excess electrons in the p-region to be:

$$(a) \quad V_a = 0.3 \text{ V}, \quad \underline{N_p = 1.78 \times 10^4}$$

$$(b) \quad V_a = 0.4 \text{ V}, \quad \underline{N_p = 8.46 \times 10^5}$$

$$(c) \quad V_a = 0.5 \text{ V}, \quad \underline{N_p = 4.02 \times 10^7}$$

Similarly, the total number of excess holes in the n-region is found to be:

$$N_n = AL_p p_{nO} \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right]$$

We find that

$$D_p = 12.4 \text{ cm}^2 / \text{s} \quad \text{and} \quad L_p = 11.1 \text{ } \mu\text{m}$$

Also

$$p_{nO} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Then

$$N_n = (2.50 \times 10^{-2}) \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right]$$

So

$$(a) \quad V_a = 0.3 \text{ V}, \quad \underline{N_n = 2.68 \times 10^3}$$

$$(b) \quad V_a = 0.4 \text{ V}, \quad \underline{N_n = 1.27 \times 10^5}$$

$$(c) \quad V_a = 0.5 \text{ V}, \quad \underline{N_n = 6.05 \times 10^6}$$

8.15

$$I \propto n_i^2 \exp \left(\frac{eV_a}{kT} \right) \propto \exp \left(\frac{-E_g}{kT} \right) \exp \left(\frac{eV_a}{kT} \right)$$

Then

$$I \propto \exp \left(\frac{eV_a - E_g}{kT} \right)$$

so

$$\frac{I_1}{I_2} = \frac{\exp \left(\frac{eV_{a1} - E_{g1}}{kT} \right)}{\exp \left(\frac{eV_{a2} - E_{g2}}{kT} \right)}$$

or

$$\frac{I_1}{I_2} = \exp \left(\frac{eV_{a1} - eV_{a2} - E_{g1} + E_{g2}}{kT} \right)$$

We have

$$\frac{10 \times 10^{-3}}{10 \times 10^{-6}} = \exp \left(\frac{0.255 - 0.32 - 0.525 + E_{g2}}{0.0259} \right)$$

or

$$10^3 = \exp \left(\frac{E_{g2} - 0.59}{0.0259} \right)$$

Then

$$E_{g2} = 0.59 + (0.0259) \ln(10^3)$$

which yields

$$E_{g2} = 0.769 \text{ eV}$$

8.16

(a) We have

$$I_s = Aen_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

which can be written in the form

$$\begin{aligned} I_s &= C'n_i^2 \\ &= C'N_{CO}N_{VO} \left(\frac{T}{300} \right)^3 \exp\left(\frac{-E_g}{kT} \right) \end{aligned}$$

or

$$I_s = CT^3 \exp\left(\frac{-E_g}{kT} \right)$$

(b)

Taking the ratio

$$\begin{aligned} \frac{I_{s2}}{I_{s1}} &= \left(\frac{T_2}{T_1} \right)^3 \cdot \frac{\exp\left(\frac{-E_g}{kT_2} \right)}{\exp\left(\frac{-E_g}{kT_1} \right)} \\ &= \left(\frac{T_2}{T_1} \right)^3 \cdot \exp\left[+E_g \left(\frac{1}{kT_1} - \frac{1}{kT_2} \right) \right] \end{aligned}$$

$$\text{For } T_1 = 300K, kT_1 = 0.0259, \frac{1}{kT_1} = 38.61$$

$$\text{For } T_2 = 400K, kT_2 = 0.03453, \frac{1}{kT_2} = 28.96$$

(i) Germanium, $E_g = 0.66 \text{ eV}$

$$\frac{I_{s2}}{I_{s1}} = \left(\frac{400}{300} \right)^3 \exp[(0.66)(38.61 - 28.96)]$$

or

$$\frac{I_{s2}}{I_{s1}} = 1383$$

(ii) Silicon, $E_g = 1.12 \text{ eV}$

$$\frac{I_{s2}}{I_{s1}} = \left(\frac{400}{300} \right)^3 \cdot \exp[(1.12)(38.61 - 28.96)]$$

or

$$\frac{I_{s2}}{I_{s1}} = 1.17 \times 10^5$$

8.17

Computer Plot

8.18

One condition:

$$\left| \frac{I_f}{I_r} \right| = \frac{J_s \exp\left(\frac{eV_a}{kT} \right)}{J_s} = \exp\left(\frac{eV_a}{kT} \right) = 10^4$$

or

$$\frac{kT}{e} = \frac{V_a}{\ln(10^4)} = \frac{0.5}{\ln(10^4)}$$

or

$$\frac{kT}{e} = 0.05429 = (0.0259) \left(\frac{T}{300} \right)$$

which yields

$$T = 629K$$

Second condition:

$$\begin{aligned} I_s &= A \left(\frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p} \right) \\ &= Aen_i^2 \left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d} \right) \\ &= AeN_c N_v \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right] \exp\left(\frac{-E_g}{kT} \right) \end{aligned}$$

which becomes

$$\begin{aligned} 10^{-6} &= (10^{-4})(1.6 \times 10^{-19})(2.8 \times 10^{19})(1.04 \times 10^{19}) \\ &\times \left(\frac{1}{5 \times 10^{18}} \sqrt{\frac{25}{10^{-7}}} + \frac{1}{10^{15}} \sqrt{\frac{10}{10^{-7}}} \right) \exp\left(\frac{-E_g}{kT} \right) \end{aligned}$$

or

$$\exp\left(\frac{+E_g}{kT} \right) = 4.66 \times 10^{10}$$

For $E_g = 1.10 \text{ eV}$,

$$kT = \frac{E_g}{\ln(4.66 \times 10^{10})} = \frac{1.10}{\ln(4.66 \times 10^{10})}$$

or

$$kT = 0.04478 \text{ eV} = (0.0259) \left(\frac{T}{300} \right)$$

Then

$$T = 519K$$

This second condition yields a smaller temperature, so the maximum temperature is

$$T = 519K$$

8.19

(a) We can write for the n-region

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

The general solution is

$$\delta p_n = A \exp(+x/L_p) + B \exp(-x/L_p)$$

The boundary condition at $x = x_n$ gives

$$\begin{aligned} \delta p_n(x_n) &= p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right] \\ &= A \exp(+x_n/L_p) + B \exp(-x_n/L_p) \end{aligned}$$

and the boundary condition at $x = x_n + W_n$ gives

$$\begin{aligned} \delta p_n(x_n + W_n) &= 0 \\ &= A \exp[(x_n + W_n)/L_p] + B \exp[-(x_n + W_n)/L_p] \end{aligned}$$

From this equation, we have

$$A = -B \exp[-2(x_n + W_n)/L_p]$$

Then, from the first boundary condition, we obtain

$$\begin{aligned} p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right] \\ &= B \exp[-(x_n + 2W_n)/L_p] + B \exp(-x_n/L_p) \\ &= B \exp(-x_n/L_p) [1 - \exp(-2W_n/L_p)] \end{aligned}$$

We then obtain

$$B = \frac{p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right]}{\exp(-x_n/L_p) [1 - \exp(-2W_n/L_p)]}$$

which can be written in the form

$$B = \frac{p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right] \cdot \exp[(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

Also

$$A = \frac{-p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right] \cdot \exp[-(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

The solution can now be written as

$$\begin{aligned} \delta p_n &= \frac{p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right]}{2 \sinh\left(\frac{W_n}{L_p}\right)} \\ &\times \left\{ \exp\left[\frac{(x_n + W_n - x)}{L_p}\right] - \exp\left[\frac{-(x_n + W_n - x)}{L_p}\right] \right\} \\ \text{or finally,} \\ \delta p_n &= p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right] \cdot \frac{\sinh\left(\frac{x_n + W_n - x}{L_p}\right)}{\sinh\left(\frac{W_n}{L_p}\right)} \end{aligned}$$

(b)

$$\begin{aligned} J_p &= -eD_p \frac{d(\delta p_n)}{dx} \Big|_{x=x_n} \\ &= \frac{-eD_p p_{n0} \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right]}{\sinh\left(\frac{W_n}{L_p}\right)} \\ &\quad \times \left(\frac{-1}{L_p} \right) \cosh\left(\frac{x_n + W_n - x}{L_p}\right) \Big|_{x=x_n} \end{aligned}$$

Then

$$J_p = \frac{eD_p p_{n0}}{L_p} \coth\left(\frac{W_n}{L_p}\right) \cdot \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right]$$

8.20

$$I_D \propto n_i^2 \exp\left(\frac{V_D}{V_t}\right)$$

For the temperature range $300 \leq T \leq 320K$, neglect the change in N_C and N_V

So

$$\begin{aligned} I_D &\propto \exp\left(\frac{-E_g}{kT}\right) \cdot \exp\left(\frac{eV_D}{kT}\right) \\ &\propto \exp\left[\frac{-(E_g - eV_D)}{kT}\right] \end{aligned}$$

Taking the ratio of currents, but maintaining I_D a constant, we have

$$1 = \frac{\exp\left[\frac{-(E_g - eV_{D1})}{kT_1}\right]}{\exp\left[\frac{-(E_g - eV_{D2})}{kT_2}\right]} \Rightarrow$$

$$\frac{E_g - eV_{D1}}{kT_1} = \frac{E_g - eV_{D2}}{kT_2}$$

We have

$$T = 300K, V_{D1} = 0.60V \text{ and}$$

$$kT_1 = 0.0259 eV, \frac{kT_1}{e} = 0.0259V$$

$$T = 310K$$

$$kT_2 = 0.02676 eV, \frac{kT_2}{e} = 0.02676V$$

$$T = 320K$$

$$kT_3 = 0.02763 eV, \frac{kT_3}{e} = 0.02763V$$

So, for $T = 310K$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D2}}{0.02676}$$

which yields

$$V_{D2} = 0.5827V$$

For $T = 320K$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D3}}{0.02763}$$

which yields

$$V_{D3} = 0.5653V$$

8.21

Computer Plot

8.22

$$g_d = \frac{e}{kT} \cdot I_D = \frac{2 \times 10^{-3}}{0.0259}$$

or

$$g_d = 0.0772 S$$

Also

$$C_d = \frac{1}{2} \left(\frac{e}{kT} \right) (I_{pO} \tau_{pO} + I_{nO} \tau_{nO})$$

We have

$$\tau_{pO} = \tau_{nO} = 10^{-6} s$$

$$I_{pO} + I_{nO} = 2 \times 10^{-3} A$$

Then

$$C_d = \frac{(2 \times 10^{-3})(10^{-6})}{2(0.0259)} \Rightarrow$$

$$C_d = 3.86 \times 10^{-8} F$$

Then

$$Y = g_d + j\omega C_d$$

or

$$Y = 0.0772 + j\omega(3.86 \times 10^{-8})$$

8.23 For a p^+n diode

$$g_d = \frac{I_{DQ}}{V_t}, \quad C_d = \frac{I_{DQ} \tau_{pO}}{2V_t}$$

Now

$$g_d = \frac{10^{-3}}{0.0259} = 3.86 \times 10^{-2} S$$

and

$$C_d = \frac{(10^{-3})(10^{-7})}{2(0.0259)} = 1.93 \times 10^{-9} F$$

Now

$$Z = \frac{1}{Y} = \frac{1}{g_d + j\omega C_d} = \frac{g_d - j\omega C_d}{g_d^2 + \omega^2 C_d^2}$$

We have $\omega = 2\pi f$,

We find:

$$f = 10 \text{ kHz} : Z = 25.9 - j0.0814$$

$$f = 100 \text{ kHz} : Z = 25.9 - j0.814$$

$$f = 1 \text{ MHz} : Z = 23.6 - j7.41$$

$$f = 10 \text{ MHz} : Z = 2.38 - j7.49$$

8.24

(b)

Two capacitances will be equal at some forward-bias voltage.

For a forward-bias voltage, the junction capacitance is

$$C_j = A \left[\frac{e \epsilon N_a N_d}{2(V_{bi} - V_a)(N_a + N_d)} \right]^{1/2}$$

The diffusion capacitance is

$$C_d = \left(\frac{1}{2V_t} \right) (I_{pO} \tau_{pO} + I_{nO} \tau_{nO})$$

where

$$I_{pO} = \frac{A e n_i^2}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

and

$$I_{nO} = \frac{Aen_i^2}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

We find

$$D_p = (320)(0.0259) = 8.29 \text{ cm}^2 / \text{s}$$

$$D_n = (850)(0.0259) = 22.0 \text{ cm}^2 / \text{s}$$

and

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{17})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.7363 \text{ V}$$

Now, we obtain

$$C_j = (10^{-4}) \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(V_{bi} - V_a)} \times \frac{(5 \times 10^{15})(10^{17})}{(5 \times 10^{15} + 10^{17})} \right]^{1/2}$$

or

$$C_j = (10^{-4}) \left[\frac{3.945 \times 10^{-16}}{(V_{bi} - V_a)} \right]^{1/2}$$

We also obtain

$$I_{pO} = \frac{(10^{-4})(1.6 \times 10^{-19})(1.5 \times 10^{10})^2}{10^{17}} \sqrt{\frac{8.29}{10^{-7}}} \times \exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{pO} = 3.278 \times 10^{-16} \exp\left(\frac{V_a}{V_t}\right)$$

Also

$$I_{nO} = \frac{(10^{-4})(1.6 \times 10^{-19})(1.5 \times 10^{10})^2}{5 \times 10^{15}} \sqrt{\frac{22}{10^{-6}}} \times \exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{nO} = 3.377 \times 10^{-15} \exp\left(\frac{V_a}{V_t}\right)$$

We can now write

$$C_d = \frac{1}{2(0.0259)} \left[(3.278 \times 10^{-16})(10^{-7}) + (3.377 \times 10^{-15})(10^{-6}) \right] \cdot \exp\left(\frac{V_a}{V_t}\right)$$

or

$$C_d = 6.583 \times 10^{-20} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

We want to set $C_j = C_d$

So

$$(10^{-4}) \left[\frac{3.945 \times 10^{-16}}{0.7363 - V_a} \right]^{1/2} = 6.583 \times 10^{-20} \cdot \exp\left(\frac{V_a}{0.0259}\right)$$

By trial and error, we find

$$V_a = 0.463 \text{ V}$$

At this voltage,

$$C_j = C_d \approx 3.8 \text{ pF}$$

8.25

For a p^+n diode, $I_{pO} \gg I_{nO}$, then

$$C_d = \left(\frac{1}{2V_t} \right) (I_{pO} \tau_{pO})$$

Now

$$\frac{\tau_{pO}}{2V_t} = 2.5 \times 10^{-6} \text{ F / A}$$

Then

$$\tau_{pO} = 2(0.0259)(2.5 \times 10^{-6})$$

or

$$\tau_{pO} = 1.3 \times 10^{-7} \text{ s}$$

At 1 mA,

$$C_d = (2.5 \times 10^{-6})(10^{-3}) \Rightarrow$$

$$C_d = 2.5 \times 10^{-9} \text{ F}$$

8.26

$$(a) \quad C_d = \frac{1}{2} \left(\frac{e}{kT} \right) A (I_{pO} \tau_{pO} + I_{nO} \tau_{nO})$$

For a one-sided n^+p diode, $I_{nO} \gg I_{pO}$, then

$$C_d = \frac{1}{2} \left(\frac{e}{kT} \right) A (I_{nO} \tau_{nO})$$

so

$$10^{-12} = \frac{1}{2} \left(\frac{1}{0.0259} \right) (10^{-3}) (I_{nO}) (10^{-7})$$

or

$$(b) \quad \underline{I_{nO} = I_D = 0.518 \text{ mA}}$$

$$I_{nO} = A \frac{e D_n n_{pO}}{L_n} \exp \left(\frac{V_a}{V_t} \right)$$

We find

$$L_n = \sqrt{D_n \tau_{nO}} = 15.8 \text{ } \mu\text{m} \text{ and}$$

$$n_{pO} = \frac{n_i^2}{N_a} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Then

$$\begin{aligned} 0.518 \times 10^{-3} \\ = \frac{(1.6 \times 10^{-19})(25)(2.25 \times 10^4)(10^{-3})}{15.8 \times 10^{-4}} \exp \left(\frac{V_a}{V_t} \right) \end{aligned}$$

or

$$0.518 \times 10^{-3} = 5.70 \times 10^{-14} \exp \left(\frac{V_a}{0.0259} \right)$$

We find

$$(c) \quad \underline{V_a = 0.594 \text{ V}}$$

$$g_d = \left(\frac{e}{kT} \right) I_D = \frac{1}{r_d} \Rightarrow$$

$$r_d = \frac{0.0259}{0.518 \times 10^{-3}}$$

or

$$\underline{r_d = 50 \text{ } \Omega}$$

8.27

(a) p-region

$$R_p = \frac{\rho_p L}{A} = \frac{L}{\sigma_p A} = \frac{L}{A(e\mu_p N_a)}$$

so

$$R_p = \frac{0.2}{(10^{-2})(1.6 \times 10^{-19})(480)(10^{16})}$$

or

$$R_p = 26 \text{ } \Omega$$

n-region

$$R_n = \frac{\rho_n L}{A} = \frac{L}{\sigma_n A} = \frac{L}{A(e\mu_n N_d)}$$

so

$$R_n = \frac{0.10}{(10^{-2})(1.6 \times 10^{-19})(1350)(10^{15})}$$

or

$$R_n = 46.3 \text{ } \Omega$$

The total series resistance is

$$R = R_p + R_n = 26 + 46.3 \Rightarrow$$

$$\underline{R = 72.3 \text{ } \Omega}$$

(b)

$$V = IR \Rightarrow 0.1 = I(72.3)$$

or

$$\underline{I = 1.38 \text{ mA}}$$

8.28

$$\begin{aligned} R &= \frac{\rho_n L(n)}{A(n)} + \frac{\rho_p L(p)}{A(p)} \\ &= \frac{(0.2)(10^{-2})}{2 \times 10^{-5}} + \frac{(0.1)(10^{-2})}{2 \times 10^{-5}} \end{aligned}$$

or

$$\underline{R = 150 \text{ } \Omega}$$

We can write

$$V = I_D R + V_t \ln \left(\frac{I_D}{I_s} \right)$$

(a) (i) $I_D = 1 \text{ mA}$

$$V = (10^{-3})(150) + (0.0259) \ln \left(\frac{10^{-3}}{10^{-10}} \right)$$

or

$$\underline{V = 0.567 \text{ V}}$$

(ii) $I_D = 10 \text{ mA}$

$$V = (10 \times 10^{-3})(150) + (0.0259) \ln \left(\frac{10 \times 10^{-3}}{10^{-10}} \right)$$

or $V = 1.98 \text{ V}$

(b)

For $R = 0$

(i) $I_D = 1 \text{ mA}$

$$V = (0.0259) \ln \left(\frac{10^{-3}}{10^{-10}} \right) \Rightarrow$$

$$\underline{V = 0.417 \text{ V}}$$

(ii) $I_D = 10 \text{ mA}$

$$V = (0.0259) \ln \left(\frac{10 \times 10^{-3}}{10^{-10}} \right) \Rightarrow$$

$$\underline{V = 0.477 \text{ V}}$$

8.29

$$r_d = 48 \, \Omega = \frac{1}{g_d} \Rightarrow g_d = 0.0208$$

We have

$$g_d = \frac{e}{kT} \cdot I_D \Rightarrow I_D = (0.0208)(0.0259)$$

or

$$I_D = 0.539 \, \text{mA}$$

Also

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) \Rightarrow V_a = V_t \ln\left(\frac{I_D}{I_S}\right)$$

so

$$V_a = (0.0259) \ln\left(\frac{0.539 \times 10^{-3}}{2 \times 10^{-11}}\right) \Rightarrow$$

$$V_a = 0.443 \, \text{V}$$

8.30

$$(a) \quad \frac{1}{r_d} = \frac{dI_D}{dV_a} = I_S \left(\frac{1}{V_t}\right) \exp\left(\frac{V_a}{V_t}\right)$$

or

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259} \cdot \exp\left(\frac{0.020}{0.0259}\right)$$

which yields

$$r_d = 1.2 \times 10^{11} \, \Omega$$

(b)

For $V_a = -0.020 \, \text{V}$,

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259} \cdot \exp\left(\frac{-0.020}{0.0259}\right)$$

or

$$r_d = 5.6 \times 10^{11} \, \Omega$$

8.31

Ideal reverse-saturation current density

$$J_S = \frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}$$

We find

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \, \text{cm}^{-3}$$

and

$$p_{nO} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \, \text{cm}^{-3}$$

Also

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(200)(10^{-8})} = 14.2 \, \mu\text{m}$$

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(6)(10^{-8})} = 2.45 \, \mu\text{m}$$

Then

$$J_S = \frac{(1.6 \times 10^{-19})(200)(3.24 \times 10^{-4})}{14.2 \times 10^{-4}} + \frac{(1.6 \times 10^{-19})(6)(3.24 \times 10^{-4})}{2.45 \times 10^{-4}}$$

so

$$J_S = 8.57 \times 10^{-18} \, \text{A/cm}^2$$

Reverse-biased generation current density

$$J_{\text{gen}} = \frac{en_i W}{2\tau_o}$$

We have

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left[\frac{(10^{16})(10^{16})}{(1.8 \times 10^6)^2}\right]$$

or

$$V_{bi} = 1.16 \, \text{V}$$

And

$$W = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} = \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.16 + 5)}{1.6 \times 10^{-19}} \times \left[\frac{10^{16} + 10^{16}}{(10^{16})(10^{16})} \right] \right]^{1/2}$$

or

$$W = 1.34 \times 10^{-4} \, \text{cm}$$

Then

$$J_{\text{gen}} = \frac{(1.6 \times 10^{-19})(1.8 \times 10^6)(1.34 \times 10^{-4})}{2(10^{-8})}$$

or

$$J_{\text{gen}} = 1.93 \times 10^{-9} \, \text{A/cm}^2$$

Generation current dominates in GaAs reverse-biased junctions.

8.32

(a) We can write

$$J_s = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

$$= n_i^2 (1.6 \times 10^{19}) \left[\frac{1}{10^{16}} \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{10^{16}} \sqrt{\frac{10}{5 \times 10^{-7}}} \right]$$

or

$$J_s = n_i^2 (1.85 \times 10^{-31})$$

We also have

$$J_{gen} = \frac{en_i W}{2\tau_o}$$

For $V_{bi} + V_R = 5V$, we find $W = 1.14 \times 10^{-4} \text{ cm}$

So

$$J_{gen} = \frac{(1.6 \times 10^{-19})(1.14 \times 10^{-4})n_i}{2(5 \times 10^{-7})}$$

or

$$J_{gen} = n_i (1.82 \times 10^{-17})$$

When $J_s = J_{gen}$,

$$1.85 \times 10^{-31} n_i = 1.82 \times 10^{-17}$$

which yields

$$n_i = 9.88 \times 10^{13} \text{ cm}^{-3}$$

We have

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$(9.88 \times 10^{13})^2 = (2.8 \times 10^{19})(1.04 \times 10^{19}) \left(\frac{T}{300}\right)^3$$

$$\times \exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

By trial and error, we find

$$T = 505K$$

At this temperature

$$J_s = J_{gen} = (1.82 \times 10^{-17})(9.88 \times 10^{13}) \Rightarrow$$

$$J_s = J_{gen} = 1.8 \times 10^{-3} \text{ A / cm}^2$$

(b)

$$J_s \exp\left(\frac{V_a}{V_t}\right) = J_{gen} \exp\left(\frac{V_a}{2V_t}\right)$$

At $T = 300K$

$$J_s = (1.5 \times 10^{10})^2 (1.85 \times 10^{-31})$$

or

$$J_s = 4.16 \times 10^{-11} \text{ A / cm}^2$$

and

$$J_{gen} = (1.5 \times 10^{10})(1.82 \times 10^{-17}) \Rightarrow$$

or

$$J_{gen} = 2.73 \times 10^{-7} \text{ A / cm}^2$$

Then we can write

$$\exp\left(\frac{V_a}{2V_t}\right) = \frac{J_{gen}}{J_s} = \frac{2.73 \times 10^{-7}}{4.16 \times 10^{-11}} = 6.56 \times 10^3$$

so that

$$V_a = 2(0.0259) \ln(6.56 \times 10^3) \Rightarrow$$

$$V_a = 0.455 V$$

8.33

(a) We can write

$$J_s = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

We find

$$D_n = (3000)(0.0259) = 77.7 \text{ cm}^2 / s$$

$$D_p = (200)(0.0259) = 5.18 \text{ cm}^2 / s$$

Then

$$J_s = (1.6 \times 10^{-19})(1.8 \times 10^6)^2 \left[\frac{1}{10^{17}} \sqrt{\frac{77.7}{10^{-8}}} + \frac{1}{10^{17}} \sqrt{\frac{5.18}{10^{-8}}} \right]$$

or

$$J_s = 5.75 \times 10^{-19} \text{ A / cm}^2$$

so

$$I_s = A J_s = (10^{-3})(5.75 \times 10^{-19})$$

or

$$I_s = 5.75 \times 10^{-22} \text{ A}$$

We also have

$$I_{gen} = \frac{en_i W A}{2\tau_o}$$

Now

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$= (0.0259) \ln \left[\frac{(10^{17})(10^{17})}{(1.8 \times 10^6)^2} \right]$$

or

$$V_{bi} = 1.28 \text{ V}$$

Also

$$W = \left[\frac{2 \in (V_{bi} + V_R) \left(\frac{N_a + N_d}{N_a N_d} \right)}{e} \right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.28 + 5)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{10^{17} + 10^{17}}{(10^{17})(10^{17})} \right) \right]^{1/2}$$

or

$$W = 0.427 \times 10^{-4} \text{ cm}$$

so

$$I_{gen} = \frac{(1.6 \times 10^{-19})(1.8 \times 10^6)(0.427 \times 10^{-4})(10^{-3})}{2(10^{-8})}$$

or

$$I_{gen} = 6.15 \times 10^{-13} \text{ A}$$

The total reverse-bias current

$$I_R = I_S + I_{gen} = 5.75 \times 10^{-22} + 6.15 \times 10^{-13}$$

or

$$I_R \approx 6.15 \times 10^{-13} \text{ A}$$

Forward Bias: Ideal diffusion current

For $V_a = 0.3 \text{ V}$

$$I_D = I_S \exp \left(\frac{V_a}{V_t} \right) = (5.75 \times 10^{-22}) \exp \left(\frac{0.3}{0.0259} \right)$$

or

$$I_D = 6.17 \times 10^{-17} \text{ A}$$

For $V_a = 0.5 \text{ V}$

$$I_D = (5.75 \times 10^{-22}) \exp \left(\frac{0.5}{0.0259} \right)$$

or

$$I_D = 1.39 \times 10^{-13} \text{ A}$$

Recombination current

For $V_a = 0.3 \text{ V}$:

$$W = \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.28 - 0.3) \left(\frac{2 \times 10^{17}}{10^{34}} \right)}{1.6 \times 10^{-19}} \right]^{1/2}$$

or

$$W = 0.169 \times 10^{-4} \text{ cm}$$

Then

$$I_{rec} = \frac{en_i W A}{2\tau_o} \exp \left(\frac{V_a}{2V_t} \right)$$

$$= \frac{(1.6 \times 10^{-19})(1.8 \times 10^6)(0.169 \times 10^{-4})(10^{-3})}{2(10^{-8})} \\ \times \exp \left[\frac{0.3}{2(0.0259)} \right]$$

or

$$I_{rec} = 7.96 \times 10^{-11} \text{ A}$$

For $V_a = 0.5 \text{ V}$

$$W = \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.28 - 0.5) \left(\frac{2 \times 10^{17}}{10^{34}} \right)}{1.6 \times 10^{-19}} \right]^{1/2}$$

or

$$W = 0.150 \times 10^{-4} \text{ cm}$$

Then

$$I_{rec} = \frac{(1.6 \times 10^{-19})(1.8 \times 10^6)(0.15 \times 10^{-4})(10^{-3})}{2(10^{-8})} \\ \times \exp \left[\frac{0.5}{2(0.0259)} \right]$$

or

$$I_{rec} = 3.36 \times 10^{-9} \text{ A}$$

Total forward-bias current:

For $V_a = 0.3 \text{ V}$;

$$I_D = 6.17 \times 10^{-17} + 7.96 \times 10^{-11}$$

or

$$I_D \approx 7.96 \times 10^{-11} \text{ A}$$

For $V_a = 0.5 \text{ V}$

$$I_D = 1.39 \times 10^{-13} + 3.36 \times 10^{-9}$$

or

$$I_D \approx 3.36 \times 10^{-9} \text{ A}$$

(b)

Reverse-bias; ratio of generation to ideal diffusion current:

$$\frac{I_{gen}}{I_S} = \frac{6.15 \times 10^{-13}}{5.75 \times 10^{-22}}$$

Ratio = 1.07×10^9
Forward bias: Ratio of recombination to ideal
diffusion current:

For $V_a = 0.3 \text{ V}$

$$\frac{I_{rec}}{I_D} = \frac{7.96 \times 10^{-11}}{6.17 \times 10^{-17}}$$

Ratio = 1.29×10^6
For $V_a = 0.5 \text{ V}$

$$\frac{I_{rec}}{I_D} = \frac{3.36 \times 10^{-9}}{1.39 \times 10^{-13}}$$

$$\text{Ratio} = 2.42 \times 10^4$$

8.34

Computer Plot

8.35

Computer Plot

8.36

Computer Plot

8.37

We have that

$$R = \frac{np - n_i^2}{\tau_{pO}(n + n') + \tau_{nO}(p + p')}$$

Let $\tau_{pO} = \tau_{nO} = \tau_O$ and $n' = p' = n_i$

We can write

$$n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

We also have

$$(E_{Fn} - E_{Fi}) + (E_{Fi} - E_{Fp}) = eV_a$$

so that

$$(E_{Fi} - E_{Fp}) = eV_a - (E_{Fn} - E_{Fi})$$

Then

$$p = n_i \exp\left[\frac{eV_a - (E_{Fn} - E_{Fi})}{kT}\right]$$

$$= n_i \exp\left(\frac{eV_a}{kT}\right) \cdot \exp\left[\frac{-(E_{Fn} - E_{Fi})}{kT}\right]$$

Define

$$\eta_a = \frac{eV_a}{kT} \quad \text{and} \quad \eta = \left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Then the recombination rate can be written as

$$R = \frac{(n_i e^\eta)(n_i e^{\eta_s} \cdot e^{-\eta}) - n_i^2}{\tau_O [n_i e^\eta + n_i + n_i e^{\eta_s} \cdot e^{-\eta} + n_i]}$$

or

$$R = \frac{n_i (e^{\eta_s} - 1)}{\tau_O (2 + e^\eta + e^{\eta_s} \cdot e^{-\eta})}$$

To find the maximum recombination rate, set

$$\frac{dR}{d\eta} = 0$$

$$= \frac{n_i (e^{\eta_s} - 1)}{\tau_O} \cdot \frac{d}{dx} [2 + e^\eta + e^{\eta_s} \cdot e^{-\eta}]^{-1}$$

or

$$0 = \frac{n_i (e^{\eta_s} - 1)}{\tau_O} \cdot (-1) [2 + e^\eta + e^{\eta_s} \cdot e^{-\eta}]^{-2}$$

$$\times [e^\eta - e^{\eta_s} \cdot e^{-\eta}]$$

which simplifies to

$$0 = \frac{-n_i (e^{\eta_s} - 1)}{\tau_O} \cdot \frac{[e^\eta - e^{\eta_s} \cdot e^{-\eta}]}{[2 + e^\eta + e^{\eta_s} \cdot e^{-\eta}]^2}$$

The denominator is not zero, so we have

$$e^\eta - e^{\eta_s} \cdot e^{-\eta} = 0 \Rightarrow$$

$$e^{2\eta} = e^{\eta_s} \Rightarrow \eta = \frac{1}{2} \eta_s$$

Then the maximum recombination rate becomes

$$R_{\max} = \frac{n_i (e^{\eta_s} - 1)}{\tau_O [2 + e^{\eta_s/2} + e^{\eta_s} \cdot e^{-\eta_s/2}]}$$

$$= \frac{n_i (e^{\eta_s} - 1)}{\tau_O [2 + e^{\eta_s/2} + e^{\eta_s/2}]}$$

or

$$R_{\max} = \frac{n_i (e^{\eta_s} - 1)}{2\tau_O (e^{\eta_s/2} + 1)}$$

which can be written as

$$R_{\max} = \frac{n_i \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]}{2\tau_O \left[\exp\left(\frac{eV_a}{2kT}\right) + 1 \right]}$$

If $V_a \gg \left(\frac{kT}{e}\right)$, then we can neglect the (-1)

term in the numerator and the (+1) term in the denominator so we finally have

$$R_{\max} = \frac{n_i}{2\tau_o} \exp\left(\frac{eV_a}{2kT}\right)$$

Q.E.D.

8.38

We have

$$J_{\text{gen}} = \int_0^W eGdx$$

In this case, $G = g' = 4 \times 10^{19} \text{ cm}^{-3} \text{ s}^{-1}$, that is a constant through the space charge region. Then

$$J_{\text{gen}} = eg'W$$

We find

$$\begin{aligned} V_{bi} &= V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \\ &= (0.0259) \ln\left[\frac{(5 \times 10^{15})(5 \times 10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.659 \text{ V} \end{aligned}$$

and

$$\begin{aligned} W &= \left[\frac{2 \in (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.659 + 10)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{5 \times 10^{15} + 5 \times 10^{15}}{(5 \times 10^{15})(5 \times 10^{15})} \right) \right]^{1/2} \end{aligned}$$

or

$$W = 2.35 \times 10^{-4} \text{ cm}$$

Then

$$J_{\text{gen}} = (1.6 \times 10^{-19})(4 \times 10^{19})(2.35 \times 10^{-4})$$

or

$$J_{\text{gen}} = 1.5 \times 10^{-3} \text{ A / cm}^2$$

8.39

$$\begin{aligned} J_s &= en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{po}}} \right] \\ &= (1.6 \times 10^{-19})(1.5 \times 10^{10})^2 \left[\frac{1}{3 \times 10^{16}} \sqrt{\frac{18}{10^{-7}}} \right. \\ &\quad \left. + \frac{1}{10^{18}} \sqrt{\frac{6}{10^{-7}}} \right] \end{aligned}$$

or

$$J_s = 1.64 \times 10^{-11} \text{ A / cm}^2$$

Now

$$J_D = J_s \exp\left(\frac{V_D}{V_t}\right)$$

Also

$$J = 0 = J_G - J_D$$

or

$$0 = 25 \times 10^{-3} - 1.64 \times 10^{-11} \exp\left(\frac{V_D}{V_t}\right)$$

which yields

$$\exp\left(\frac{V_D}{V_t}\right) = 1.52 \times 10^9$$

or

$$V_D = V_t \ln(1.52 \times 10^9)$$

so

$$V_D = 0.548 \text{ V}$$

8.40

$$V_B = \frac{\in E_{\text{crit}}^2}{2eN_B}$$

or

$$30 = \frac{(11.7)(8.85 \times 10^{-14})(4 \times 10^5)^2}{2(1.6 \times 10^{-19})N_B}$$

which yields

$$N_B = N_d = 1.73 \times 10^{16} \text{ cm}^{-3}$$

8.41

For the breakdown voltage, we need

$N_d = 3 \times 10^{15} \text{ cm}^{-3}$ and for this doping, we find

$\mu_p = 430 \text{ cm}^2 / \text{V} \cdot \text{s}$. Then

$$D_p = (430)(0.0259) = 11.14 \text{ cm}^2 / \text{s}$$

For the p^+n junction,

$$\begin{aligned} J_s &= en_i^2 \cdot \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{po}}} \\ &= \frac{(1.6 \times 10^{-19})(1.5 \times 10^{10})^2}{3 \times 10^{15}} \sqrt{\frac{11.14}{10^{-7}}} \end{aligned}$$

or

$$J_s = 1.27 \times 10^{-10} \text{ A / cm}^2$$

Then

$$I = J_s A \exp\left(\frac{V_a}{V_t}\right)$$

$$2 \times 10^{-3} = (1.27 \times 10^{-10}) A \exp\left(\frac{0.65}{0.0259}\right)$$

Finally

$$A = 1.99 \times 10^{-4} \text{ cm}^2$$

8.42

GaAs, n^+p , and $N_a = 10^{16} \text{ cm}^{-3}$

From Figure 8.25

$$V_B \approx 75 \text{ V}$$

8.43

$$E_{\max} = \frac{eN_d x_n}{\epsilon}$$

We can write

$$x_n = \frac{E_{\max} \epsilon}{eN_d}$$

$$= \frac{(4 \times 10^5)(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(5 \times 10^{16})}$$

or

$$x_n = 5.18 \times 10^{-5} \text{ cm}$$

We find

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{16})(5 \times 10^{16})}{(1.5 \times 10^{10})^2} \right] = 0.778 \text{ V}$$

Now

$$x_n = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2}$$

or

$$(5.18 \times 10^{-5})^2 = \left[\frac{2(11.7)(8.85 \times 10^{-14})}{1.6 \times 10^{-19}} \right. \\ \left. \times (V_{bi} + V_R) \left(\frac{5 \times 10^{16}}{5 \times 10^{16}} \right) \left(\frac{1}{5 \times 10^{16} + 5 \times 10^{16}} \right) \right]$$

which yields

$$2.68 \times 10^{-9} = 1.29 \times 10^{-10} (V_{bi} + V_R)$$

so

$$V_{bi} + V_R = 20.7 \Rightarrow V_R = 19.9 \text{ V}$$

8.44

For a silicon p^+n junction with

$$N_d = 5 \times 10^{15} \text{ cm}^{-3} \text{ and } V_B \approx 100 \text{ V}$$

Neglecting V_{bi} compared to V_B

$$x_n \approx \left[\frac{2 \epsilon V_B}{eN_d} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(100)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$$

or

$$x_n (\text{min}) = 5.09 \text{ } \mu\text{m}$$

8.45

We find

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{18})(10^{18})}{(1.5 \times 10^{10})^2} \right] = 0.933 \text{ V}$$

Now

$$E_{\max} = \frac{eN_d x_n}{\epsilon}$$

so

$$10^6 = \frac{(1.6 \times 10^{-19})(10^{18})x_n}{(11.7)(8.85 \times 10^{-14})}$$

which yields

$$x_n = 6.47 \times 10^{-6} \text{ cm}$$

Now

$$x_n = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2}$$

Then

$$(6.47 \times 10^{-6})^2 = \left[\frac{2(11.7)(8.85 \times 10^{-14})}{1.6 \times 10^{-19}} \right. \\ \left. \times (V_{bi} + V_R) \left(\frac{10^{18}}{10^{18}} \right) \left(\frac{1}{10^{18} + 10^{18}} \right) \right]$$

which yields

$$V_{bi} + V_R = 6.468 \text{ V}$$

or

$$V_R = 5.54 \text{ V}$$

8.46

Assume silicon: For an n^+p junction

$$x_p = \left[\frac{2 \epsilon (V_{bi} + V_R)}{eN_a} \right]^{1/2}$$

Assume $V_{bi} \ll V_R$

(a)

For $x_p = 75 \mu m$

Then

$$(75 \times 10^{-4})^2 = \frac{2(11.7)(8.85 \times 10^{-14})V_R}{(1.6 \times 10^{-19})(10^{15})}$$

which yields $V_R = 4.35 \times 10^3 V$

(b)

For $x_p = 150 \mu m$, we find

$$V_R = 1.74 \times 10^4 V$$

From Figure 8.25, the breakdown voltage is approximately 300 V. So, in each case, breakdown is reached first.

8.47

Impurity gradient

$$a = \frac{2 \times 10^{18}}{2 \times 10^{-4}} = 10^{22} cm^{-4}$$

From the figure

$$V_B = 15 V$$

8.48

(a) If $\frac{I_R}{I_F} = 0.2$

Then we have

$$erf \sqrt{\frac{t_s}{\tau_{pO}}} = \frac{I_F}{I_F + I_R} = \frac{1}{1 + \frac{I_R}{I_F}} = \frac{1}{1 + 0.2}$$

or

$$erf \sqrt{\frac{t_s}{\tau_{pO}}} = 0.833$$

We find

$$\sqrt{\frac{t_s}{\tau_{pO}}} = 0.978 \Rightarrow \frac{t_s}{\tau_{pO}} = 0.956$$

(b)

If $\frac{I_R}{I_F} = 1.0$, then

$$erf \sqrt{\frac{t_s}{\tau_{pO}}} = \frac{1}{1 + 1} = 0.5$$

which yields

$$\frac{t_s}{\tau_{pO}} = 0.228$$

8.49

We want

$$\frac{t_s}{\tau_{pO}} = 0.2$$

Then

$$erf \sqrt{\frac{t_s}{\tau_{pO}}} = \frac{1}{1 + \frac{I_R}{I_F}} = erf \sqrt{0.2}$$

where

$$erf \sqrt{0.2} = erf(0.447) = 0.473$$

We obtain

$$\frac{I_R}{I_F} = \frac{1}{0.473} - 1 \Rightarrow \frac{I_R}{I_F} = 1.11$$

We have

$$erf \sqrt{\frac{t_2}{\tau_{pO}}} + \frac{\exp\left(\frac{-t_2}{\tau_{pO}}\right)}{\sqrt{\pi\left(\frac{t_2}{\tau_{pO}}\right)}} = 1 + (0.1)\left(\frac{I_R}{I_F}\right) = 1.11$$

By trial and error,

$$\frac{t_2}{\tau_{pO}} = 0.65$$

8.50

$C_j = 18 pF$ at $V_R = 0$

$C_j = 4.2 pF$ at $V_R = 10 V$

We have $\tau_{nO} = \tau_{pO} = 10^{-7} s$, $I_F = 2 mA$

And $I_R \approx \frac{V_R}{R} = \frac{10}{10} = 1 mA$

So

$$t_s \approx \tau_{pO} \ln\left(1 + \frac{I_F}{I_R}\right) = (10^{-7}) \ln\left(1 + \frac{2}{1}\right)$$

or

$$t_s = 1.1 \times 10^{-7} s$$

Also

$$C_{avg} = \frac{18 + 4.2}{2} = 11.1 pF$$

The time constant is

$$\tau_s = RC_{avg} = (10^4)(11.1 \times 10^{-12}) = 1.11 \times 10^{-7} \text{ s}$$

Now

$$\text{Turn-off time} = t_s + \tau_s = (1.1 + 1.11) \times 10^{-7} \text{ s}$$

Or

$$\underline{2.21 \times 10^{-7} \text{ s}}$$

8.51

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{19})^2}{(1.5 \times 10^{10})^2} \right] = 1.14 \text{ V}$$

We find

$$W = \left[\frac{2 \epsilon (V_{bi} - V_a)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(1.14 - 0.40)}{1.6 \times 10^{-19}} \times \left(\frac{5 \times 10^{19} + 5 \times 10^{19}}{(5 \times 10^{19})^2} \right) \right]^{1/2}$$

which yields

$$\underline{W = 6.19 \times 10^{-7} \text{ cm} = 61.9 \text{ \AA}}$$

8.52
Sketch

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Chapter 9

Problem Solutions

9.1

(a) We have

$$\begin{aligned} e\phi_n &= eV_t \ln\left(\frac{N_c}{N_d}\right) \\ &= (0.0259) \ln\left(\frac{2.8 \times 10^{19}}{10^{16}}\right) = 0.206 \text{ eV} \end{aligned}$$

(c)

$$\phi_{BO} = \phi_m - \chi = 4.28 - 4.01$$

or

$$\phi_{BO} = 0.27 \text{ V}$$

and

$$V_{bi} = \phi_{BO} - \phi_n = 0.27 - 0.206$$

or

$$V_{bi} = 0.064 \text{ V}$$

Also

$$\begin{aligned} x_d &= \left[\frac{2 \epsilon V_{bi}}{eN_d} \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.064)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2} \end{aligned}$$

or

$$x_d = 9.1 \times 10^{-6} \text{ cm}$$

Then

$$\begin{aligned} |E_{\max}| &= \frac{eN_d x_d}{\epsilon} \\ &= \frac{(1.6 \times 10^{-19})(10^{16})(9.1 \times 10^{-6})}{(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$|E_{\max}| = 1.41 \times 10^4 \text{ V / cm}$$

(d)

Using the figure, $\phi_{Bn} = 0.55 \text{ V}$

So

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344 \text{ V}$$

We then find

$$x_d = 2.11 \times 10^{-5} \text{ cm} \quad \text{and} \quad E_{\max} = 3.26 \times 10^4 \text{ V / cm}$$

9.2

$$(a) \quad \phi_{BO} = \phi_m - \chi = 5.1 - 4.01$$

or

$$\phi_{BO} = 1.09 \text{ V}$$

(b)

$$\begin{aligned} \phi_n &= V_t \ln\left(\frac{N_c}{N_d}\right) \\ &= (0.0259) \ln\left(\frac{2.8 \times 10^{19}}{10^{15}}\right) = 0.265 \text{ V} \end{aligned}$$

Then

$$V_{bi} = \phi_{BO} - \phi_n = 1.09 - 0.265$$

or

$$V_{bi} = 0.825 \text{ V}$$

(c)

$$\begin{aligned} W = x_d &= \left[\frac{2 \epsilon V_{bi}}{eN_d} \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.825)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2} \end{aligned}$$

or

$$W = 1.03 \times 10^{-4} \text{ cm}$$

(d)

$$\begin{aligned} |E_{\max}| &= \frac{eN_d x_d}{\epsilon} \\ &= \frac{(1.6 \times 10^{-19})(10^{15})(1.03 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$|E_{\max}| = 1.59 \times 10^4 \text{ V / cm}$$

9.3

(a) Gold on n-type GaAs

$$\chi = 4.07 \text{ V} \quad \text{and} \quad \phi_m = 5.1 \text{ V}$$

$$\phi_{BO} = \phi_m - \chi = 5.1 - 4.07$$

and

$$\phi_{BO} = 1.03 \text{ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{5 \times 10^{16}} \right)$$

or

$$\phi_n = 0.0580 \text{ V}$$

(c)

$$V_{bi} = \phi_{BO} - \phi_n = 1.03 - 0.058$$

or

$$V_{bi} = 0.972 \text{ V}$$

(d)

$$x_d = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(0.972 + 5)}{(1.6 \times 10^{-19})(5 \times 10^{16})} \right]^{1/2}$$

or

$$x_d = 0.416 \text{ } \mu\text{m}$$

(e)

$$|E_{\max}| = \frac{e N_d x_d}{\epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(5 \times 10^{16})(0.416 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 2.87 \times 10^5 \text{ V / cm}$$

9.4

$\phi_{Bn} = 0.86 \text{ V}$ and $\phi_n = 0.058 \text{ V}$ (Problem 9.3)

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.86 - 0.058$$

or

$$V_{bi} = 0.802 \text{ V}$$

and

$$x_d = \left[\frac{2 \epsilon (V_{bi} + V_R)}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(0.802 + 5)}{(1.6 \times 10^{-19})(5 \times 10^{16})} \right]^{1/2}$$

or]

$$x_d = 0.410 \text{ } \mu\text{m}$$

Also

$$|E_{\max}| = \frac{e N_d x_d}{\epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(5 \times 10^{16})(0.410 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 2.83 \times 10^5 \text{ V / cm}$$

9.5

Gold, n-type silicon junction. From the figure,

$$\phi_{Bn} = 0.81 \text{ V}$$

For $N_d = 5 \times 10^{15} \text{ cm}^{-3}$, we have

$$\phi_n = V_t \ln \left(\frac{N_c}{N_d} \right)$$

$$= (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{5 \times 10^{15}} \right) = \phi_n = 0.224 \text{ V}$$

Then

$$V_{bi} = 0.81 - 0.224 = 0.586 \text{ V}$$

(a)

Now

$$C' = \left[\frac{e \epsilon N_d}{2(V_{bi} + V_R)} \right]^{1/2}$$

$$= \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(5 \times 10^{15})}{2(0.586 + 4)} \right]^{1/2}$$

or

$$C' = 9.50 \times 10^{-9} \text{ F / cm}^2$$

For $A = 5 \times 10^{-4} \text{ cm}^2$, $C = C'A$

So

$$C = 4.75 \text{ pF}$$

(b)

For $N_d = 5 \times 10^{16} \text{ cm}^{-3}$, we find

$$\phi_n = (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{5 \times 10^{16}} \right) = 0.164 \text{ V}$$

Then

$$V_{bi} = 0.81 - 0.164 = 0.646 \text{ V}$$

Now

$$C' = \left[\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(5 \times 10^{16})}{2(0.646 + 4)} \right]^{1/2}$$

or

$$C' = 2.99 \times 10^{-8} \text{ F / cm}^2$$

and

$$C = C'A$$

so

$$C = 15 \text{ pF}$$

9.6

(a) From the figure, $V_{bi} = 0.90 \text{ V}$

(b) We find

$$\frac{\Delta \left(\frac{1}{C'} \right)^2}{\Delta V_R} = \frac{3 \times 10^{15} - 0}{2 - (-0.9)} = 1.03 \times 10^{15}$$

and

$$1.03 \times 10^{15} = \frac{2}{e \in N_d}$$

Then we can write

$$N_d = \frac{2}{(1.6 \times 10^{-19})(13.1)(8.85 \times 10^{-14})(1.03 \times 10^{15})}$$

or

$$N_d = 1.05 \times 10^{16} \text{ cm}^{-3}$$

(c)

$$\begin{aligned} \phi_n &= V_t \ln \left(\frac{N_c}{N_d} \right) \\ &= (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{1.05 \times 10^{16}} \right) \end{aligned}$$

or

$$\phi_n = 0.0985 \text{ V}$$

(d)

$$\phi_{Bn} = V_{bi} + \phi_n = 0.90 + 0.0985$$

or

$$\phi_{Bn} = 0.9985 \text{ V}$$

9.7

From the figure, $\phi_{Bn} = 0.55 \text{ V}$

(a)

$$\begin{aligned} \phi_n &= V_t \ln \left(\frac{N_c}{N_d} \right) \\ &= (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{10^{16}} \right) = 0.206 \text{ V} \end{aligned}$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344 \text{ V}$$

We find

$$x_d = \left[\frac{2 \in V_{bi}}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.344)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_d = 0.211 \text{ } \mu\text{m}$$

Also

$$\begin{aligned} |E_{\max}| &= \frac{e N_d x_d}{\in} \\ &= \frac{(1.6 \times 10^{-19})(10^{16})(0.211 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$|E_{\max}| = 3.26 \times 10^4 \text{ V / cm}$$

(b)

$$\Delta \phi = \sqrt{\frac{e E}{4 \pi \in}} = \left[\frac{(1.6 \times 10^{-19})(3.26 \times 10^4)}{4 \pi (11.7)(8.85 \times 10^{-14})} \right]^{1/2}$$

or

$$\Delta \phi = 20.0 \text{ mV}$$

Also

$$\begin{aligned} x_m &= \sqrt{\frac{e}{16 \pi \in E}} \\ &= \left[\frac{(1.6 \times 10^{-19})}{16 \pi (11.7)(8.85 \times 10^{-14})(3.26 \times 10^4)} \right]^{1/2} \end{aligned}$$

or

$$x_m = 0.307 \times 10^{-6} \text{ cm}$$

(c)

For $V_R = 4 \text{ V}$

$$x_d = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.344 + 4)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_d = 0.75 \text{ } \mu\text{m}$$

and

$$|E_{\max}| = \frac{(1.6 \times 10^{-19})(10^{16})(0.75 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 1.16 \times 10^5 \text{ V / cm}$$

We find

$$\Delta \phi = \sqrt{\frac{e E}{4 \pi \in}} \Rightarrow \Delta \phi = 37.8 \text{ mV}$$

and

$$x_m = \sqrt{\frac{e}{16\pi \epsilon E}} \Rightarrow x_m = 0.163 \times 10^{-6} \text{ cm}$$

9.8

We have

$$-\phi(x) = \frac{-e}{16\pi \epsilon x} - Ex$$

or

$$e\phi(x) = \frac{e^2}{16\pi \epsilon x} + Eex$$

Now

$$\frac{d(e\phi(x))}{dx} = 0 = \frac{-e^2}{16\pi \epsilon x^2} + Ee$$

Solving for x^2 , we find

$$x^2 = \frac{e}{16\pi \epsilon E}$$

or

$$x = x_m = \sqrt{\frac{e}{16\pi \epsilon E}}$$

Substituting this value of $x_m = x$ into the equation for the potential, we find

$$\Delta\phi = \frac{e}{16\pi \epsilon \sqrt{\frac{e}{16\pi \epsilon E}}} + E\sqrt{\frac{e}{16\pi \epsilon E}}$$

which yields

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \epsilon}}$$

9.9

Gold, n-type GaAs, from the figure $\phi_{Bn} = 0.87 \text{ V}$

(a)

$$\begin{aligned} \phi_n &= V_t \ln\left(\frac{N_c}{N_d}\right) \\ &= (0.0259) \ln\left(\frac{4.7 \times 10^{17}}{5 \times 10^{16}}\right) = 0.058 \text{ V} \end{aligned}$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.058$$

or

$$V_{bi} = 0.812 \text{ V}$$

Also

$$x_d = \left[\frac{2 \epsilon V_{bi}}{eN_d} \right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(0.812)}{(1.6 \times 10^{-19})(5 \times 10^{16})} \right]^{1/2}$$

or

$$x_d = 0.153 \text{ } \mu\text{m}$$

Then

$$\begin{aligned} |E_{\max}| &= \frac{eN_d x_d}{\epsilon} \\ &= \left[\frac{(1.6 \times 10^{-19})(5 \times 10^{16})(0.153 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})} \right] \end{aligned}$$

or

$$|E_{\max}| = 1.06 \times 10^5 \text{ V / cm}$$

(b)

We want $\Delta\phi$ to be 7% or ϕ_{Bn} ,

So

$$\Delta\phi = (0.07)(0.87) = 0.0609 \text{ V}$$

Now

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \epsilon}} \Rightarrow E = \frac{(\Delta\phi^2)(4\pi \epsilon)}{e}$$

so

$$E = \frac{(0.0609)^2 (4\pi)(13.1)(8.85 \times 10^{-14})}{1.6 \times 10^{-19}}$$

or

$$E_{\max} = 3.38 \times 10^5 \text{ V / cm}$$

Now

$$E_{\max} = \frac{eN_d x_d}{\epsilon} \Rightarrow x_d = \frac{\epsilon E}{eN_d}$$

so

$$x_d = \frac{(13.1)(8.85 \times 10^{-14})(3.38 \times 10^5)}{(1.6 \times 10^{-19})(5 \times 10^{16})}$$

or

$$x_d = 0.49 \text{ } \mu\text{m}$$

Then

$$x_d = 0.49 \times 10^{-4} = \left[\frac{2 \epsilon (V_{bi} + V_R)}{eN_d} \right]^{1/2}$$

or we can write

$$\begin{aligned} (V_{bi} + V_R) &= \frac{eN_d x_d^2}{2 \epsilon} \\ &= \frac{(1.6 \times 10^{-19})(5 \times 10^{16})(0.49 \times 10^{-4})^2}{2(13.1)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$V_{bi} + V_R = 8.28 \text{ V} = 0.812 + V_R$$

or

$$\underline{V_R = 7.47 \text{ V}}$$

9.10

Computer Plot

9.11

(a) $\phi_{BO} = \phi_m - \chi = 5.2 - 4.07$

or

$$\underline{\phi_{BO} = 1.13 \text{ V}}$$

(b)

We have

$$(E_g - e\phi_o - e\phi_{Bn}) = \frac{1}{eD_{it}} \sqrt{2e \epsilon N_d (\phi_{Bn} - \phi_n)} - \frac{\epsilon_i}{eD_{it} \delta} [\phi_m - (\chi + \phi_{Bn})]$$

which becomes

$$\begin{aligned} e(1.43 - 0.60 - \phi_{Bn}) &= \frac{1}{e \left(\frac{10^{13}}{e} \right)} \left[2(1.6 \times 10^{-19})(13.1)(8.85 \times 10^{-14}) \right. \\ &\quad \left. \times (10^{16})(\phi_{Bn} - 0.10) \right]^{1/2} \\ &\quad - \frac{(8.85 \times 10^{-14})}{e \left(\frac{10^{13}}{e} \right) (25 \times 10^{-8})} [5.2 - (4.07 + \phi_{Bn})] \end{aligned}$$

or

$$\begin{aligned} 0.83 - \phi_{Bn} &= 0.038 \sqrt{\phi_{Bn} = 0.10 - 0.221(1.13 - \phi_{Bn})} \end{aligned}$$

We then find

$$\underline{\phi_{Bn} = 0.858 \text{ V}}$$

(c)

If $\phi_m = 4.5 \text{ V}$, then

$$\phi_{BO} = \phi_m - \chi = 4.5 - 4.07$$

or

$$\underline{\phi_{BO} = 0.43 \text{ V}}$$

From part (b), we have

$$\begin{aligned} 0.83 - \phi_{Bn} &= 0.038 \sqrt{\phi_{Bn} = 0.10 - 0.221[4.5 - (4.07 + \phi_{Bn})]} \end{aligned}$$

We then find

$$\underline{\phi_{Bn} = 0.733 \text{ V}}$$

With interface states, the barrier height is less sensitive to the metal work function.

9.12

We have that

$$\begin{aligned} (E_g - e\phi_o - e\phi_{Bn}) &= \frac{1}{eD_{it}} \sqrt{2e \epsilon N_d (\phi_{Bn} - \phi_n)} \\ &\quad - \frac{\epsilon_i}{eD_{it} \delta} [\phi_m - (\chi + \phi_{Bn})] \end{aligned}$$

Let $eD_{it} = D'_{it} (cm^{-2} eV^{-1})$. Then we can write

$$\begin{aligned} e(1.12 - 0.230 - 0.60) &= \frac{1}{D'_{it}} \left[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14}) \right. \\ &\quad \left. \times (5 \times 10^{16})(0.60 - 0.164) \right]^{1/2} \\ &\quad - \frac{(8.85 \times 10^{-14})}{D'_{it} (20 \times 10^{-8})} [4.75 - (4.01 + 0.60)] \end{aligned}$$

We find that

$$\underline{D'_{it} = 4.97 \times 10^{11} \text{ cm}^{-2} eV^{-1}}$$

9.13

$$\begin{aligned} \text{(a) } \phi_n &= V_t \ln \left(\frac{N_c}{N_d} \right) \\ &= (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{10^{16}} \right) \end{aligned}$$

or

$$\underline{\phi_n = 0.206 \text{ V}}$$

(b)

$$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.206$$

or

$$\underline{V_{bi} = 0.684 \text{ V}}$$

(c)

$$J_{ST} = A^* T^2 \exp \left(\frac{-e\phi_{Bn}}{kT} \right)$$

For silicon, $A^* = 120 \text{ A} / \text{cm}^2 / ^\circ K^2$

Then

$$J_{ST} = (120)(300)^2 \exp \left(\frac{-0.89}{0.0259} \right)$$

or

$$\underline{J_{ST} = 1.3 \times 10^{-8} \text{ A} / \text{cm}^2}$$

(d)

$$J_n = J_{ST} \exp\left(\frac{eV_a}{kT}\right)$$

or

$$V_a = V_t \ln\left(\frac{J_n}{J_{ST}}\right) = (0.0259) \ln\left(\frac{2}{1.3 \times 10^{-8}}\right)$$

or

$$\underline{V_a = 0.488 \text{ V}}$$

9.14

(a) From the figure, $\phi_{Bn} = 0.68 \text{ V}$

Then

$$\begin{aligned} J_{ST} &= A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right) \\ &= (120)(300)^2 \exp\left(\frac{-0.68}{0.0259}\right) \end{aligned}$$

or

$$J_{ST} = 4.28 \times 10^{-5} \text{ A / cm}^2$$

$$\text{For } I = 10^{-3} \text{ A} \Rightarrow J_n = \frac{10^{-3}}{5 \times 10^{-4}} = 2 \text{ A / cm}^2$$

We have

$$\begin{aligned} V_a &= V_t \ln\left(\frac{J_n}{J_{ST}}\right) \\ &= (0.0259) \ln\left(\frac{2}{4.28 \times 10^{-5}}\right) \end{aligned}$$

or

$$\underline{V_a = 0.278 \text{ V}}$$

$$\text{For } I = 10 \text{ mA} \Rightarrow J_n = 20 \text{ A / cm}^2$$

And

$$V_a = (0.0259) \ln\left(\frac{20}{4.28 \times 10^{-5}}\right)$$

or

$$\underline{V_a = 0.338 \text{ V}}$$

$$\text{For } I = 100 \text{ mA} \Rightarrow J_n = 200 \text{ A / cm}^2$$

And

$$V_a = (0.0259) \ln\left(\frac{200}{4.28 \times 10^{-5}}\right)$$

or

$$\underline{V_a = 0.398 \text{ V}}$$

(b)

For $T = 400 \text{ K}$, $\phi_{Bn} = 0.68 \text{ V}$

Now

$$J_{ST} = (120)(400)^2 \exp\left[\frac{-0.68}{(0.0259)(400/300)}\right]$$

or

$$J_{ST} = 5.39 \times 10^{-2} \text{ A / cm}^2$$

For $I = 1 \text{ mA}$,

$$V_a = (0.0259) \left(\frac{400}{300}\right) \ln\left(\frac{2}{5.39 \times 10^{-2}}\right)$$

or

$$\underline{V_a = 0.125 \text{ V}}$$

For $I = 10 \text{ mA}$,

$$V_a = (0.0259) \left(\frac{400}{300}\right) \ln\left[\frac{20}{5.39 \times 10^{-2}}\right]$$

or

$$\underline{V_a = 0.204 \text{ V}}$$

For $I = 100 \text{ mA}$,

$$V_a = (0.0259) \left(\frac{400}{300}\right) \ln\left(\frac{200}{5.39 \times 10^{-2}}\right)$$

or

$$\underline{V_a = 0.284 \text{ V}}$$

9.15

(a) From the figure, $\phi_{Bn} = 0.86 \text{ V}$

$$\begin{aligned} J_{ST} &= A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right) \\ &= (1.12)(300)^2 \exp\left(\frac{-0.86}{0.0259}\right) \end{aligned}$$

or

$$J_{ST} = 3.83 \times 10^{-10} \text{ A / cm}^2$$

Now

$$J_n = J_{ST} \exp\left(\frac{V_a}{V_t}\right)$$

and we can write, for $J_n = 5 \text{ A / cm}^2$

$$\begin{aligned} V_a &= V_t \ln\left(\frac{J_n}{J_{ST}}\right) \\ &= (0.0259) \ln\left(\frac{5}{3.83 \times 10^{-10}}\right) \end{aligned}$$

or

$$\underline{V_a = 0.603 \text{ V}}$$

(b)

For $J_n = 10 \text{ A/cm}^2$

$$V_a = (0.0259) \ln \left(\frac{10}{3.83 \times 10^{-10}} \right) = 0.621 \text{ V}$$

so

$$\Delta V_a = 0.621 - 0.603 \Rightarrow$$

$$\Delta V_a = 18 \text{ mV}$$

9.16

Computer Plot

9.17

From the figure, $\phi_{Bn} = 0.86 \text{ V}$

$$J_{ST} = A^* T^2 \exp \left(\frac{-\phi_{Bn}}{V_t} \right) \exp \left(\frac{\Delta \phi}{V_t} \right)$$

$$= (120)(300)^2 \exp \left(\frac{-0.68}{0.0259} \right) \exp \left(\frac{\Delta \phi}{V_t} \right)$$

or

$$J_{ST} = 4.28 \times 10^{-5} \exp \left(\frac{\Delta \phi}{V_t} \right)$$

We have

$$\Delta \phi = \sqrt{\frac{eE}{4\pi \epsilon}}$$

Now

$$\phi_n = V_t \ln \left(\frac{N_c}{N_d} \right)$$

$$= (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{10^{16}} \right) = 0.206 \text{ V}$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.68 - 0.206 = 0.474 \text{ V}$$

(a)

We find for $V_R = 2 \text{ V}$,

$$x_d = \left[\frac{2 \epsilon (V_{bi} + V_R)}{eN_d} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(2.474)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_d = 0.566 \text{ } \mu\text{m}$$

Then

$$|E_{\max}| = \frac{eN_d x_d}{\epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(10^{16})(0.566 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 8.75 \times 10^4 \text{ V/cm}$$

Now

$$\Delta \phi = \left[\frac{(1.6 \times 10^{-19})(8.75 \times 10^4)}{4\pi(11.7)(8.85 \times 10^{-14})} \right]^{1/2}$$

or

$$\Delta \phi = 0.0328 \text{ V}$$

Then

$$J_{R1} = 4.28 \times 10^{-5} \exp \left(\frac{0.0328}{0.0259} \right)$$

or

$$J_{R1} = 1.52 \times 10^{-4} \text{ A/cm}^2$$

For $A = 10^{-4} \text{ cm}^2$, then

$$I_{R1} = 1.52 \times 10^{-8} \text{ A}$$

(b)

For $V_R = 4 \text{ V}$,

$$x_d = \left[\frac{2(11.7)(8.85 \times 10^{-14})(4.474)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_d = 0.761 \text{ } \mu\text{m}$$

$$|E_{\max}| = \frac{(1.6 \times 10^{-19})(10^{16})(0.761 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{\max}| = 1.18 \times 10^5 \text{ V/cm}$$

and

$$\Delta \phi = \left[\frac{(1.6 \times 10^{-19})(1.18 \times 10^5)}{4\pi(11.7)(8.85 \times 10^{-14})} \right]^{1/2}$$

or

$$\Delta \phi = 0.0381 \text{ V}$$

Now

$$J_{R2} = 4.28 \times 10^{-5} \exp \left(\frac{0.0381}{0.0259} \right)$$

or

$$J_{R2} = 1.86 \times 10^{-4} \text{ A/cm}^2$$

Finally,

$$I_{R2} = 1.86 \times 10^{-8} \text{ A}$$

9.18

We have that

$$J_{s \rightarrow m}^- = \int_{E_c}^{\infty} v_x dn$$

The incremental electron concentration is given by

$$dn = g_c(E) f_F(E) dE$$

We have

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c}$$

and, assuming the Boltzmann approximation

$$f_F(E) = \exp\left[\frac{-(E - E_F)}{kT}\right]$$

Then

$$dn = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \cdot \exp\left[\frac{-(E - E_F)}{kT}\right] dE$$

If the energy above E_c is kinetic energy, then

$$\frac{1}{2} m_n^* v^2 = E - E_c$$

We can then write

$$\sqrt{E - E_c} = v \sqrt{\frac{m_n^*}{2}}$$

and

$$dE = \frac{1}{2} m_n^* \cdot 2v dv = m_n^* v dv$$

We can also write

$$\begin{aligned} E - E_F &= (E - E_c) + (E_c - E_F) \\ &= \frac{1}{2} m_n^* v^2 + e\phi_n \end{aligned}$$

so that

$$dn = 2 \left(\frac{m_n^*}{h} \right)^3 \exp\left(\frac{-e\phi_n}{kT}\right) \cdot \exp\left(\frac{-m_n^* v^2}{2kT}\right) \cdot 4\pi v^2 dv$$

We can write

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

The differential volume element is

$$4\pi v^2 dv = dv_x dv_y dv_z$$

The current is due to all x-directed velocities that are greater than v_{ox} and for all y- and z-directed velocities. Then

$$\begin{aligned} J_{s \rightarrow m}^- &= 2 \left(\frac{m_n^*}{h} \right)^3 \exp\left(\frac{-e\phi_n}{kT}\right) \\ &\quad \times \int_{v_{ox}}^{\infty} v_x \exp\left(\frac{-m_n^* v_x^2}{2kT}\right) dv_x \\ &\quad \times \int_{-\infty}^{+\infty} \exp\left(\frac{-m_n^* v_y^2}{2kT}\right) dv_y \times \int_{-\infty}^{+\infty} \exp\left(\frac{-m_n^* v_z^2}{2kT}\right) dv_z \end{aligned}$$

We can write that

$$\frac{1}{2} m_n^* v_{ox}^2 = e(V_{bi} - V_a)$$

Make a change of variables:

$$\frac{m_n^* v_x^2}{2kT} = \alpha^2 + \frac{e(V_{bi} - V_a)}{kT}$$

or

$$v_x^2 = \frac{2kT}{m_n^*} \left[\alpha^2 + \frac{e(V_{bi} - V_a)}{kT} \right]$$

Taking the differential, we find

$$v_x dv_x = \left(\frac{2kT}{m_n^*} \right) \alpha d\alpha$$

We may note that when $v_x = v_{ox}$, $\alpha = 0$.

Other change of variables:

$$\frac{m_n^* v_y^2}{2kT} = \beta^2 \Rightarrow v_y = \left(\frac{2kT}{m_n^*} \right)^{1/2} \cdot \beta$$

$$\frac{m_n^* v_z^2}{2kT} = \gamma^2 \Rightarrow v_z = \left(\frac{2kT}{m_n^*} \right)^{1/2} \cdot \gamma$$

Substituting the new variables, we have

$$\begin{aligned} J_{s \rightarrow m}^- &= 2 \left(\frac{m_n^*}{h} \right)^3 \cdot \left(\frac{2kT}{m_n^*} \right)^2 \exp\left(\frac{-e\phi_n}{kT}\right) \\ &\quad \times \exp\left[\frac{-e(V_{bi} - V_a)}{kT}\right] \cdot \int_0^{\infty} \alpha \exp(-\alpha^2) d\alpha \\ &\quad \times \int_{-\infty}^{+\infty} \exp(-\beta^2) d\beta \cdot \int_{-\infty}^{+\infty} \exp(-\gamma^2) d\gamma \end{aligned}$$

9.19

For the Schottky diode,

$$J_{ST} = 3 \times 10^{-8} \text{ A/cm}^2, A = 5 \times 10^{-4} \text{ cm}^2$$

For $I = 1 \text{ mA}$,

$$J = \frac{10^{-3}}{5 \times 10^{-4}} = 2 \text{ A/cm}^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_{ST}}\right)$$

$$= (0.0259) \ln\left(\frac{2}{3 \times 10^{-8}}\right)$$

or

$$V_a = 0.467 \text{ V (Schottky diode)}$$

For the pn junction, $J_s = 3 \times 10^{-12} \text{ A/cm}^2$

Then

$$V_a = (0.0259) \ln\left(\frac{2}{3 \times 10^{-12}}\right)$$

or

$$V_a = 0.705 \text{ V (pn junction diode)}$$

9.20

For the pn junction diode,

$$J_s = 5 \times 10^{-12} \text{ A/cm}^2, A = 8 \times 10^{-4} \text{ cm}^2$$

For $I = 1.2 \text{ mA}$,

$$J = \frac{1.2 \times 10^{-3}}{8 \times 10^{-4}} = 1.5 \text{ A/cm}^2$$

Then

$$V_a = V_t \ln\left(\frac{J}{J_s}\right)$$

$$= (0.0259) \ln\left(\frac{1.5}{5 \times 10^{-12}}\right) = 0.684 \text{ V}$$

For the Schottky diode, the applied voltage will be less, so

$$V_a = 0.684 - 0.265 = 0.419 \text{ V}$$

We have

$$I = A J_{ST} \exp\left(\frac{V_a}{V_t}\right)$$

so

$$1.2 \times 10^{-3} = A (7 \times 10^{-8}) \exp\left(\frac{0.419}{0.0259}\right)$$

which yields

$$A = 1.62 \times 10^{-3} \text{ cm}^2$$

9.21

(a) Diodes in parallel:

We can write

$$I_s = I_{ST} \exp\left(\frac{V_{as}}{V_t}\right) \text{ (Schottky diode)}$$

and

$$I_{PN} = I_s \exp\left(\frac{V_{apn}}{V_t}\right) \text{ (pn junction diode)}$$

We have $I_s + I_{PN} = 0.5 \times 10^{-3} \text{ A}$, $V_{as} = V_{apn}$

Then

$$0.5 \times 10^{-3} = (I_{ST} + I_s) \exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t \ln\left(\frac{0.5 \times 10^{-3}}{I_s + I_{ST}}\right)$$

$$= (0.0259) \ln\left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-8} + 10^{-12}}\right) = 0.239 \text{ V}$$

Now

$$I_s = 5 \times 10^{-8} \exp\left(\frac{0.239}{0.0259}\right)$$

or

$$I_s \approx 0.5 \times 10^{-3} \text{ A (Schottky diode)}$$

and

$$I_{PN} = 10^{-12} \exp\left(\frac{0.239}{0.0259}\right)$$

or

$$I_{PN} = 1.02 \times 10^{-8} \text{ A (pn junction diode)}$$

(b) Diodes in Series:

We obtain,

$$V_{as} = (0.0259) \ln\left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-8}}\right)$$

or

$$V_{as} = 0.239 \text{ V (Schottky diode)}$$

and

$$V_{apn} = (0.0259) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-12}}\right)$$

or

$$V_{apn} = 0.519 \text{ V (pn junction diode)}$$

9.22

(a) For $I = 0.8 \text{ mA}$, we find

$$J = \frac{0.8 \times 10^{-3}}{7 \times 10^{-4}} = 1.14 \text{ A/cm}^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_s}\right)$$

For the pn junction diode,

$$V_a = (0.0259) \ln \left(\frac{1.14}{3 \times 10^{-12}} \right)$$

or

$$V_a = 0.691 \text{ V}$$

For the Schottky diode,

$$V_a = (0.0259) \ln \left(\frac{1.14}{4 \times 10^{-8}} \right)$$

or

$$V_a = 0.445 \text{ V}$$

(b)

For the pn junction diode,

$$J_s \propto n_i^2 \propto \left(\frac{T}{300} \right)^3 \exp \left(\frac{-E_g}{kT} \right)$$

Then

$$\frac{J_s(400)}{J_s(300)} = \left(\frac{400}{300} \right)^3 \exp \left[\frac{-E_g}{(0.0259)(400/300)} + \frac{E_g}{0.0259} \right]$$

or

$$= 2.37 \exp \left[\frac{1.12}{0.0259} - \frac{1.12}{0.03453} \right]$$

We find

$$\frac{J_s(400)}{J_s(300)} = 1.16 \times 10^5$$

Now

$$I = (7 \times 10^{-4})(1.16 \times 10^5)(3 \times 10^{-12}) \exp \left(\frac{0.691}{0.03453} \right)$$

or

$$I = 120 \text{ mA}$$

For the Schottky diode

$$J_{ST} \propto T^2 \exp \left(\frac{-e\phi_{BO}}{kT} \right)$$

Now

$$\frac{J_{ST}(400)}{J_{ST}(300)} = \left(\frac{400}{300} \right)^2 \exp \left[\frac{-\phi_{BO}}{(0.0259)(400/300)} + \frac{\phi_{BO}}{0.0259} \right]$$

or

$$= 1.78 \exp \left[\frac{0.82}{0.0259} - \frac{0.82}{0.03453} \right]$$

We obtain

$$\frac{J_{ST}(400)}{J_{ST}(300)} = 4.85 \times 10^3$$

and so

$$I = (7 \times 10^{-4})(4.85 \times 10^3)(4 \times 10^{-8}) \exp \left(\frac{0.445}{0.03453} \right)$$

or

$$I = 53.7 \text{ mA}$$

9.23

Computer Plot

9.24

We have

$$R_c = \frac{\left(\frac{kT}{e} \right) \cdot \exp \left(\frac{e\phi_{Bn}}{kT} \right)}{A^* T^2}$$

which can be rewritten as

$$\ln \left[\frac{R_c A^* T^2}{(kT/e)} \right] = \frac{e\phi_{Bn}}{kT}$$

so

$$\begin{aligned} \phi_{Bn} &= \left(\frac{kT}{e} \right) \cdot \ln \left[\frac{R_c A^* T^2}{(kT/e)} \right] \\ &= (0.0259) \ln \left[\frac{(10^{-5})(120)(300)^2}{0.0259} \right] \end{aligned}$$

or

$$\phi_{Bn} = 0.216 \text{ V}$$

9.25

(b) We need $\phi_n = \phi_m - \chi_s = 4.2 - 4.0 = 0.20 \text{ V}$

And

$$\phi_n = V_t \ln \left(\frac{N_c}{N_d} \right)$$

or

$$0.20 = (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{N_d} \right)$$

which yields

$$N_d = 1.24 \times 10^{16} \text{ cm}^{-3}$$

(c)

$$\text{Barrier height} = 0.20 \text{ V}$$

9.26

We have that

$$E = \frac{-eN_d}{\epsilon} (x_n - x)$$

Then

$$\phi = -\int E dx = \frac{eN_d}{\epsilon} \left(x_n \cdot x - \frac{x^2}{2} \right) + C_2$$

Let $\phi = 0$ at $x = 0 \Rightarrow C_2 = 0$

So

$$\phi = \frac{eN_d}{\epsilon} \left(x_n \cdot x - \frac{x^2}{2} \right)$$

At $x = x_n$, $\phi = V_{bi}$, so

$$\phi = V_{bi} = \frac{eN_d}{\epsilon} \cdot \frac{x_n^2}{2}$$

or

$$x_n = \sqrt{\frac{2\epsilon V_{bi}}{eN_d}}$$

Also

$$V_{bi} = \phi_{BO} - \phi_n$$

where

$$\phi_n = V_t \ln \left(\frac{N_c}{N_d} \right)$$

For

$$\phi = \frac{\phi_{BO}}{2} = \frac{0.70}{2} = 0.35 \text{ V}$$

we have

$$0.35 = \frac{(1.6 \times 10^{-19}) N_d}{(11.7)(8.85 \times 10^{-14})} \left[x_n (50 \times 10^{-8}) - \frac{(50 \times 10^{-8})^2}{2} \right]$$

or

$$0.35 = 7.73 \times 10^{-14} N_d (x_n - 25 \times 10^{-8})$$

We have

$$x_n = \left[\frac{2(11.7)(8.85 \times 10^{-14}) V_{bi}}{(1.6 \times 10^{-19}) N_d} \right]^{1/2}$$

and

$$V_{bi} = 0.70 - \phi_n$$

By trial and error,

$$N_d = 3.5 \times 10^{18} \text{ cm}^{-3}$$

9.27

$$\begin{aligned} \text{(b) } \phi_{BO} &= \phi_p = V_t \ln \left(\frac{N_v}{N_a} \right) \\ &= (0.0259) \ln \left(\frac{1.04 \times 10^{19}}{5 \times 10^{16}} \right) \Rightarrow \\ \phi_{BO} &= 0.138 \text{ V} \end{aligned}$$

9.28

Sketches

9.29

Sketches

9.30

Electron affinity rule

$$\Delta E_c = e(\chi_n - \chi_p)$$

For GaAs, $\chi = 4.07$; and for AlAs, $\chi = 3.5$,

If we assume a linear extrapolation between GaAs and AlAs, then for

$$Al_{0.3}Ga_{0.7}As \Rightarrow \chi = 3.90$$

Then

$$|E_c| = 4.07 - 3.90 \Rightarrow$$

$$|E_c| = 0.17 \text{ eV}$$

9.31

Consider an n-P heterojunction in thermal equilibrium. Poisson's equation is

$$\frac{d^2 \phi}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{dE}{dx}$$

In the n-region,

$$\frac{dE_n}{dx} = \frac{\rho(x)}{\epsilon_n} = \frac{eN_{dn}}{\epsilon_n}$$

For uniform doping, we have

$$E_n = \frac{eN_{dn}x}{\epsilon_n} + C_1$$

The boundary condition is

$E_n = 0$ at $x = -x_n$, so we obtain

$$C_1 = \frac{eN_{dn}x_n}{\epsilon_n}$$

Then

$$E_n = \frac{eN_{dn}}{\epsilon_n} (x + x_n)$$

In the P-region,

$$\frac{dE_p}{dx} = -\frac{eN_{aP}}{\epsilon_p}$$

which gives

$$E_p = -\frac{eN_{aP}x}{\epsilon_p} + C_2$$

We have the boundary condition that

$E_p = 0$ at $x = x_p$ so that

$$C_2 = \frac{eN_{aP}x_p}{\epsilon_p}$$

Then

$$E_p = \frac{eN_{aP}}{\epsilon_p}(x_p - x)$$

Assuming zero surface charge density at $x = 0$, the electric flux density D is continuous, so

$$\epsilon_n E_n(0) = \epsilon_p E_p(0)$$

which yields

$$N_{dn}x_n = N_{aP}x_p$$

We can determine the electric potential as

$$\begin{aligned}\phi_n(x) &= -\int E_n dx \\ &= -\left[\frac{eN_{dn}x^2}{2\epsilon_n} + \frac{eN_{dn}x_n x}{\epsilon_n} \right] + C_3\end{aligned}$$

Now

$$\begin{aligned}V_{bin} &= |\phi_n(0) - \phi_n(-x_n)| \\ &= C_3 - \left[C_3 - \frac{eN_{dn}x_n^2}{2\epsilon_n} + \frac{eN_{dn}x_n^2}{\epsilon_n} \right]\end{aligned}$$

or

$$V_{bin} = \frac{eN_{dn}x_n^2}{2\epsilon_n}$$

Similarly on the P-side, we find

$$V_{biP} = \frac{eN_{aP}x_p^2}{2\epsilon_p}$$

We have that

$$V_{bi} = V_{bin} + V_{biP} = \frac{eN_{dn}x_n^2}{2\epsilon_n} + \frac{eN_{aP}x_p^2}{2\epsilon_p}$$

We can write

$$x_p = x_n \left(\frac{N_{dn}}{N_{aP}} \right)$$

Substituting and collecting terms, we find

$$V_{bi} = \left[\frac{e\epsilon_p N_{dn} N_{aP} + e\epsilon_n N_{dn}^2}{2\epsilon_n \epsilon_p N_{aP}} \right] \cdot x_n^2$$

Solving for x_n , we have

$$x_n = \left[\frac{2\epsilon_n \epsilon_p N_{aP} V_{bi}}{eN_{dn}(\epsilon_p N_{aP} + \epsilon_n N_{dn})} \right]^{1/2}$$

Similarly on the P-side, we have

$$x_p = \left[\frac{2\epsilon_n \epsilon_p N_{dn} V_{bi}}{eN_{aP}(\epsilon_p N_{aP} + \epsilon_n N_{dn})} \right]^{1/2}$$

The total space charge width is then

$$W = x_n + x_p$$

Substituting and collecting terms, we obtain

$$W = \left[\frac{2\epsilon_n \epsilon_p V_{bi} (N_{aP} + N_{dn})}{eN_{dn} N_{aP} (\epsilon_n N_{dn} + \epsilon_p N_{aP})} \right]^{1/2}$$

Chapter 10

Problem Solutions

10.1

Sketch

10.2

Sketch

10.3

$$(a) \quad |I_S| = \frac{eD_n A_{BE} n_{BO}}{x_B} = \frac{(1.6 \times 10^{-19})(20)(10^{-4})(10^4)}{10^{-4}}$$

$$\text{or } I_S = 3.2 \times 10^{-14} \text{ A}$$

(b)

$$(i) \quad i_C = 3.2 \times 10^{-14} \exp\left(\frac{0.5}{0.0259}\right) \Rightarrow$$

$$\underline{i_C = 7.75 \mu A}$$

$$(ii) \quad i_C = 3.2 \times 10^{-14} \exp\left(\frac{0.6}{0.0259}\right) \Rightarrow$$

$$\underline{i_C = 0.368 \text{ mA}}$$

$$(iii) \quad i_C = 3.2 \times 10^{-14} \exp\left(\frac{0.7}{0.0259}\right) \Rightarrow$$

$$\underline{i_C = 17.5 \text{ mA}}$$

10.4

$$(a) \quad \beta = \frac{\alpha}{1 - \alpha} = \frac{0.9920}{1 - 0.9920} \Rightarrow \underline{\beta = 124}$$

(b) From 10.3b

$$(i) \quad \text{For } i_C = 7.75 \mu A; i_B = \frac{i_C}{\beta} = \frac{7.75}{124} \Rightarrow$$

$$\underline{i_B = 0.0625 \mu A},$$

$$i_E = \left(\frac{1 + \beta}{\beta}\right) \cdot i_C = \left(\frac{125}{124}\right)(7.75) \Rightarrow$$

$$\underline{i_E = 7.81 \mu A}$$

$$(ii) \quad \text{For } i_C = 0.368 \text{ mA}, \underline{i_B = 2.97 \mu A},$$

$$\underline{i_E = 0.371 \text{ mA}}$$

$$(iii) \quad \text{For } i_C = 17.5 \text{ mA}, \underline{i_B = 0.141 \text{ mA}},$$

$$\underline{i_E = 17.64 \text{ mA}}$$

10.5

$$(a) \quad \beta = \frac{i_C}{i_B} = \frac{510}{6} \Rightarrow \underline{\beta = 85}$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{85}{86} \Rightarrow \underline{\alpha = 0.9884}$$

$$i_E = i_C + i_B = 510 + 6 \Rightarrow \underline{i_E = 516 \mu A}$$

(b)

$$\beta = \frac{2.65}{0.05} \Rightarrow \underline{\beta = 53}$$

$$\alpha = \frac{53}{54} \Rightarrow \underline{\alpha = 0.9815}$$

$$i_E = 2.65 + 0.05 \Rightarrow \underline{i_E = 2.70 \text{ mA}}$$

10.6

(c) For $i_B = 0.05 \text{ mA}$,

$$i_C = \beta i_B = (100)(0.05) \Rightarrow \underline{i_C = 5 \text{ mA}}$$

We have

$$v_{CE} = V_{CC} - i_C R = 10 - (5)(1)$$

or

$$\underline{v_{CE} = 5 \text{ V}}$$

10.7

$$(b) \quad V_{CC} = I_C R + V_{CB} + V_{BE}$$

so

$$10 = I_C(2) + 0 + 0.6$$

or

$$\underline{I_C = 4.7 \text{ mA}}$$

10.8

(a)

$$n_{pO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

At $x = 0$,

$$n_p(0) = n_{pO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

or we can write

$$V_{BE} = V_t \ln\left(\frac{n_p(0)}{n_{pO}}\right)$$

We want $n_p(0) = 10\% \times 10^{16} = 10^{15} \text{ cm}^{-3}$,

So

$$V_{BE} = (0.0259) \ln \left(\frac{10^{15}}{2.25 \times 10^4} \right)$$

or

$$\underline{V_{BE} = 0.635 \text{ V}}$$

(b)

At $x' = 0$,

$$p_n(0) = p_{n0} \exp \left(\frac{V_{BE}}{V_t} \right)$$

where

$$p_{n0} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

Then

$$p_n(0) = 2.25 \times 10^3 \exp \left(\frac{0.635}{0.0259} \right) \Rightarrow$$

$$\underline{p_n(0) = 10^{14} \text{ cm}^{-3}}$$

(c)

From the B-C space charge region,

$$x_{p1} = \left[\frac{2 \in (V_{bi} + V_{R1}) \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right)}{e} \right]^{1/2}$$

We find

$$V_{bi1} = (0.0259) \ln \left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.635 \text{ V}$$

Then

$$x_{p1} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 3)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{10^{15}}{10^{16}} \right) \left(\frac{1}{10^{15} + 10^{16}} \right) \right]^{1/2}$$

or

$$x_{p1} = 0.207 \text{ } \mu\text{m}$$

We find

$$V_{bi2} = (0.0259) \ln \left[\frac{(10^{17})(10^{16})}{(1.5 \times 10^{10})^2} \right] = 0.754 \text{ V}$$

Then

$$x_{p2} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.754 - 0.635)}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{10^{17}}{10^{16}} \right) \left(\frac{1}{10^{17} + 10^{16}} \right) \right]^{1/2}$$

or

$$x_{p2} = 0.118 \text{ } \mu\text{m}$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.10 - 0.207 - 0.118$$

or

$$\underline{x_B = 0.775 \text{ } \mu\text{m}}$$

10.9

$$(a) \quad p_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{17}} \Rightarrow$$

$$p_{EO} = 4.5 \times 10^2 \text{ cm}^{-3}$$

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{16}} \Rightarrow$$

$$n_{BO} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{10^{15}} \Rightarrow$$

$$p_{CO} = 2.25 \times 10^5 \text{ cm}^{-3}$$

(b)

$$n_B(0) = n_{BO} \exp \left(\frac{V_{BE}}{V_t} \right) \\ = (2.25 \times 10^4) \exp \left(\frac{0.625}{0.0259} \right)$$

or

$$\underline{n_B(0) = 6.80 \times 10^{14} \text{ cm}^{-3}}$$

Also

$$p_E(0) = p_{EO} \exp \left(\frac{V_{BE}}{V_t} \right) \\ = (4.5 \times 10^2) \exp \left(\frac{0.625}{0.0259} \right)$$

or

$$\underline{p_E(0) = 1.36 \times 10^{13} \text{ cm}^{-3}}$$

10.10

$$(a) \quad n_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{10^{18}} \Rightarrow$$

$$n_{EO} = 2.25 \times 10^2 \text{ cm}^{-3}$$

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} \Rightarrow$$

$$\underline{p_{BO} = 4.5 \times 10^3 \text{ cm}^{-3}}$$

$$n_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{10^{15}} \Rightarrow$$

$$\underline{n_{CO} = 2.25 \times 10^5 \text{ cm}^{-3}}$$

(b)

$$p_B(0) = p_{BO} \exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (4.5 \times 10^3) \exp\left(\frac{0.650}{0.0259}\right)$$

or

$$\underline{p_B(0) = 3.57 \times 10^{14} \text{ cm}^{-3}}$$

Also

$$n_E(0) = n_{EO} \exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25 \times 10^2) \exp\left(\frac{0.650}{0.0259}\right)$$

or

$$\underline{n_E(0) = 1.78 \times 10^{13} \text{ cm}^{-3}}$$

10.11

We have

$$\frac{d(\delta n_B)}{dx} = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)} \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \right.$$

$$\left. \times \left(\frac{-1}{L_B}\right) \cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B} \cosh\left(\frac{x}{L_B}\right) \right\}$$

At $x = 0$,

$$\frac{d(\delta n_B)}{dx} \Big|_{(0)} = \frac{-n_{BO}}{L_B \sinh\left(\frac{x_B}{L_B}\right)} \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \right.$$

$$\left. \times \cosh\left(\frac{x_B}{L_B}\right) + 1 \right\}$$

At $x = x_B$,

$$\frac{d(\delta n_B)}{dx} \Big|_{(x_B)} = \frac{-n_{BO}}{L_B \sinh\left(\frac{x_B}{L_B}\right)}$$

$$\times \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] + \cosh\left(\frac{x_B}{L_B}\right) \right\}$$

Taking the ratio,

$$\frac{\frac{d(\delta n_B)}{dx} \Big|_{(x_B)}}{\frac{d(\delta n_B)}{dx} \Big|_{(0)}} = \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] + \cosh\left(\frac{x_B}{L_B}\right)}{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cosh\left(\frac{x_B}{L_B}\right) + 1}$$

$$\approx \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)}$$

(a) For $\frac{x_B}{L_B} = 0.1 \Rightarrow \text{Ratio} = \underline{0.9950}$

(b) For $\frac{x_B}{L_B} = 1.0 \Rightarrow \text{Ratio} = \underline{0.648}$

(c) For $\frac{x_B}{L_B} = 10 \Rightarrow \text{Ratio} = \underline{9.08 \times 10^{-5}}$

10.12

In the base of the transistor, we have

$$D_B \frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B \tau_{BO}}$

The general solution to the differential equation is of the form,

$$\delta n_B(x) = A \exp\left(\frac{x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we have

$$\delta n_B(0) = A + B = n_B(0) - n_{BO}$$

$$= n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right]$$

Also

$$\delta n_B(x_B) = A \exp\left(\frac{x_B}{L_B}\right) + B \exp\left(\frac{-x_B}{L_B}\right) = -n_{BO}$$

From the first boundary condition, we can write

$$A = n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] - B$$

Substituting into the second boundary condition equation, we find

$$B \left[\exp\left(\frac{x_B}{L_B}\right) - \exp\left(\frac{-x_B}{L_B}\right) \right] = n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \exp\left(\frac{x_B}{L_B}\right) + n_{BO}$$

which can be written as

$$B = \frac{n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \exp\left(\frac{x_B}{L_B}\right) + n_{BO}}{2 \sinh\left(\frac{x_B}{L_B}\right)}$$

We then find

$$A = \frac{-n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \exp\left(\frac{-x_B}{L_B}\right) - n_{BO}}{2 \sinh\left(\frac{x_B}{L_B}\right)}$$

10.13

In the base of the pnp transistor, we have

$$D_B \frac{d^2(\delta p_B(x))}{dx^2} - \frac{\delta p_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta p_B(x))}{dx^2} - \frac{\delta p_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B \tau_{BO}}$

The general solution is of the form

$$\delta p_B(x) = A \exp\left(\frac{x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we can write

$$\begin{aligned} \delta p_B(0) &= A + B = p_B(0) - p_{BO} \\ &= p_{BO} \left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] \end{aligned}$$

Also

$$\delta p_B(x_B) = A \exp\left(\frac{x_B}{L_B}\right) + B \exp\left(\frac{-x_B}{L_B}\right) = -p_{BO}$$

From the first boundary condition equation, we find

$$A = p_{BO} \left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] - B$$

Substituting into the second boundary equation

$$B = \frac{p_{BO} \left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] \cdot \exp\left(\frac{x_B}{L_B}\right) + p_{BO}}{2 \sinh\left(\frac{x_B}{L_B}\right)}$$

and then we obtain

$$A = \frac{-p_{BO} \left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] \cdot \exp\left(\frac{-x_B}{L_B}\right) - p_{BO}}{2 \sinh\left(\frac{x_B}{L_B}\right)}$$

Substituting the expressions for A and B into the general solution and collecting terms, we obtain

$$\begin{aligned} \delta p_B(x) &= p_{BO} \\ &\times \left\{ \frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] \cdot \sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)} \right\} \end{aligned}$$

10.14

For the idealized straight line approximation, the total minority carrier concentration is given by

$$n_B(x) = n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) \right] \cdot \left(\frac{x_B - x}{x_B} \right)$$

The excess concentration is

$$\delta n_B = n_B(x) - n_{BO}$$

so for the idealized case, we can write

$$\delta n_{BO}(x) = n_{BO} \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) \right] \cdot \left(\frac{x_B - x}{x_B} \right) - 1 \right\}$$

At $x = \frac{1}{2} x_B$, we have

$$\delta n_{BO}\left(\frac{1}{2} x_B\right) = n_{BO} \left\{ \frac{1}{2} \left[\exp\left(\frac{V_{BE}}{V_t}\right) \right] - 1 \right\}$$

For the actual case, we have

$$\begin{aligned} \delta n_B\left(\frac{1}{2} x_B\right) &= n_{BO} \\ &\times \left\{ \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \sinh\left(\frac{x_B}{2L_B}\right) - \sinh\left(\frac{x_B}{2L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)} \right\} \end{aligned}$$

(a) For $\frac{x_B}{L_B} = 0.1$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.0500208$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 0.100167$$

Then

$$\begin{aligned} & \frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)} \\ &= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right] \cdot (0.50 - 0.49937) - 1.0 + 0.99875}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1} \end{aligned}$$

which becomes

$$= \frac{(0.00063)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.00125)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

If we assume that $\exp\left(\frac{V_{BE}}{V_t}\right) \gg 1$, then we find

that the ratio is

$$\frac{0.00063}{0.50} = 0.00126 \Rightarrow \underline{0.126\%}$$

(b)

For $\frac{x_B}{L_B} = 1.0$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.5211$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then

$$\begin{aligned} & \frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)} \\ &= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right] (0.50 - 0.4434) - 1.0 + 0.8868}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1} \end{aligned}$$

which becomes

$$\frac{(0.0566)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.1132)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

Assuming that $\exp\left(\frac{V_{BE}}{V_t}\right) \gg 1$

Then the ratio is

$$= \frac{0.0566}{0.50} = 0.1132 \Rightarrow \underline{11.32\%}$$

10.15

The excess hole concentration at $x = 0$ is

$$\delta p_B(0) = p_{BO} \left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] = 8 \times 10^{14} \text{ cm}^{-3}$$

and the excess hole concentration at $x = x_B$ is

$$\delta p_B(x_B) = -p_{BO} = -2.25 \times 10^4 \text{ cm}^{-3}$$

From the results of problem 10.13, we can write

$$\delta p(x) = p_{BO} \times \left\{ \frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1 \right] \cdot \sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)} \right\}$$

or

$$\begin{aligned} \delta p_B(x) &= \\ &= \frac{(8 \times 10^{14}) \sinh\left(\frac{x_B - x}{L_B}\right) - (2.25 \times 10^4) \sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)} \end{aligned}$$

Let $x_B = L_B = 10 \mu\text{m}$, so that

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then, we can find $\delta p_B(x)$ for (a) the ideal linear approximation and for (b) the actual distribution as follow:

x	(a) δp_B	(b) δp_B
0	$8x10^{14}$	$8x10^{14}$
$0.25L_B$	$6x10^{14}$	$5.6x10^{14}$
$0.50L_B$	$4x10^{14}$	$3.55x10^{14}$
$0.75L_B$	$2x10^{14}$	$1.72x10^{14}$
$1.0L_B$	$-2.25x10^4$	$-2.25x10^4$

(c)

For the ideal case when $x_B \ll L_B$, then

$J(0) = J(x_B)$, so that

$$\frac{J(x_B)}{J(0)} = 1$$

For the case when $x_B = L_B = 10 \mu\text{m}$

$$J(0) = \frac{eD_B}{\sinh\left(\frac{x_B}{L_B}\right)} \frac{d}{dx} \left\{ (8x10^{14}) \sinh\left(\frac{x_B - x}{L_B}\right) - (2.25x10^4) \sinh\left(\frac{x}{L_B}\right) \right\} \Big|_{x=0}$$

or

$$J(0) = \frac{eD_B}{\sinh(1)} \left\{ \frac{-1}{L_B} (8x10^{14}) \cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B} (2.25x10^4) \cosh\left(\frac{x}{L_B}\right) \right\} \Big|_{x=0}$$

which becomes

$$= \frac{-eD_B}{L_B \sinh(1)} \cdot \left\{ (8x10^{14}) \cosh(1) + (2.25x10^4) \cosh(0) \right\}$$

We find

$$J(0) = \frac{-(1.6x10^{-19})(10)}{(10x10^{-4})(1.175)} \times \left[(8x10^{14})(1.543) + (2.25x10^4)(1) \right]$$

or

$$J(0) = -1.68 \text{ A / cm}^2$$

Now

$$J(x_B) = \frac{-eD_B}{L_B \sinh(1)} \left\{ (8x10^{14}) \cosh(0) + (2.25x10^4) \cosh(1) \right\}$$

or

$$= \frac{-(1.6x10^{-19})(10)}{(10x10^{-4})(1.175)} \times \left[(8x10^{14})(1) + (2.25x10^4)(1.543) \right]$$

We obtain

$$J(x_B) = -1.089 \text{ A / cm}^2$$

Then

$$\frac{J(x_B)}{J(0)} = \frac{-1.089}{-1.68} \Rightarrow \frac{J(x_B)}{J(0)} = 0.648$$

10.16

(a) npn transistor biased in saturation

$$D_B \frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B \tau_{BO}}$

The general solution is of the form

$$\delta n_B(x) = A \exp\left(\frac{x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

If $x_B \ll L_B$, then also $x \ll L_B$ so that

$$\begin{aligned} \delta n_B(x) &\approx A \left(1 + \frac{x}{L_B} \right) + B \left(1 - \frac{x}{L_B} \right) \\ &= (A + B) + (A - B) \left(\frac{x}{L_B} \right) \end{aligned}$$

which can be written as

$$\delta n_B(x) = C + D \left(\frac{x}{L_B} \right)$$

The boundary conditions are

$$\delta n_B(0) = C = n_{BO} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right]$$

and

$$\delta n_B(x_B) = C + D \left(\frac{x_B}{L_B} \right) = n_{BO} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right]$$

Then the coefficient D can be written as

$$D = \left(\frac{L_B}{x_B} \right) \left\{ n_{BO} \left[\exp \left(\frac{V_{BC}}{V_t} \right) - 1 \right] - n_{BO} \left[\exp \left(\frac{V_{BE}}{V_t} \right) - 1 \right] \right\}$$

The excess electron concentration is then given by

$$\delta n_B(x) = n_{BO} \left\{ \left[\exp \left(\frac{V_{BE}}{V_t} \right) - 1 \right] \cdot \left(1 - \frac{x}{L_B} \right) + \left[\exp \left(\frac{V_{BC}}{V_t} \right) - 1 \right] \cdot \left(\frac{x}{x_B} \right) \right\}$$

(b)

The electron diffusion current density is

$$\begin{aligned} J_n &= eD_B \frac{d(\delta n_B(x))}{dx} \\ &= eD_B n_{BO} \left\{ \left[\exp \left(\frac{V_{BE}}{V_t} \right) - 1 \right] \cdot \left(\frac{-1}{x_B} \right) + \left[\exp \left(\frac{V_{BC}}{V_t} \right) - 1 \right] \cdot \left(\frac{1}{x_B} \right) \right\} \end{aligned}$$

or

$$J_n = -\frac{eD_B n_{BO}}{x_B} \left\{ \exp \left(\frac{V_{BE}}{V_t} \right) - \exp \left(\frac{V_{BC}}{V_t} \right) \right\}$$

(c)

The total excess charge in the base region is

$$\begin{aligned} Q_{nB} &= -e \int_0^{x_B} \delta n_B(x) dx \\ &= -en_{BO} \left\{ \left[\exp \left(\frac{V_{BE}}{V_t} \right) - 1 \right] \cdot \left(x - \frac{x^2}{2x_B} \right) + \left[\exp \left(\frac{V_{BC}}{V_t} \right) - 1 \right] \cdot \left(\frac{x^2}{2x_B} \right) \right\} \bigg|_0^{x_B} \end{aligned}$$

which yields

$$Q_{nB} = \frac{-en_{BO}x_B}{2} \left\{ \left[\exp \left(\frac{V_{BE}}{V_t} \right) - 1 \right] + \left[\exp \left(\frac{V_{BC}}{V_t} \right) - 1 \right] \right\}$$

10.17

(a) Extending the results of problem 10.16 to a pnp transistor, we can write

$$J_p = \frac{eD_B p_{BO}}{x_B} \left[\exp \left(\frac{V_{EB}}{V_t} \right) - \exp \left(\frac{V_{CB}}{V_t} \right) \right]$$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

Then

$$\begin{aligned} 165 &= \frac{(1.6 \times 10^{-19})(10)(2.25 \times 10^3)}{0.7 \times 10^{-4}} \\ &\quad \times \left[\exp \left(\frac{0.75}{0.0259} \right) - \exp \left(\frac{V_{CB}}{V_t} \right) \right] \end{aligned}$$

or

$$3.208 \times 10^{12} = 3.768 \times 10^{12} - \exp \left(\frac{V_{CB}}{V_t} \right)$$

which yields

$$\begin{aligned} V_{CB} &= (0.0259) \ln(0.56 \times 10^{12}) \Rightarrow \\ V_{CB} &= 0.70 \text{ V} \end{aligned}$$

(b)

$$\begin{aligned} V_{EC}(\text{sat}) &= V_{EB} - V_{CB} = 0.75 - 0.70 \Rightarrow \\ V_{EC}(\text{sat}) &= 0.05 \text{ V} \end{aligned}$$

(c)

Again, extending the results of problem 10.16 to a pnp transistor, we can write

$$\begin{aligned} Q_{pB} &= \frac{ep_{BO}x_B}{2} \left\{ \left[\exp \left(\frac{V_{EB}}{V_t} \right) - 1 \right] + \left[\exp \left(\frac{V_{CB}}{V_t} \right) - 1 \right] \right\} \\ &= \frac{(1.6 \times 10^{-19})(2.25 \times 10^3)(0.7 \times 10^{-4})}{2} \\ &\quad \times [3.768 \times 10^{12} + 0.56 \times 10^{12}] \end{aligned}$$

or

$$Q_{pB} = 5.45 \times 10^{-8} \text{ C / cm}^2$$

or

$$\frac{Q_{pB}}{e} = 3.41 \times 10^{11} \text{ holes / cm}^2$$

(d)

In the collector, we have

$$\delta n_p(x) = n_{pO} \left[\exp \left(\frac{V_{CB}}{V_t} \right) - 1 \right] \cdot \exp \left(\frac{-x}{L_C} \right)$$

The total number of excess electrons in the collector is

$$\begin{aligned}
 N_{\text{coll}} &= \int_0^{\infty} \delta n_p(x) dx \\
 &= -n_{\text{PO}} L_C \left[\exp\left(\frac{V_{CB}}{V_t}\right) - 1 \right] \cdot \exp\left(\frac{-x}{L_C}\right) \Big|_0^{\infty} \\
 &= n_{\text{PO}} L_C \left[\exp\left(\frac{V_{CB}}{V_t}\right) - 1 \right]
 \end{aligned}$$

We have

$$n_{\text{PO}} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}$$

Then the total number of electrons is

$$N_{\text{coll}} = (4.5 \times 10^4)(35 \times 10^{-4})(0.56 \times 10^{12})$$

or

$$N_{\text{coll}} = 8.82 \times 10^{13} \text{ electrons / cm}^2$$

10.18

$$(b) \quad n_{\text{BO}} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

and

$$p_{\text{CO}} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{7 \times 10^{15}} = 3.21 \times 10^4 \text{ cm}^{-3}$$

At $x = x_B$,

$$\begin{aligned}
 n_B(x_B) &= n_{\text{BO}} \exp\left(\frac{V_{BC}}{V_t}\right) \\
 &= (2.25 \times 10^3) \exp\left(\frac{0.565}{0.0259}\right)
 \end{aligned}$$

or

$$n_B(x_B) = 6.7 \times 10^{12} \text{ cm}^{-3}$$

At $x'' = 0$,

$$\begin{aligned}
 p_C(0) &= p_{\text{CO}} \exp\left(\frac{V_{BC}}{V_t}\right) \\
 &= (3.21 \times 10^4) \exp\left(\frac{0.565}{0.0259}\right)
 \end{aligned}$$

or

$$p_C(0) = 9.56 \times 10^{13} \text{ cm}^{-3}$$

(c)

From the B-C space-charge region,

$$V_{b1} = (0.0259) \ln \left[\frac{(10^{17})(7 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.745 \text{ V}$$

Then

$$\begin{aligned}
 x_{p1} &= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.745 - 0.565)}{1.6 \times 10^{-19}} \right. \\
 &\quad \left. \times \left(\frac{7 \times 10^{15}}{10^{17}} \right) \left(\frac{1}{7 \times 10^{15} + 10^{17}} \right) \right\}^{1/2}
 \end{aligned}$$

or

$$x_{p1} = 1.23 \times 10^{-6} \text{ cm}$$

From the B-E space-charge region,

$$V_{b2} = (0.0259) \ln \left[\frac{(10^{19})(10^{17})}{(1.5 \times 10^{10})^2} \right] = 0.933 \text{ V}$$

Then

$$\begin{aligned}
 x_{p2} &= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.933 + 2)}{1.6 \times 10^{-19}} \right. \\
 &\quad \left. \times \left(\frac{10^{19}}{10^{17}} \right) \left(\frac{1}{10^{19} + 10^{17}} \right) \right\}^{1/2}
 \end{aligned}$$

or

$$x_{p2} = 1.94 \times 10^{-5} \text{ cm}$$

Now

$$x_B = x_{\text{BO}} - x_{p1} - x_{p2} = 1.20 - 0.0123 - 0.194$$

or

$$x_B = 0.994 \text{ } \mu\text{m}$$

10.19

Low injection limit is reached when

$p_C(0) = (0.10)N_C$, so that

$$p_C(0) = (0.10)(5 \times 10^{14}) = 5 \times 10^{13} \text{ cm}^{-3}$$

We have

$$p_{\text{CO}} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{14}} = 4.5 \times 10^5 \text{ cm}^{-3}$$

Also

$$p_C(0) = p_{\text{CO}} \exp\left(\frac{V_{CB}}{V_t}\right)$$

or

$$\begin{aligned}
 V_{CB} &= V_t \ln \left(\frac{p_C(0)}{p_{\text{CO}}} \right) \\
 &= (0.0259) \ln \left(\frac{5 \times 10^{13}}{4.5 \times 10^5} \right)
 \end{aligned}$$

or

$$V_{CB} = 0.48 \text{ V}$$

10.20

(a)

$$\alpha = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}} = \frac{1.18}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\alpha = 0.787}$$

(b)

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}} = \frac{1.20}{1.20 + 0.10} \Rightarrow \underline{\gamma = 0.923}$$

(c)

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.18}{1.20} \Rightarrow \underline{\alpha_T = 0.983}$$

(d)

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} = \frac{1.20 + 0.10}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\delta = 0.867}$$

(e)

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.787}{1 - 0.787}$$

or

$$\underline{\underline{\beta = 3.69}}$$

10.21

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right) = (2.25 \times 10^3) \exp\left(\frac{0.50}{0.0259}\right)$$

or

$$\underline{n_B(0) = 5.45 \times 10^{11} \text{ cm}^{-3}}$$

As a good approximation,

$$I_C = \frac{eD_B A n_B(0)}{x_B} = \frac{(1.6 \times 10^{-19})(20)(10^{-3})(5.45 \times 10^{11})}{10^{-4}}$$

or

$$\underline{I_C = 17.4 \mu A}$$

(b)

Base transport factor

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)}$$

We find

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(20)(10^{-7})} = 1.41 \times 10^{-3} \text{ cm}$$

so that

$$\alpha_T = \frac{1}{\cosh(1/14.1)} \Rightarrow \underline{\alpha_T = 0.9975}$$

Emitter injection efficiency

Assuming $D_E = D_B$, $x_B = x_E$, and $L_E = L_B$;

then

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} = \frac{1}{1 + \frac{10^{17}}{10^{18}}} \Rightarrow \underline{\gamma = 0.909}$$

Then

$$\alpha = \gamma \alpha_T \delta = (0.909)(0.9975)(1) \Rightarrow \underline{\alpha = 0.9067}$$

and

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.9067}{1 - 0.9067} \Rightarrow \underline{\beta = 9.72}$$

For $I_E = 1.5 \text{ mA}$,

$$I_C = \alpha I_E = (0.9067)(1.5) \Rightarrow \underline{I_C = 1.36 \text{ mA}}$$

(c)

For $I_B = 2 \mu A$,

$$I_C = \beta I_B = (9.72)(2) \Rightarrow \underline{I_C = 19.4 \mu A}$$

10.22

(a) We have

$$J_{nE} = \frac{eD_B n_{BO}}{L_B} \left\{ \frac{1}{\sinh\left(\frac{x_B}{L_B}\right)} + \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right]}{\tanh\left(\frac{x_B}{L_B}\right)} \right\}$$

We find that

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(15)(5 \times 10^{-8})} = 8.66 \times 10^{-4} \text{ cm}$$

Then

$$J_{nE} = \frac{(1.6 \times 10^{-19})(15)(4.5 \times 10^3)}{8.66 \times 10^{-4}} \times \left\{ \frac{1}{\sinh\left(\frac{0.70}{8.66}\right)} + \frac{\exp\left(\frac{0.60}{0.0259}\right)}{\tanh\left(\frac{0.70}{8.66}\right)} \right\}$$

or

$$J_{nE} = 1.79 \text{ A/cm}^2$$

We also have

$$J_{pE} = \frac{eD_E p_{EO}}{L_E} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \frac{1}{\tanh\left(\frac{x_E}{L_E}\right)}$$

Also

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$

and

$$L_E = \sqrt{D_E \tau_{EO}} = \sqrt{(8)(10^{-8})} = 2.83 \times 10^{-4} \text{ cm}$$

Then

$$J_{pE} = \frac{(1.6 \times 10^{-19})(8)(2.25 \times 10^2)}{2.83 \times 10^{-4}} \times \left[\exp\left(\frac{0.60}{0.0259}\right) - 1 \right] \cdot \frac{1}{\tanh\left(\frac{0.8}{2.83}\right)}$$

or

$$J_{pE} = 0.0425 \text{ A/cm}^2$$

We can find

$$J_{nC} = \frac{eD_B n_{BO}}{L_B} \left\{ \frac{\left[\exp\left(\frac{0.60}{0.0259}\right) - 1 \right]}{\sinh\left(\frac{x_B}{L_B}\right)} + \frac{1}{\tanh\left(\frac{x_B}{L_B}\right)} \right\} = \frac{(1.6 \times 10^{-19})(15)(4.5 \times 10^3)}{8.66 \times 10^{-4}} \times \left\{ \frac{\exp\left(\frac{0.60}{0.0259}\right)}{\sinh\left(\frac{0.7}{8.66}\right)} + \frac{1}{\tanh\left(\frac{0.7}{8.66}\right)} \right\}$$

or

$$J_{nC} = 1.78 \text{ A/cm}^2$$

The recombination current is

$$J_R = J_{rO} \exp\left(\frac{eV_{BE}}{2kT}\right) = (3 \times 10^{-8}) \exp\left(\frac{0.60}{2(0.0259)}\right)$$

or

$$J_R = 3.22 \times 10^{-3} \text{ A/cm}^2$$

(b)

Using the calculated currents, we find

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}} = \frac{1.79}{1.79 + 0.0425} \Rightarrow \gamma = 0.977$$

We find

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.78}{1.79} \Rightarrow \alpha_T = 0.994$$

and

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} = \frac{1.79 + 0.0425}{1.79 + 0.00322 + 0.0425}$$

or

$$\delta = 0.998$$

Then

$$\alpha = \gamma \alpha_T \delta = (0.977)(0.994)(0.998) \Rightarrow \alpha = 0.969$$

Now

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.969}{1 - 0.969} \Rightarrow \beta = 31.3$$

10.23

$$(a) \quad \gamma = \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} \approx 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

or

$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i)

$$\begin{aligned} \frac{\gamma(B)}{\gamma(A)} &= \frac{1 - \frac{2N_{BO}}{N_E} \cdot K}{1 - \frac{N_{BO}}{N_E} \cdot K} \\ &\approx \left(1 - \frac{2N_{BO}}{N_E} \cdot K \right) \left(1 + \frac{N_{BO}}{N_E} \cdot K \right) \\ &\approx 1 - \frac{2N_{BO}}{N_E} \cdot K + \frac{N_{BO}}{N_E} \cdot K \end{aligned}$$

or

$$\frac{\gamma(B)}{\gamma(A)} \approx 1 - \frac{N_{BO}}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)

$$\frac{\gamma(C)}{\gamma(A)} = 1$$

(b) (i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)

$$\begin{aligned} \frac{\alpha_T(C)}{\alpha_T(A)} &= \frac{\left(1 - \frac{1}{2} \cdot \frac{(x_{BO}/2)}{L_B}\right)^2}{\left(1 - \frac{1}{2} \cdot \frac{x_{BO}}{L_B}\right)^2} \\ &\approx \frac{\left(1 - \frac{x_{BO}}{2L_B}\right)}{\left(1 - \frac{x_{BO}}{L_B}\right)} \approx \left(1 - \frac{x_{BO}}{2L_B}\right) \left(1 + \frac{x_{BO}}{L_B}\right) \\ &\approx 1 - \frac{x_{BO}}{2L_B} + \frac{x_{BO}}{L_B} \end{aligned}$$

or

$$\frac{\alpha_T(C)}{\alpha_T(A)} \approx 1 + \frac{x_{BO}}{2L_B}$$

(c) Neglect any change in space charge width.
Then

$$\begin{aligned} \delta &= \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)} \\ &\approx 1 - \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right) \end{aligned}$$

(i)

$$\begin{aligned} \frac{\delta(B)}{\delta(A)} &= \frac{1 - \frac{K}{J_{sOB}}}{1 - \frac{K}{J_{sOA}}} \approx \left(1 - \frac{K}{J_{sOB}}\right) \left(1 + \frac{K}{J_{sOA}}\right) \\ &\approx 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}} \end{aligned}$$

Now

$$J_{sO} \propto n_{BO} = \frac{n_i^2}{N_B}$$

so

$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{2N_{BO}K}{C} + \frac{N_{BO}K}{C} = 1 - \frac{N_{BO}K}{C}$$

Then

$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{J_{rO} \exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(ii)

We find

$$\frac{\delta(C)}{\delta(A)} \approx 1 + \frac{J_{rO} \exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(d)

Device C has the largest β . Base transport factor as well as the recombination factor increases.

10.24

(a)

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} = \frac{1}{1 + K \cdot \frac{N_B}{N_E}}$$

or

$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i) Then

$$\begin{aligned} \frac{\gamma(B)}{\gamma(A)} &= \frac{1 - K \cdot \frac{N_B}{2N_{EO}}}{1 - K \cdot \frac{N_B}{N_{EO}}} \\ &\approx \left(1 - K \cdot \frac{N_B}{2N_{EO}}\right) \cdot \left(1 + K \cdot \frac{N_B}{N_{EO}}\right) \\ &\approx 1 - K \cdot \frac{N_B}{2N_{EO}} + K \cdot \frac{N_B}{N_{EO}} \end{aligned}$$

or

$$= 1 + K \cdot \frac{N_B}{2N_{EO}}$$

or

$$\frac{\gamma(B)}{\gamma(A)} = 1 + \frac{N_B}{2N_{EO}} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)
Now

$$\gamma = \frac{1}{1 + K' \cdot \frac{x_B}{x_E}} \approx 1 - K' \cdot \frac{x_B}{x_E}$$

Then

$$\begin{aligned} \frac{\gamma(C)}{\gamma(A)} &= \frac{1 - K' \cdot \frac{x_B}{(x_{EO}/2)}}{1 - K' \cdot \frac{x_B}{x_{EO}}} \\ &\approx \left(1 - K' \cdot \frac{2x_B}{x_{EO}}\right) \cdot \left(1 + K' \cdot \frac{x_B}{x_{EO}}\right) \\ &\approx 1 - 2K' \cdot \frac{x_B}{x_{EO}} + K' \cdot \frac{x_B}{x_{EO}} \\ &= 1 - K' \cdot \frac{x_B}{x_{EO}} \end{aligned}$$

or

$$\frac{\gamma(C)}{\gamma(A)} = 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_{EO}}$$

(b)

$$\alpha_T = 1 - \frac{1}{2} \left(\frac{x_B}{L_B} \right)^2$$

so

(i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

and

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = 1$$

(c)

Neglect any change in space charge width

$$\begin{aligned} \delta &= \frac{1}{1 + \frac{J_{rO}}{J_{SO}} \exp\left(\frac{-V_{BE}}{2V_t}\right)} \\ &= \frac{1}{1 + \frac{k}{J_{SO}}} \approx 1 - \frac{k}{J_{SO}} \end{aligned}$$

(i)

$$\begin{aligned} \frac{\delta(B)}{\delta(A)} &= \frac{1 - \frac{k}{J_{SOB}}}{1 - \frac{k}{J_{SOA}}} \approx \left(1 - \frac{k}{J_{SOB}}\right) \left(1 + \frac{k}{J_{SOA}}\right) \\ &\approx 1 - \frac{k}{J_{SOB}} + \frac{k}{J_{SOA}} \end{aligned}$$

Now

$$J_{SO} \propto \frac{1}{N_E x_E}$$

so

(i)

$$\frac{\delta(B)}{\delta(A)} = 1 - k'(2N_{EO}) + k'(N_{EO})$$

or

$$\frac{\delta(B)}{\delta(A)} = 1 - k' \cdot (N_{EO})$$

(recombination factor decreases)

(ii)

We have

$$\frac{\delta(C)}{\delta(A)} = 1 - k'' \cdot \left(\frac{x_{EO}}{2}\right) + k'' \cdot (x_{EO})$$

or

$$\frac{\delta(C)}{\delta(A)} = 1 + \frac{1}{2} k'' \cdot x_{EO}$$

(recombination factor increases)

10.25

(b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

Then

$$\begin{aligned} n_B(0) &= n_{BO} \exp\left(\frac{V_{BC}}{V_t}\right) \\ &= (2.25 \times 10^3) \exp\left(\frac{0.6}{0.0259}\right) = 2.59 \times 10^{13} \text{ cm}^{-3} \end{aligned}$$

Now

$$\begin{aligned} J_{nC} &= \frac{eD_B n_B(0)}{x_B} \\ &= \frac{(1.6 \times 10^{-19})(20)(2.59 \times 10^{13})}{10^{-4}} \end{aligned}$$

or

$$J_{nC} = 0.829 \text{ A/cm}^2$$

Assuming a long collector,

$$J_{pC} = \frac{eD_C p_{nO}}{L_C} \exp\left(\frac{V_{BC}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

and

$$L_C = \sqrt{D_C \tau_{CO}} = \sqrt{(15)(2 \times 10^{-7})} = 1.73 \times 10^{-3} \text{ cm}$$

Then

$$J_{pC} = \frac{(1.6 \times 10^{-19})(15)(2.25 \times 10^4)}{1.73 \times 10^{-3}} \exp\left(\frac{0.6}{0.0259}\right)$$

or

$$J_{pC} = 0.359 \text{ A/cm}^2$$

The collector current is

$$I_C = (J_{nC} + J_{pC}) \cdot A = (0.829 + 0.359)(10^{-3})$$

or

$$I_C = 1.19 \text{ mA}$$

The emitter current is

$$I_E = J_{nC} \cdot A = (0.829)(10^{-3})$$

or

$$I_E = 0.829 \text{ mA}$$

10.26

(a)

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)} \quad \beta = \frac{\alpha_T}{1 - \alpha_T}$$

x_B/L_B	α_T	β
0.01	0.99995	19,999
0.10	0.995	199
1.0	0.648	1.84
10.0	0.0000908	≈ 0

(b)

For $D_E = D_B$, $L_E = L_B$, $x_E = x_B$, we have

$$\gamma = \frac{1}{1 + (p_{EO}/n_{BO})} = \frac{1}{1 + (N_B/N_C)}$$

and

$$\beta = \frac{\gamma}{1 - \gamma}$$

N_B/N_E	γ	β
0.01	0.990	99
0.10	0.909	9.99
1.0	0.50	1.0
10.0	0.0909	0.10

(c)

For $x_B/L_B < 0.10$, the value of β is unreasonably large, which means that the base transport factor is not the limiting factor. For $x_B/L_B > 1.0$, the value of β is very small, which means that the base transport factor will probably be the limiting factor.

If $N_B/N_E < 0.01$, the emitter injection efficiency is probably not the limiting factor. If, however, $N_B/N_E > 0.01$, then the current gain is small and the emitter injection efficiency is probably the limiting factor.

10.27

We have

$$J_{sO} = \frac{eD_B n_{BO}}{L_B \tanh(x_B/L_B)}$$

Now

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8 \times 10^{-4} \text{ cm}$$

Then

$$J_{sO} = \frac{(1.6 \times 10^{-19})(25)(2.25 \times 10^3)}{(15.8 \times 10^{-4}) \tanh(0.7/15.8)}$$

or

$$J_{sO} = 1.3 \times 10^{-10} \text{ A/cm}^2$$

Now

$$\begin{aligned} \delta &= \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)} \\ &= \frac{1}{1 + \frac{2 \times 10^{-9}}{1.3 \times 10^{-10}} \cdot \exp\left(\frac{-V_{BE}}{2(0.0259)}\right)} \end{aligned}$$

or

(a)

$$\delta = \frac{1}{1 + (15.38) \exp\left(\frac{-V_{BE}}{0.0518}\right)}$$

and

(b)

$$\beta = \frac{\delta}{1 - \delta}$$

Now

$\frac{V_{BE}}{V_T}$	δ	β
0.20	0.755	3.08
0.40	0.993	142
0.60	0.99986	7,142

(c)

If $V_{BE} < 0.4 V$, the recombination factor is likely the limiting factor in the current gain.

10.28

$$\text{For } \beta = 120 = \frac{\alpha}{1 - \alpha} \Rightarrow \alpha = \frac{\beta}{1 + \beta}$$

So

$$\alpha = \frac{120}{121} = 0.9917$$

Now

$$\alpha = \gamma \alpha_T \delta = 0.9917 = (0.998) x^2$$

where

$$x = \alpha_T = \gamma = 0.9968$$

We have

$$\alpha_T = \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)} = 0.9968$$

which means

$$\frac{x_B}{L_B} = 0.0801$$

We find

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8 \mu m$$

Then

$$x_B (\text{max}) = (0.0801)(15.8) \Rightarrow x_B (\text{max}) = 1.26 \mu m$$

We also have

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}} \cdot \frac{D_E}{D_B} \cdot \frac{L_B}{L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

where

$$L_E = \sqrt{D_E \tau_{EO}} = \sqrt{(10)(5 \times 10^{-8})} = 7.07 \mu m$$

Then

$$0.9968 = \frac{1}{1 + \frac{p_{EO}}{n_{BO}} \cdot \left(\frac{10}{25}\right) \left(\frac{15.8}{7.07}\right) \frac{\tanh(1.26/15.8)}{\tanh(0.5/7.07)}}$$

which yields

$$\frac{p_{EO}}{n_{BO}} = 0.003186 = \frac{N_B}{N_E}$$

Finally

$$N_E = \frac{N_B}{0.003186} = \frac{10^{16}}{0.003186} \Rightarrow N_E = 3.14 \times 10^{18} \text{ cm}^{-3}$$

10.29

(a) We have $J_{rO} = 5 \times 10^{-8} \text{ A/cm}^2$

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8 \mu m$$

Then

$$J_{sO} = \frac{e D_B n_{BO}}{L_B \tanh(x_B/L_B)} = \frac{(1.6 \times 10^{-19})(25)(4.5 \times 10^3)}{(15.8 \times 10^{-4}) \tanh(x_B/L_B)}$$

or

$$J_{sO} = \frac{1.14 \times 10^{-11}}{\tanh(x_B/L_B)}$$

We have

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_T}\right)}$$

For $T = 300 K$ and $V_{BE} = 0.55 V$,

$$\delta = 0.995 =$$

$$\frac{1}{1 + \left(\frac{5 \times 10^{-8}}{1.14 \times 10^{-11}}\right) \cdot \tanh\left(\frac{x_B}{L_B}\right) \cdot \exp\left(\frac{-0.55}{2(0.0259)}\right)}$$

which yields

$$\frac{x_B}{L_B} = 0.047$$

or

$$x_B = (0.047)(15.8 \times 10^{-4}) \Rightarrow$$

$$\underline{x_B = 0.742 \mu\text{m}}$$

(b)

For $T = 400\text{K}$ and $J_{rO} = 5 \times 10^{-8} \text{ A/cm}^2$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = \left(\frac{400}{300}\right)^3 \cdot \frac{\exp\left[\frac{-E_g}{(0.0259)(400/300)}\right]}{\exp\left[\frac{-E_g}{(0.0259)}\right]}$$

For $E_g = 1.12 \text{ eV}$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = 1.17 \times 10^5$$

or

$$n_{BO}(400) = (1.17 \times 10^5)(4.5 \times 10^3)$$

$$= 5.27 \times 10^8 \text{ cm}^{-3}$$

Then

$$J_{sO} = \frac{(1.6 \times 10^{-19})(25)(5.27 \times 10^8)}{(15.8 \times 10^{-4}) \tanh(0.742/15.8)}$$

or

$$J_{sO} = 2.84 \times 10^{-5} \text{ A/cm}^2$$

Finally,

$$\delta = \frac{1}{1 + \frac{5 \times 10^{-8}}{2.84 \times 10^{-5}} \cdot \exp\left[\frac{-0.55}{2(0.0259)(400/300)}\right]}$$

or

$$\underline{\delta = 0.9999994}$$

10.30

Computer plot

10.31

Computer plot

10.32

Computer plot

10.33

Computer plot

10.34

Metallurgical base width = $1.2 \mu\text{m} = x_B + x_n$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

and

$$p_B(0) = p_{BO} \exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25 \times 10^4) \exp\left(\frac{0.625}{0.0259}\right)$$

$$= 6.8 \times 10^{14} \text{ cm}^{-3}$$

Now

$$J_p = eD_B \frac{dp_B}{dx} = eD_B \left(\frac{p_B(0)}{x_B}\right)$$

$$= \frac{(1.6 \times 10^{-19})(10)(6.8 \times 10^{14})}{x_B}$$

or

$$J_p = \frac{1.09 \times 10^{-3}}{x_B}$$

We have

$$x_n = \left\{ \frac{2 \in (V_{bi} + V_R)}{e} \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right) \right\}^{1/2}$$

and

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.635 \text{ V}$$

We can write

$$x_n = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_R)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{10^{15}}{10^{16}} \right) \left(\frac{1}{10^{15} + 10^{16}} \right) \right\}^{1/2}$$

or

$$x_n = \left\{ (1.177 \times 10^{-10})(V_{bi} + V_R) \right\}^{1/2}$$

We know

$$x_B = 1.2 \times 10^{-4} - x_n$$

For $V_R = V_{BC} = 5 \text{ V}$

$$x_n = 0.258 \times 10^{-4} \text{ cm} \Rightarrow x_B = 0.942 \times 10^{-4} \text{ cm}$$

Then

$$\underline{J_p = 11.6 \text{ A/cm}^2}$$

For $V_R = V_{BC} = 10 \text{ V}$,

$$x_n = 0.354 \times 10^{-4} \text{ cm} \Rightarrow x_B = 0.846 \times 10^{-4} \text{ cm}$$

Then

$$J_p = 12.9 \text{ A / cm}^2$$

$$\text{For } V_R = V_{BC} = 15 \text{ V},$$

$$x_n = 0.429 \times 10^{-4} \text{ cm} \Rightarrow x_B = 0.771 \times 10^{-4} \text{ cm}$$

Then

$$J_p = 14.1 \text{ A / cm}^2$$

(b)

We can write

$$J_p = g'(V_{EC} + V_A)$$

where

$$g' = \frac{\Delta J_p}{\Delta V_{EC}} = \frac{\Delta J_p}{\Delta V_{BC}} = \frac{14.1 - 11.6}{10}$$

or

$$g' = 0.25 \text{ mA / cm}^2 / \text{V}$$

Now

$$J_p = 11.6 \text{ A / cm}^2 \text{ at}$$

$$V_{EC} = V_{BC} + V_{EB} = 5 + 0.626 = 5.626 \text{ V}$$

Then

$$11.6 = (0.25)(5.625 + V_A)$$

which yields

$$V_A = 40.8 \text{ V}$$

10.35 We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{16}} = 7.5 \times 10^3 \text{ cm}^{-3}$$

and

$$\begin{aligned} n_B(0) &= n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right) \\ &= (7.5 \times 10^3) \exp\left(\frac{0.7}{0.0259}\right) \end{aligned}$$

or

$$n_B(0) = 4.10 \times 10^{15} \text{ cm}^{-3}$$

We have

$$\begin{aligned} J &= eD_B \frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B} \\ &= \frac{(1.6 \times 10^{-19})(20)(4.10 \times 10^{15})}{x_B} \end{aligned}$$

or

$$J = \frac{1.312 \times 10^{-2}}{x_B}$$

Neglecting the space charge width at the B-E junction, we have

$$x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(3 \times 10^{16})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.705 \text{ V}$$

and

$$\begin{aligned} x_p &= \left\{ \frac{2 \epsilon (V_{bi} + V_{CB})}{e} \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right) \right\}^{1/2} \\ &= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{CB})}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{5 \times 10^{15}}{3 \times 10^{16}} \right) \left(\frac{1}{5 \times 10^{15} + 3 \times 10^{16}} \right) \right\}^{1/2} \end{aligned}$$

or

$$x_p = \left\{ (6.163 \times 10^{-11})(V_{bi} + V_{CB}) \right\}^{1/2}$$

Now, for $V_{CB} = 5 \text{ V}$, $x_p = 0.1875 \text{ } \mu\text{m}$, and

For $V_{CB} = 10 \text{ V}$, $x_p = 0.2569 \text{ } \mu\text{m}$

(a)

$$x_{BO} = 1.0 \text{ } \mu\text{m}$$

For $V_{CB} = 5 \text{ V}$, $x_B = 1.0 - 0.1875 = 0.8125 \text{ } \mu\text{m}$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.8125 \times 10^{-4}} = 161.5 \text{ A / cm}^2$$

For $V_{CB} = 10 \text{ V}$, $x_B = 1.0 - 0.2569 = 0.7431 \text{ } \mu\text{m}$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.7431 \times 10^{-4}} = 176.6 \text{ A / cm}^2$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}} (V_{CE} + V_A)$$

where

$$\begin{aligned} \frac{\Delta J}{\Delta V_{CE}} &= \frac{\Delta J}{\Delta V_{CB}} = \frac{176.6 - 161.5}{5} \\ &= 3.02 \text{ A / cm}^2 / \text{V} \end{aligned}$$

Then

$$\begin{aligned} 161.5 &= 3.02(5.7 + V_A) \Rightarrow \\ V_A &= 47.8 \text{ V} \end{aligned}$$

(b)

$$x_{BO} = 0.80 \mu\text{m}$$

$$\text{For } V_{CB} = 5 \text{ V}, x_B = 0.80 - 0.1875 = 0.6125 \mu\text{m}$$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.6125 \times 10^{-4}} = 214.2 \text{ A/cm}^2$$

$$\text{For } V_{CB} = 10 \text{ V}, x_B = 0.80 - 0.2569 = 0.5431 \mu\text{m}$$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.5431 \times 10^{-4}} = 241.6 \text{ A/cm}^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{241.6 - 214.2}{5} \\ = 5.48 \text{ A/cm}^2 / \text{V}$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}} (V_{CE} + V_A)$$

or

$$214.2 = 5.48(5.7 + V_A) \Rightarrow \\ \underline{V_A = 33.4 \text{ V}}$$

(c)

$$x_{BO} = 0.60 \mu\text{m}$$

$$\text{For } V_{CB} = 5 \text{ V}, x_B = 0.60 - 0.1875 = 0.4124 \mu\text{m}$$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.4125 \times 10^{-4}} = 318.1 \text{ A/cm}^2$$

$$\text{For } V_{CB} = 10 \text{ V}, x_B = 0.60 - 0.2569 = 0.3431 \mu\text{m}$$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.3431 \times 10^{-4}} = 382.4 \text{ A/cm}^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{382.4 - 318.1}{5} \\ = 12.86 \text{ A/cm}^2 / \text{V}$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}} (V_{CE} + V_A)$$

so

$$318.1 = 12.86(5.7 + V_A) \Rightarrow \\ \underline{V_A = 19.0 \text{ V}}$$

10.36

Neglect the B-E space charge region

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right) \\ = 2.25 \times 10^3 \exp\left(\frac{0.60}{0.0259}\right) = 2.59 \times 10^{13} \text{ cm}^{-3}$$

$$J = eD_B \frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B} \\ = \frac{(1.6 \times 10^{-19})(20)(2.59 \times 10^{13})}{x_B}$$

or

$$J = \frac{8.29 \times 10^{-5}}{x_B}$$

(a)

$$\text{Now } x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{16})(10^{17})}{(1.5 \times 10^{10})^2} \right] = 0.754 \text{ V}$$

Also

$$x_p = \left[\frac{2 \epsilon (V_{bi} + V_{CB})}{e} \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right) \right]^{1/2} \\ = \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{CB})}{1.6 \times 10^{-19}} \right. \\ \left. \times \left(\frac{10^{16}}{10^{17}} \right) \left(\frac{1}{10^{16} + 10^{17}} \right) \right]^{1/2}$$

or

$$x_p = [(1.177 \times 10^{-11})(V_{bi} + V_{CB})]^{1/2}$$

$$\text{For } V_{CB} = 1 \text{ V}, x_p(1) = 4.544 \times 10^{-6} \text{ cm}$$

$$\text{For } V_{CB} = 5 \text{ V}, x_p(5) = 8.229 \times 10^{-6} \text{ cm}$$

Now

$$x_B = x_{BO} - x_p = 1.1 \times 10^{-4} - x_p$$

Then

$$\text{For } V_{CB} = 1 \text{ V}, x_B(1) = 1.055 \mu\text{m}$$

$$\text{For } V_{CB} = 5 \text{ V}, x_B(5) = 1.018 \mu\text{m}$$

So

$$\Delta x_B = 1.055 - 1.018 \Rightarrow$$

or

$$\Delta x_B = 0.037 \mu m$$

(b)

Now

$$J(1) = \frac{8.29 \times 10^{-5}}{1.055 \times 10^{-4}} = 0.7858 \text{ A/cm}^2$$

and

$$J(5) = \frac{8.29 \times 10^{-5}}{1.018 \times 10^{-4}} = 0.8143 \text{ A/cm}^2$$

and

$$\Delta J = 0.8143 - 0.7858$$

or

$$\Delta J = 0.0285 \text{ A/cm}^2$$

10.37

Let $x_E = x_B$, $L_E = L_B$, $D_E = D_B$

Then the emitter injection efficiency is

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}}} = \frac{1}{1 + \frac{n_i^2}{N_E} \cdot \frac{N_B}{n_{iB}^2}}$$

where $n_{iB}^2 = n_i^2$

For no bandgap narrowing, $n_{iE}^2 = n_i^2$.

With bandgap narrowing, $n_{iE}^2 = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right)$,

Then

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right)}$$

(a)

No bandgap narrowing, so $\Delta E_g = 0$.

$$\alpha = \gamma \alpha_T \delta = \gamma (0.995)^2. \text{ We find}$$

$\frac{N_E}{\text{cm}^{-3}}$	γ	α	β
E17	0.5	0.495	0.980
E18	0.909	0.8999	8.99
E19	0.990	0.980	49
E20	0.9990	0.989	89.9

(b)

Taking into account bandgap narrowing, we find

$\frac{N_E}{\text{cm}^{-3}}$	$\frac{\Delta E_g \text{ (meV)}}{\text{cm}^{-3}}$	γ	α	β
E17	0	0.5	0.495	0.98
E18	25	0.792	0.784	3.63
E19	80	0.820	0.812	4.32
E20	230	0.122	0.121	0.14

10.38

(a) We have

$$\gamma = \frac{1}{1 + \frac{p_{EO} D_E L_B}{n_{BO} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

For $x_E = x_B$, $L_E = L_B$, $D_E = D_B$, we obtain

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}}} = \frac{1}{1 + \frac{(n_i^2/N_E) \exp(\Delta E_g/kT)}{(n_i^2/N_B)}}$$

or

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right)}$$

For $N_E = 10^{19} \text{ cm}^{-3}$, we have $\Delta E_g = 80 \text{ meV}$.

Then

$$0.996 = \frac{1}{1 + \frac{N_B}{10^{19}} \exp\left(\frac{0.080}{0.0259}\right)}$$

which yields

$$N_B = 1.83 \times 10^{15} \text{ cm}^{-3}$$

(b)

Neglecting bandgap narrowing, we would have

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} \Rightarrow 0.996 = \frac{1}{1 + \frac{N_B}{10^{19}}}$$

which yields

$$N_B = 4.02 \times 10^{16} \text{ cm}^{-3}$$

10.39

(a)

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{(S/2)}{e \mu_p N_B (L x_B)}$$

Then

$$R = \frac{4 \times 10^{-4}}{(1.6 \times 10^{-19})(400)(10^{16})(100 \times 10^{-4})(0.7 \times 10^{-4})}$$

or

$$R = 893 \, \Omega$$

(b)

$$V = IR = (10 \times 10^{-6})(893) \Rightarrow$$

$$V = 8.93 \, mV$$

(c)

At $x = 0$,

$$n_p(0) = n_{p0} \exp\left(\frac{V_{BE}}{V_t}\right)$$

and at $x = \frac{S}{2}$,

$$n'_p(0) = n_{p0} \exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)$$

Then

$$\begin{aligned} \frac{n'_p(0)}{n_p(0)} &= \frac{n_{p0} \exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)}{n_{p0} \exp\left(\frac{V_{BE}}{V_t}\right)} \\ &= \exp\left(\frac{-0.00893}{0.0259}\right) = 0.7084 \end{aligned}$$

or

$$\frac{n'_p(0)}{n_p(0)} = 70.8\%$$

10.40

From problem 10.39(c), we have

$$\frac{n'_p(0)}{n_p(0)} = \exp\left(\frac{-V}{V_t}\right)$$

where V is the voltage drop across the $S/2$ length. Now

$$0.90 = \exp\left(\frac{-V}{0.0259}\right)$$

which yields $V = 2.73 \, mV$

We have

$$R = \frac{V}{I} = \frac{2.73 \times 10^{-3}}{10 \times 10^{-6}} = 273 \, \Omega$$

We can also write

$$R = \frac{S/2}{e\mu_p N_B (Lx_B)}$$

Solving for S , we find

$$\begin{aligned} S &= 2R\mu_p eN_B Lx_B \\ &= 2(273)(400)(1.6 \times 10^{-19})(10^{16}) \\ &\quad \times (100 \times 10^{-4})(0.7 \times 10^{-4}) \end{aligned}$$

or

$$S = 2.45 \, \mu m$$

10.41

(a)

$$N_B = N_B(0) \exp\left(\frac{-ax}{x_B}\right)$$

where

$$a = \ln\left(\frac{N_B(0)}{N_B(x_B)}\right) > 0$$

and is a constant. In thermal equilibrium

$$J_p = e\mu_p N_B E - eD_p \frac{dN_B}{dx} = 0$$

so that

$$E = \frac{D_p}{\mu_p} \cdot \frac{1}{N_B} \cdot \frac{dN_B}{dx} = \left(\frac{kT}{e}\right) \cdot \frac{1}{N_B} \cdot \frac{dN_B}{dx}$$

which becomes

$$\begin{aligned} E &= \left(\frac{kT}{e}\right) \cdot \frac{1}{N_B} \cdot N_B(0) \cdot \left(\frac{-a}{x_B}\right) \cdot \exp\left(\frac{-ax}{x_B}\right) \\ &= \left(\frac{kT}{e}\right) \cdot \left(\frac{-a}{x_B}\right) \cdot \frac{1}{N_B} \cdot N_B \end{aligned}$$

or

$$E = -\left(\frac{a}{x_B}\right) \left(\frac{kT}{e}\right)$$

which is a constant.

(b)

The electric field is in the negative x -direction which will aid the flow of minority carrier electrons across the base.

(c)

$$J_n = e\mu_n nE + eD_n \frac{dn}{dx}$$

Assuming no recombination in the base, J_n will be a constant across the base. Then

$$\frac{dn}{dx} + \left(\frac{\mu_n}{D_n}\right) nE = \frac{J_n}{eD_n} = \frac{dn}{dx} + n \left(\frac{E}{V_t}\right)$$

where $V_t = \left(\frac{kT}{e}\right)$

The homogeneous solution to the differential equation is found from

$$\frac{dn_H}{dx} + An_H = 0$$

where $A = \frac{E}{V_t}$

The solution is of the form

$$n_H = n_H(0) \exp(-Ax)$$

The particular solution is found from

$$n_p \cdot A = B$$

where $B = \frac{J_n}{eD_n}$

The particular solution is then

$$n_p = \frac{B}{A} = \frac{\left(\frac{J_n}{eD_n}\right)}{\left(\frac{E}{V_t}\right)} = \frac{J_n V_t}{eD_n E} = \frac{J_n}{e\mu_n E}$$

The total solution is then

$$n = \frac{J_n}{e\mu_n E} + n_H(0) \exp(-Ax)$$

and

$$n(0) = n_{p0} \exp\left(\frac{V_{BE}}{V_t}\right) = \frac{n_i^2}{N_B(0)} \exp\left(\frac{V_{BE}}{V_t}\right)$$

Then

$$n_H(0) = \frac{n_i^2}{N_B(0)} \exp\left(\frac{V_{BE}}{V_t}\right) - \frac{J_n}{e\mu_n E}$$

10.42

- (a) The basic pn junction breakdown voltage from the figure for $N_C = 5 \times 10^{15} \text{ cm}^{-3}$ is approximately $BV_{CBO} = 90 \text{ V}$.

(b)

We have

$$BV_{CEO} = BV_{CBO} \sqrt[n]{1 - \alpha}$$

For $n = 3$ and $\alpha = 0.992$, we obtain

$$BV_{CEO} = 90 \cdot \sqrt[3]{1 - 0.992} = (90)(0.20)$$

or

$$BV_{CEO} = 18 \text{ V}$$

(c)

The B-E breakdown voltage, for

$N_B = 10^{17} \text{ cm}^{-3}$, is approximately,

$$BV_{BE} = 12 \text{ V}$$

10.43

We want $BV_{CEO} = 60 \text{ V}$

So then

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \Rightarrow 60 = \frac{BV_{CBO}}{\sqrt[3]{50}}$$

which yields

$$BV_{CBO} = 221 \text{ V}$$

For this breakdown voltage, we need

$$N_C \approx 1.5 \times 10^{15} \text{ cm}^{-3}$$

The depletion width into the collector at this voltage is

$$x_C = x_n = \left\{ \frac{2 \epsilon (V_{bi} + V_{BC})}{e} \left(\frac{N_B}{N_C} \right) \left(\frac{1}{N_B + N_C} \right) \right\}^{1/2}$$

We find

$$V_{bi} = (0.0259) \ln \left[\frac{(1.5 \times 10^{15})(10^{16})}{(1.5 \times 10^{10})^2} \right] = 0.646 \text{ V}$$

and $V_{BC} = BV_{CEO} = 60 \text{ V}$

so that

$$x_C = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.646 + 60)}{1.6 \times 10^{-19}} \times \left(\frac{10^{16}}{1.5 \times 10^{15}} \right) \left(\frac{1}{10^{16} + 1.5 \times 10^{15}} \right) \right\}^{1/2}$$

or

$$x_C = 6.75 \text{ } \mu\text{m}$$

10.44

$$V_{bi} = (0.0259) \ln \left[\frac{(3 \times 10^{16})(5 \times 10^{17})}{(1.5 \times 10^{10})^2} \right] = 0.824 \text{ V}$$

At punch-through, we have

$$x_B = 0.70 \times 10^{-4} = x_p(V_{BC} = V_{th}) - x_p(V_{BC} = 0)$$

$$= \left\{ \frac{2 \epsilon (V_{bi} + V_{pt})}{e} \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right) \right\}^{1/2} - \left\{ \frac{2 \epsilon V_{bi}}{e} \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right) \right\}^{1/2}$$

which can be written as

$$0.70 \times 10^{-4} = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{pt})}{1.6 \times 10^{-19}} \times \left(\frac{5 \times 10^{17}}{3 \times 10^{16}} \right) \left(\frac{1}{5 \times 10^{17} + 3 \times 10^{16}} \right) \right\}^{1/2}$$

$$- \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.824)}{1.6 \times 10^{-19}} \times \left(\frac{5 \times 10^{17}}{3 \times 10^{16}} \right) \left(\frac{1}{5 \times 10^{17} + 3 \times 10^{16}} \right) \right\}^{1/2}$$

which becomes

$$0.70 \times 10^{-4} = (0.202 \times 10^{-4}) \sqrt{V_{bi} + V_{pt}} - (0.183 \times 10^{-4})$$

We obtain

$$V_{bi} + V_{pt} = 19.1 \text{ V}$$

or

$$V_{pt} = 18.3 \text{ V}$$

Considering the junction alone, avalanche breakdown would occur at approximately $BV \approx 25 \text{ V}$.

10.45

(a) Neglecting the B-E junction depletion width,

$$V_{pt} = \frac{eW_B^2}{2\epsilon} \cdot \frac{N_B(N_C + N_B)}{N_C}$$

$$= \left\{ \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2}{2(11.7)(8.85 \times 10^{-14})} \cdot \frac{(10^{17})(10^{17} + 7 \times 10^{15})}{(7 \times 10^{15})} \right\}$$

or

$$V_{pt} = 295 \text{ V}$$

However, actual junction breakdown for these doping concentrations is $\approx 70 \text{ V}$. So punch-through will not be reached.

10.46

At punch-through,

$$x_O = \left\{ \frac{2\epsilon(V_{bi} + V_{pt})}{e} \cdot \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_B + N_C} \right) \right\}^{1/2}$$

Since $V_{pt} = 25 \text{ V}$, we can neglect V_{bi} .

Then we have

$$(0.75 \times 10^{-4}) = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(25)}{1.6 \times 10^{-19}} \times \left(\frac{10^{16}}{N_B} \right) \left(\frac{1}{10^{16} + N_B} \right) \right\}^{1/2}$$

We obtain

$$N_B = 1.95 \times 10^{16} \text{ cm}^{-3}$$

10.47

$$V_{CE}(sat) = \left(\frac{kT}{e} \right) \cdot \ln \left[\frac{I_C(1 - \alpha_R) + I_B}{\alpha_F I_B - (1 - \alpha_F) I_C} \cdot \frac{\alpha_F}{\alpha_R} \right]$$

We can write

$$\exp \left(\frac{V_{CE}(sat)}{0.0259} \right) = \frac{(1)(1 - 0.2) + I_B}{(0.99)I_B - (1 - 0.99)(1)} \left(\frac{0.99}{0.20} \right)$$

or

$$\exp \left(\frac{V_{CE}(sat)}{0.0259} \right) = \left(\frac{0.8 + I_B}{0.99I_B - 0.01} \right) (4.95)$$

(a)

For $V_{CE}(sat) = 0.30 \text{ V}$, we find

$$\exp \left(\frac{0.30}{0.0259} \right) = 1.0726 \times 10^5$$

$$= \left(\frac{0.8 + I_B}{0.99I_B - 0.01} \right) (4.95)$$

We find

$$I_B = 0.01014 \text{ mA}$$

(b)

For $V_{CE}(sat) = 0.20 \text{ V}$, we find

$$I_B = 0.0119 \text{ mA}$$

(c)

For $V_{CE}(sat) = 0.10 \text{ V}$, we find

$$I_B = 0.105 \text{ mA}$$

10.48

For an npn in the active mode, we have $V_{BC} < 0$,

$$\text{so that } \exp \left(\frac{V_{BC}}{V_T} \right) \approx 0.$$

Now

$$I_E + I_B + I_C = 0 \Rightarrow I_B = -(I_C + I_E)$$

Then we have

$$I_B = -\left\{ \alpha_F I_{ES} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] + I_{CS} \right\} \\ - \left\{ -\alpha_R I_{CS} - I_{ES} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \right\}$$

or

$$I_B = (1 - \alpha_F) I_{ES} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] - (1 - \alpha_R) I_{CS}$$

10.49

We can write

$$I_{ES} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \\ = \alpha_R I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] - I_E$$

Substituting, we find

$$I_C = \alpha_F \{ \alpha_R I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] - I_E \} \\ - I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right]$$

From the definition of currents, we have

 $I_E = -I_C$ for the case when $I_B = 0$. Then

$$I_C = \alpha_F \alpha_R I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] \\ + \alpha_F I_C - I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right]$$

When a C-E voltage is applied, then the B-C

becomes reverse biased, so $\exp\left(\frac{V_{BC}}{V_t}\right) \approx 0$. Then

$$I_C = -\alpha_F \alpha_R I_{CS} + \alpha_F I_C + I_{CS}$$

We find

$$I_C = I_{CEO} = \frac{I_{CS}(1 - \alpha_F \alpha_R)}{(1 - \alpha_F)}$$

10.50

We have

$$I_C = \alpha_F I_{ES} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \\ - I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right]$$

For $V_{BC} < \approx 0.1 V$, $\exp\left(\frac{V_{BC}}{V_t}\right) \approx 0$ and $I_C \approx \text{constant}$. This equation does not include the base width modulation effect.For $V_{BE} = 0.2 V$,

$$I_C = (0.98)(10^{-13}) \exp\left(\frac{0.2}{0.0250}\right) + 5 \times 10^{-13}$$

or

$$I_C = 2.22 \times 10^{-10} A$$

For $V_{BE} = 0.4 V$,

$$I_C = 5 \times 10^{-7} A$$

For $V_{BE} = 0.6 V$,

$$I_C = 1.13 \times 10^{-3} A$$

10.51

Computer Plot

10.52

(a)

$$r'_\pi = \left(\frac{kT}{e} \right) \cdot \frac{1}{I_E} = \frac{0.0259}{0.5 \times 10^{-3}} = 51.8 \Omega$$

So

$$\tau_e = r'_\pi C_{je} = (51.8)(0.8 \times 10^{-12}) \Rightarrow$$

or

$$\tau_e = 41.4 ps$$

Also

$$\tau_b = \frac{x_B^2}{2D_n} = \frac{(0.7 \times 10^{-4})^2}{2(25)} \Rightarrow$$

or

$$\tau_b = 98 ps$$

We have

$$\tau_c = r'_c (C_\mu + C_s) = (30)(2)(0.08 \times 10^{-12}) \Rightarrow$$

or

$$\tau_c = 4.8 ps$$

Also

$$\tau_d = \frac{x_{dc}^2}{v_s} = \frac{2 \times 10^{-4}}{10^{+7}} \Rightarrow$$

or

$$\tau_d = 20 ps$$

(b)

$$\tau_{ec} = \tau_e + \tau_b + \tau_c + \tau_d \\ = 41.4 + 98 + 4.8 + 20 \Rightarrow$$

or

$$\tau_{ec} = 164.2 \text{ ps}$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi(164.2 \times 10^{-12})} \Rightarrow$$

or

$$f_T = 970 \text{ MHz}$$

Also

$$f_\beta = \frac{f_T}{\beta} = \frac{970}{50} \Rightarrow$$

or

$$f_\beta = 19.4 \text{ MHz}$$

10.53

$$\tau_b = \frac{x_B^2}{2D_B} = \frac{(0.5 \times 10^{-4})^2}{2(20)} = 6.25 \times 10^{-11} \text{ s}$$

We have $\tau_b = 0.2\tau_{ec}$,

So that

$$\tau_{ec} = 3.125 \times 10^{-10} \text{ s}$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi(3.125 \times 10^{-10})} \Rightarrow$$

or

$$f_T = 509 \text{ MHz}$$

10.54

We have

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

We are given

$$\tau_b = 100 \text{ ps} \text{ and } \tau_e = 25 \text{ ps}$$

We find

$$\tau_d = \frac{x_d}{v_s} = \frac{1.2 \times 10^{-4}}{10^7} = 1.2 \times 10^{-11} \text{ s}$$

or

$$\tau_d = 12 \text{ ps}$$

Also

$$\tau_c = r_c C_c = (10)(0.1 \times 10^{-12}) = 10^{-12} \text{ s}$$

or

$$\tau_c = 1 \text{ ps}$$

Then

$$\tau_{ec} = 25 + 100 + 12 + 1 = 138 \text{ ps}$$

We obtain

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi(138 \times 10^{-12})} = 1.15 \times 10^9 \text{ Hz}$$

or

$$f_T = 1.15 \text{ GHz}$$

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Chapter 11

Problem Solutions

11.1

- (a) p-type, inversion
- (b) p-type, depletion
- (c) p-type, accumulation
- (d) n-type, inversion

11.2

- (a) For $T = 300K$

Silicon:

$$\begin{aligned}\phi_p &= V_t \ln \left(\frac{N_a}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.347 \text{ V}\end{aligned}$$

Now

$$\begin{aligned}x_{dT} &= \left[\frac{4 \in \phi_p}{eN_a} \right]^{1/2} \\ &= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}\end{aligned}$$

or

$$\underline{x_{dT} = 0.30 \text{ } \mu\text{m}}$$

Also

$$\begin{aligned}|Q'_{SD}(\text{max})| &= eN_a x_{dT} \\ &= (1.6 \times 10^{-19})(10^{16})(0.30 \times 10^{-4})\end{aligned}$$

or

$$\underline{|Q'_{SD}(\text{max})| = 4.8 \times 10^{-8} \text{ C / cm}^2}$$

GaAs:

$$\phi_p = (0.0259) \ln \left(\frac{10^{16}}{1.8 \times 10^6} \right) = 0.581 \text{ V}$$

and

$$x_{dT} = \left[\frac{4(13.1)(8.85 \times 10^{-14})(0.581)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$\underline{x_{dT} = 0.410 \text{ } \mu\text{m}}$$

Then

$$\underline{|Q'_{SD}(\text{max})| = 6.56 \times 10^{-8} \text{ C / cm}^2}$$

Germanium:

$$\phi_p = (0.0259) \ln \left(\frac{10^{16}}{2.4 \times 10^{13}} \right) = 0.156 \text{ V}$$

Then

$$x_{dT} = \left[\frac{4(16)(8.85 \times 10^{-14})(0.156)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$\underline{x_{dT} = 0.235 \text{ } \mu\text{m}}$$

Then

$$\underline{|Q'_{SD}(\text{max})| = 3.76 \times 10^{-8} \text{ C / cm}^2}$$

(b)

For $T = 200K$,

$$V_t = (0.0259) \left(\frac{200}{300} \right) = 0.01727 \text{ V}$$

Silicon: $n_i = 7.68 \times 10^4 \text{ cm}^{-3}$

We obtain $\phi_p = 0.442 \text{ V}$ and

$$\underline{x_{dT} = 0.388 \text{ } \mu\text{m}, |Q'_{SD}(\text{max})| = 5.4 \times 10^{-8} \text{ C / cm}^2}$$

GaAs: $n_i = 1.38 \text{ cm}^{-3}$

We obtain $\phi_p = 0.631 \text{ V}$ and

$$\underline{x_{dT} = 0.428 \text{ } \mu\text{m}, |Q'_{SD}(\text{max})| = 6.85 \times 10^{-8} \text{ C / cm}^2}$$

Germanium: $n_i = 2.16 \times 10^{10} \text{ cm}^{-3}$

We obtain $\phi_p = 0.225 \text{ V}$ and

$$\underline{x_{dT} = 0.282 \text{ } \mu\text{m}, |Q'_{SD}(\text{max})| = 4.5 \times 10^{-8} \text{ C / cm}^2}$$

11.3

- (a) We want $|Q'_{SD}(\text{max})| = 7.5 \times 10^{-9} \text{ C / cm}^2$

We have

$$|Q'_{SD}(\text{max})| = eN_d x_{dT}$$

where

$$x_{dT} = \left[\frac{4 \in \phi_{fn}}{eN_d} \right]^{1/2} \text{ and } \phi_{fn} = V_t \ln \left(\frac{N_d}{n_i} \right)$$

For n-type silicon,

$$\begin{aligned}|Q'_{SD}(\text{max})| &= 7.5 \times 10^{-9} = [4eN_d \in \phi_{fn}]^{1/2} \\ &= [4(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})N_d \phi_{fn}]^{1/2}\end{aligned}$$

or

$$(7.5 \times 10^{-9})^2 = (6.63 \times 10^{-31})N_d \phi_{fn}$$

which yields

$$N_d \phi_{fn} = 8.48 \times 10^{13}$$

and

$$\phi_{fn} = (0.0259) \ln \left(\frac{N_d}{1.5 \times 10^{10}} \right)$$

By trial and error

$$N_d = 3.27 \times 10^{14} \text{ cm}^{-3}$$

(b)

$$\phi_s = -2\phi_{fn}$$

where

$$\phi_{fn} = (0.0259) \ln \left(\frac{3.27 \times 10^{14}}{1.5 \times 10^{10}} \right) = 0.259 \text{ V}$$

Then

$$\phi_s = -0.518 \text{ V}$$

11.4

p-type silicon

(a) Aluminum gate

$$\phi_{ms} = \left[\phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right) \right]$$

We have

$$\phi_{fp} = V_i \ln \left(\frac{N_a}{n_i} \right) = (0.0259) \ln \left(\frac{6 \times 10^{15}}{1.5 \times 10^{10}} \right)$$

Or

$$\phi_{fp} = 0.334 \text{ V}$$

Then

$$\phi_{ms} = [3.20 - (3.25 + 0.56 + 0.334)]$$

or

$$\phi_{ms} = -0.944 \text{ V}$$

(b)

n^+ polysilicon gate:

$$\phi_{ms} = - \left(\frac{E_g}{2e} + \phi_{fp} \right) = -(0.56 + 0.334)$$

or

$$\phi_{ms} = -0.894 \text{ V}$$

(c)

p^+ polysilicon gate:

$$\phi_{ms} = \left(\frac{E_g}{2e} - \phi_{fp} \right) = (0.56 - 0.334)$$

or

$$\phi_{ms} = +0.226 \text{ V}$$

11.5

We want, for n-type silicon, $\phi_{ms} = -0.35 \text{ V}$.

(a) n^+ polysilicon gate:

$$\phi_{ms} = - \left(\frac{E_g}{2e} - \phi_{fn} \right) \Rightarrow -0.35 = -(0.56 - \phi_{fn})$$

or

$$\phi_{fn} = 0.21 = (0.0259) \ln \left(\frac{N_d}{1.5 \times 10^{10}} \right)$$

which yields

$$N_d = 4.98 \times 10^{13} \text{ cm}^{-3}$$

(b)

p^+ polysilicon gate:

$$\phi_{ms} = \left(\frac{E_g}{2e} + \phi_{fn} \right) \Rightarrow -0.35 = (0.56 + \phi_{fn})$$

or

$$\phi_{fn} = -0.91 \text{ V}$$

This is impossible, cannot use a p^+ polysilicon gate.

(c)

Aluminum gate:

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} - \phi_{fn} \right)$$

or

$$-0.35 = 3.20 - (3.25 + 0.56 - \phi_{fn})$$

which yields

$$\phi_{fn} = 0.26 = (0.0259) \ln \left(\frac{N_d}{1.5 \times 10^{10}} \right)$$

or finally,

$$N_d = 3.43 \times 10^{14} \text{ cm}^{-3}$$

11.6

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} \text{ and } C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

(a)

For $t_{ox} = 500 \text{ \AA}$

Then

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}} = 6.9 \times 10^{-8} \text{ F / cm}^2$$

n^+ polygate-to-n type silicon,

$$\phi_{ms} = - \left(\frac{E_g}{2e} - \phi_{fn} \right)$$

where

$$\phi_{fn} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

Then

$$\phi_{ms} = -(0.56 - 0.288) = -0.272 \text{ V}$$

(i) For $Q'_{ss} = 10^{10} \text{ cm}^{-2}$, we have

$$V_{FB} = -0.272 - \frac{(1.6 \times 10^{-19})(10^{10})}{6.9 \times 10^{-8}}$$

or

$$V_{FB} = -0.295 \text{ V}$$

(ii) For $Q'_{ss} = 10^{11} \text{ cm}^{-2} \Rightarrow$

$$V_{FB} = -0.504 \text{ V}$$

(iii) For $Q'_{ss} = 5 \times 10^{11} \text{ cm}^{-2} \Rightarrow$

$$V_{FB} = -1.43 \text{ V}$$

(b)

For $t_{ox} = 250 \text{ \AA}$, we find

$$C_{ox} = 1.38 \times 10^{-7} \text{ F / cm}^2$$

Then

(i) For $Q'_{ss} = 10^{10} \text{ cm}^{-2} \Rightarrow V_{FB} = -0.284 \text{ V}$

(ii) For $Q'_{ss} = 10^{11} \text{ cm}^{-2} \Rightarrow V_{FB} = -0.388 \text{ V}$

(iii) For $Q'_{ss} = 5 \times 10^{11} \text{ cm}^{-2} \Rightarrow$

$$V_{FB} = -0.852 \text{ V}$$

11.7

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right)$$

where

$$\phi_{fp} = (0.0259) \ln \left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.365 \text{ V}$$

Then

$$\phi_{ms} = 3.20 - (3.25 + 0.56 + 0.365) = -0.975 \text{ V}$$

Now

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} \Rightarrow Q'_{ss} = (\phi_{ms} - V_{FB}) C_{ox}$$

We have

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}}$$

or

$$C_{ox} = 7.67 \times 10^{-8} \text{ F / cm}^2$$

so now

$$Q'_{ss} = [-0.975 - (-1)] \cdot (7.67 \times 10^{-8})$$

or

$$Q'_{ss} = 1.92 \times 10^{-9} \text{ C / cm}^2$$

or

$$\frac{Q'_{ss}}{e} = 1.2 \times 10^{10} \text{ cm}^{-2}$$

11.8

$$V_{TN} = (|Q'_{SD}(\max)| - Q'_{ss}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp}$$

We find

$$\phi_{fp} = (0.0259) \ln \left(\frac{2 \times 10^{15}}{1.5 \times 10^{10}} \right) = 0.306 \text{ V}$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.306)}{(1.6 \times 10^{-19})(2 \times 10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.629 \text{ } \mu\text{m}$$

Also

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(2 \times 10^{15})(0.629 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 2.01 \times 10^{-8} \text{ C / cm}^2$$

and

$$Q'_{ss} = (2 \times 10^{11})(1.6 \times 10^{-19})$$

or

$$Q'_{ss} = 3.2 \times 10^{-8} \text{ C / cm}^2$$

Then

$$V_{TN} = \frac{(2.01 \times 10^{-8} - 3.2 \times 10^{-8})(450 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} + \phi_{ms} + 2(0.306)$$

or

$$V_{TN} = 0.457 + \phi_{ms}$$

(a)

For an aluminum gate:

$$\phi_{ms} = 3.20 - (3.25 + 0.56 + 0.306) = -0.916 \text{ V}$$

Then

$$V_{TN} = 0.457 - 0.916$$

or

$$V_{TN} = -0.459 \text{ V}$$

(b)

For an n^+ polygate:

$$\phi_{ms} = -(0.56 + 0.306) = -0.866 \text{ V}$$

so that

$$V_{TN} = -0.409 \text{ V}$$

(c)

For a p^+ polygate:

$$\phi_{ms} = (0.56 - 0.306) = +0.254 \text{ V}$$

so that

$$\underline{V_{TN} = +0.711 \text{ V}}$$

11.9

We find

$$\phi_{fn} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

So

$$x_{dT} = \left[\frac{4 \in \phi_{fn}}{eN_d} \right]^{1/2} \\ = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.863 \text{ } \mu\text{m}$$

Also

$$|Q'_{SD}(\text{max})| = eN_d x_{dT} \\ = (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4})$$

or

$$|Q'_{SD}(\text{max})| = 1.38 \times 10^{-8} \text{ C / cm}^2$$

We have

$$Q'_{SS} = 3.2 \times 10^{-8} \text{ C / cm}^2$$

Now

$$V_{TP} = -(|Q'_{SD}(\text{max})| + Q'_{SS}) \left(\frac{t_{ox}}{\in_{ox}} \right) + \phi_{ms} - 2\phi_{fn}$$

so

$$V_{TP} = - \frac{(1.38 \times 10^{-8} + 3.2 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} (450 \times 10^{-8}) \\ + \phi_{ms} - 2(0.288)$$

or

$$V_{TP} = -1.17 + \phi_{ms}$$

(a)

Aluminum gate:

$$\phi_{ms} = 3.20 - (3.25 + 0.56 - 0.288) = -0.322 \text{ V}$$

so

$$\underline{V_{TP} = -1.49 \text{ V}}$$

(b)

n^+ polygate:

$$\phi_{ms} = -(0.56 - 0.288) = -0.272 \text{ V}$$

so

$$\underline{V_{TP} = -1.44 \text{ V}}$$

(c)

p^+ polygate:

$$\phi_{ms} = +(0.56 + 0.288) = +0.848 \text{ V}$$

so

$$\underline{V_{TP} = -0.322 \text{ V}}$$

11.10

$$\phi_{fp} = (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}} \right) = 0.329 \text{ V}$$

Surface potential:

$$\phi_s = 2\phi_{fp} = 2(0.329) = 0.658 \text{ V}$$

We have

$$V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}} = -0.90 \text{ V}$$

Now

$$V_T = \frac{|Q'_{SD}(\text{max})|}{C_{ox}} + \phi_s + V_{FB}$$

We obtain

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a} \right]^{1/2} \\ = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.329)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.413 \text{ } \mu\text{m}$$

Then

$$|Q'_{SD}(\text{max})| = (1.6 \times 10^{-19})(5 \times 10^{15})(0.413 \times 10^{-4})$$

or

$$|Q'_{SD}(\text{max})| = 3.30 \times 10^{-8} \text{ C / cm}^2$$

We also find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

Then

$$V_T = \frac{3.30 \times 10^{-8}}{8.63 \times 10^{-8}} + 0.658 - 0.90$$

or

$$\underline{V_T = +0.140 \text{ V}}$$

11.11

$$V_{TN} = (|Q'_{SD}(\max)| - Q'_{SS}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp}$$

We have

$$\begin{aligned}\phi_{ms} &= \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right) \\ &= 3.20 - (3.25 + 0.56 + \phi_{fp})\end{aligned}$$

or

$$\phi_{ms} = -0.61 - \phi_{fp}$$

Also

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a} \right]^{1/2}$$

and

$$|Q'_{SD}(\max)| = eN_a x_{dT} = [4eN_a \in \phi_{fp}]^{1/2}$$

Then, the threshold voltage can be written as +0.80

$$\begin{aligned} &= \left\{ \left[4(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})N_a \phi_{fp} \right]^{1/2} \right. \\ &\quad \left. - 1.6 \times 10^{-8} \right\} \cdot \left[\frac{750 \times 10^{-8}}{(3.9)(8.85 \times 10^{-14})} \right] \\ &\quad - 0.61 - \phi_{fp} + 2\phi_{fp} \end{aligned}$$

which becomes

$$1.758 = 1.77 \times 10^{-8} (N_a \phi_{fp})^{1/2} + \phi_{fp}$$

We also have

$$\phi_{fp} = (0.0259) \ln \left(\frac{N_a}{1.5 \times 10^{10}} \right)$$

By trial and error

$$N_a = 1.71 \times 10^{16} \text{ cm}^{-3}$$

11.12

We have

$$V_{TP} = -(|Q'_{SD}(\max)| + Q'_{SS}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} - 2\phi_{fn}$$

We find

$$\begin{aligned}\phi_{ms} &= \phi'_m - \left(\chi' + \frac{E_g}{2e} - \phi_{fn} \right) \\ &= 3.20 - (3.25 + 0.56 - \phi_{fn})\end{aligned}$$

or

$$\phi_{ms} = -0.61 + \phi_{fn}$$

where

$$\phi_{fn} = (0.0259) \ln \left(\frac{N_d}{1.5 \times 10^{10}} \right)$$

Also

$$x_{dT} = \left[\frac{4 \in \phi_{fn}}{eN_d} \right]^{1/2}$$

Also

$$|Q'_{SD}(\max)| = eN_d x_{dT} = [4 \in eN_d \phi_{fn}]^{1/2}$$

Now the threshold voltage can be written as -1.50

$$\begin{aligned} &= - \left\{ \left[4(11.7)(8.85 \times 10^{-14})(1.6 \times 10^{-19})N_d \phi_{fn} \right]^{1/2} \right. \\ &\quad \left. + 1.6 \times 10^{-8} \right\} \cdot \left[\frac{750 \times 10^{-8}}{(3.9)(8.85 \times 10^{-14})} \right] \\ &\quad - 0.61 + \phi_{fn} - 2\phi_{fn} \end{aligned}$$

which becomes

$$0.542 = 1.77 \times 10^{-8} (N_d \phi_{fn})^{1/2} + \phi_{fn}$$

By trial and error

$$N_d = 7.7 \times 10^{14} \text{ cm}^{-3}$$

11.13

$$(a) \quad V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}}$$

Now

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

and

$$\begin{aligned}\phi_{ms} &= \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right) \\ &= 3.20 - (3.25 + 0.56 + 0.288) = -0.898 \text{ V}\end{aligned}$$

We find

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}}$$

or

$$C_{ox} = 7.67 \times 10^{-8} \text{ F / cm}^2$$

Then

$$V_{FB} = -0.898 - \frac{(3 \times 10^{11})(1.6 \times 10^{-19})}{7.67 \times 10^{-8}}$$

or

$$V_{FB} = -1.52 \text{ V}$$

(b)

We have

$$V_T = \frac{|Q'_{SD}(\max)|}{C_{ox}} + 2\phi_{fp} + V_{FB}$$

We find

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.863 \mu m$$

We obtain

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38 \times 10^{-8} \text{ C/cm}^2$$

Then

$$V_T = \frac{1.38 \times 10^{-8}}{7.67 \times 10^{-8}} + 2(0.288) - 1.52$$

or

$$V_T = -0.764 \text{ V}$$

11.14

(a) We have n-type material under the gate, so

$$x_{dT} = t_c = \left[\frac{4 \epsilon \phi_{fn}}{eN_d} \right]^{1/2}$$

where

$$\phi_{fn} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

Then

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2}$$

or

$$x_{dT} = t_c = 0.863 \mu m$$

(b)

$$V_T = -(|Q'_{SD}(\max)| + Q'_{SS}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} - 2\phi_{fp}$$

For an n^+ polygate:

$$\phi_{ms} = - \left(\frac{E_g}{2e} - \phi_{fn} \right) = -(0.56 - 0.288)$$

or

$$\phi_{ms} = -0.272 \text{ V}$$

Now

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38 \times 10^{-8} \text{ C/cm}^2$$

We find

$$Q'_{SS} = (1.6 \times 10^{-19})(10^{10}) = 1.6 \times 10^{-9} \text{ C/cm}^2$$

We then find

$$V_T = \frac{-(1.38 \times 10^{-8} + 1.6 \times 10^{-9})}{(3.9)(8.85 \times 10^{-14})} (500 \times 10^{-8}) - 0.272 - 2(0.288)$$

or

$$V_T = -1.07 \text{ V}$$

11.15

$$(b) \quad \phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right)$$

where

$$\phi'_m - \chi' = -0.20 \text{ V}$$

and

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.347 \text{ V}$$

Then

$$\phi_{ms} = -0.20 - (0.56 + 0.347)$$

or

$$\phi_{ms} = -1.11 \text{ V}$$

(c)

For $Q'_{SS} = 0$,

$$V_{TN} = |Q'_{SD}(\max)| \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp}$$

We find

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.30 \mu m$$

Now

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(10^{16})(0.30 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 4.8 \times 10^{-8} \text{ C/cm}^2$$

Then

$$V_{TN} = \frac{(4.8 \times 10^{-8})(300 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} - 1.11 + 2(0.347)$$

or

$$V_{TN} = +0.0012 \text{ V}$$

11.16

Computer plot

11.17

Computer plot

11.18

Computer plot

11.19

Computer plot

11.20

(a) For $f = 1 \text{ Hz}$

We have

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

and

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s} \right) \sqrt{\left(\frac{kT}{e} \right) \left(\frac{\epsilon_s}{eN_a} \right)}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8} + \left(\frac{3.9}{11.7} \right) \sqrt{\frac{(0.0259)(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(10^{16})}}}$$

or

$$C'_{FB} = 6.43 \times 10^{-8} \text{ F / cm}^2$$

Also

$$C'_{min} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s} \right) \cdot x_{dT}}$$

where

$$x_{dT} = \left[\frac{4 \epsilon_s \phi_{fp}}{eN_a} \right]^{1/2}$$

Now

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.347 \text{ V}$$

Then

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.30 \text{ } \mu\text{m}$$

so that

$$C'_{min} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8} + \left(\frac{3.9}{11.7} \right) (0.30 \times 10^{-4})}$$

or

$$C'_{min} = 2.47 \times 10^{-8} \text{ F / cm}^2$$

Also

$$C'(inv) = C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

(b)

For $f = 1 \text{ MHz}$, we have

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

$$C'_{FB} = 6.43 \times 10^{-8} \text{ F / cm}^2$$

$$C'_{min} = 2.47 \times 10^{-8} \text{ F / cm}^2$$

and

$$C'(inv) = C'_{min} = 2.47 \times 10^{-8} \text{ F / cm}^2$$

(c)

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

For the ideal MOS capacitor, $Q'_{ss} = 0$, then

$$V_{FB} = \phi_{ms} = 3.2 - (3.25 + 0.56 + 0.347)$$

or

$$V_{FB} = -0.957 \text{ V}$$

Also

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(10^{16})(0.30 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 4.8 \times 10^{-8} \text{ C / cm}^2$$

Now

$$V_{TN} = |Q'_{SD}(\max)| \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp} = \frac{(4.8 \times 10^{-8})(400 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} - 0.957 + 2(0.347)$$

or

$$V_{TN} = +0.293 \text{ V}$$

11.21

(a) At $f = 1 \text{ Hz}$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

Also

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{t_{ox}} \right) \sqrt{\left(\frac{kT}{e} \right) \left(\frac{\epsilon_s}{eN_a} \right)}}$$

$$= \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8} + \left(\frac{3.9}{11.7} \right) \sqrt{\frac{(0.0259)(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(5 \times 10^{14})}}}$$

or

$$C'_{FB} = 3.42 \times 10^{-8} \text{ F / cm}^2$$

Now

$$C'_{min} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s} \right) \cdot x_{dT}}$$

We find

$$\phi_{fn} = (0.0259) \ln \left(\frac{5 \times 10^{14}}{1.5 \times 10^{10}} \right) = 0.270 \text{ V}$$

and

$$x_{dT} = \left[\frac{4 \epsilon \phi_{fn}}{eN_d} \right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.270)}{(1.6 \times 10^{-19})(5 \times 10^{14})} \right]^{1/2}$$

or

$$x_{dT} = 1.18 \text{ } \mu\text{m}$$

Then

$$C'_{min} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8} + \left(\frac{3.9}{11.7} \right) (1.18 \times 10^{-4})}$$

or

$$C'_{min} = 0.797 \times 10^{-8} \text{ F / cm}^2$$

Also

$$C'(inv) = C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

(b) At $f = 1 \text{ MHz}$

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

$$C'_{FB} = 3.42 \times 10^{-8} \text{ F / cm}^2$$

$$C'_{min} = 0.797 \times 10^{-8} \text{ F / cm}^2$$

and

$$C'(inv) = C'_{min} = 0.797 \times 10^{-8} \text{ F / cm}^2$$

(c)

For the ideal oxide,

$$V_{FB} = \phi_{ms} = 3.2 - (3.25 + 0.56 - 0.27)$$

or

$$V_{FB} = -0.34 \text{ V}$$

We find

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(5 \times 10^{14})(1.18 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 0.944 \times 10^{-8} \text{ C / cm}^2$$

Then

$$V_{TP} = -|Q'_{SD}(\max)| \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} - 2\phi_{fn}$$

$$= \frac{-(0.944 \times 10^{-8})(400 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} - 0.34 - 2(0.27)$$

or

$$V_{TP} = -0.989 \text{ V}$$

11.22

The amount of fixed oxide charge at x is

$$\rho(x) \Delta x \quad (\text{C / cm}^2)$$

By lever action, the effect of this oxide charge on the flatband voltage is

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \left(\frac{x}{t_{ox}} \right) \rho(x) \Delta x$$

If we add the effect at each point, we must integrate so that

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{x \rho(x)}{t_{ox}} dx$$

11.23

(a) We have $\rho(x) = \frac{Q'_{SS}}{\Delta t}$

Then

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{x \rho(x)}{t_{ox}} dx$$

$$\approx -\frac{1}{C_{ox}} \int_{(t_{ox}-\Delta t)}^{t_{ox}} \left(\frac{t_{ox}}{t_{ox}} \right) \left(\frac{Q'_{SS}}{\Delta t} \right) dx$$

$$= -\frac{1}{C_{ox}} \left(\frac{Q'_{SS}}{\Delta t} \right) [t_{ox} - (t_{ox} - \Delta t)] = -\frac{Q'_{SS}}{C_{ox}}$$

or

$$\Delta V_{FB} = -Q'_{SS} \left(\frac{t_{ox}}{\epsilon_{ox}} \right)$$

$$= \frac{-(1.6 \times 10^{-19})(5 \times 10^{11})(750 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})}$$

or

$$\Delta V_{FB} = -1.74 \text{ V}$$

(b)

$$\text{We have } \rho(x) = \frac{Q'_{ss}}{t_{ox}} = \frac{(1.6 \times 10^{-19})(5 \times 10^{11})}{750 \times 10^{-8}} \\ = 1.067 \times 10^{-2} = \rho_o$$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{x \rho(x)}{t_{ox}} dx = -\frac{\rho_o}{C_{ox}} \int_0^{t_{ox}} x dx$$

or

$$\Delta V_{FB} = -\frac{\rho_o t_{ox}^2}{2 \epsilon_{ox}} \\ = \frac{-(1.067 \times 10^{-2})(750 \times 10^{-8})^2}{2(3.9)(8.85 \times 10^{-14})}$$

or

$$\Delta V_{FB} = -0.869 \text{ V}$$

(c)

$$\rho(x) = \rho_o \left(\frac{x}{t_{ox}} \right)$$

We find

$$\frac{1}{2} t_{ox} \rho_o = Q'_{ss} \Rightarrow \rho_o = \frac{2(1.6 \times 10^{-19})(5 \times 10^{11})}{750 \times 10^{-8}}$$

$$\text{or } \rho_o = 2.13 \times 10^{-2}$$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{1}{t_{ox}} \cdot x \cdot \rho_o \left(\frac{x}{t_{ox}} \right) dx \\ = -\frac{1}{C_{ox}} \cdot \frac{\rho_o}{(t_{ox})^2} \int_0^{t_{ox}} x^2 dx$$

which becomes

$$\Delta V_{FB} = -\frac{1}{\left(\frac{\epsilon_{ox}}{t_{ox}} \right)} \cdot \frac{\rho_o}{(t_{ox})^2} \cdot \frac{x^3}{3} \bigg|_0^{t_{ox}} = -\frac{\rho_o t_{ox}^2}{3 \epsilon_{ox}}$$

Then

$$\Delta V_{FB} = \frac{-(2.13 \times 10^{-2})(750 \times 10^{-8})^2}{3(3.9)(8.85 \times 10^{-14})}$$

$$\text{or } \Delta V_{FB} = -1.16 \text{ V}$$

11.24

Sketch

11.25

Sketch

11.26

$$(b) \quad V_{FB} = -V_{bi} = -V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ = -(0.0259) \ln \left[\frac{(10^{16})(10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{FB} = -0.695 \text{ V}$$

(c)

$$\text{Apply } V_G = -3 \text{ V}, \quad |V_{ox}| \approx 3 \text{ V}$$

For $V_G = +3 \text{ V}$,

$$\frac{dE}{dx} = -\frac{\rho}{\epsilon_s}$$

n-side: $\rho = eN_d$

$$\frac{dE}{dx} = -\frac{eN_d}{\epsilon_s} \Rightarrow E = -\frac{eN_d x}{\epsilon_s} + C_1$$

$E = 0$ at $x = -x_n$, then $C_1 = -\frac{eN_d x_n}{\epsilon_s}$, so

$$E = -\frac{eN_d}{\epsilon_s} (x + x_n) \quad \text{for } -x_n \leq x \leq 0$$

Note that at $x = 0$, $E = -\frac{eN_d x_n}{\epsilon_s}$

In the oxide, $\rho = 0$, so

$\frac{dE}{dx} = 0 \Rightarrow E = \text{constant}$. From the boundary conditions. In the oxide:

$$E = -\frac{eN_d x_n}{\epsilon_s}$$

In the p-region,

$$\frac{dE}{dx} = -\frac{\rho}{\epsilon_s} = +\frac{eN_a}{\epsilon_s} \Rightarrow E = \frac{eN_a x}{\epsilon_s} + C_2$$

$E = 0$ at $x = (t_{ox} + x_p)$, then

$$C_2 = -\frac{eN_a}{\epsilon_s}(t_{ox} + x_p), \text{ then}$$

$$E = -\frac{eN_a}{\epsilon_s}[(t_{ox} + x_p) - x]$$

$$\text{At } x = t_{ox}, E = -\frac{eN_a x_p}{\epsilon_s} = -\frac{eN_d x_n}{\epsilon_s}$$

$$\text{So that } N_a x_p = N_d x_n.$$

$$\text{Since } N_a = N_d \Rightarrow x_n = x_p$$

Now, the potential is

$$\phi = -\int E dx$$

For zero bias, we can write

$$V_n + V_{ox} + V_p = V_{bi}$$

where V_n , V_{ox} , V_p are the voltage drops across the n-region, the oxide, and the p-region, respectively. For the oxide:

$$V_{ox} = E \cdot t_{ox} = \frac{eN_d x_n t_{ox}}{\epsilon_s}$$

For the n-region:

$$V_n = \frac{eN_d}{\epsilon_s} \left(\frac{x^2}{2} + x_n \cdot x \right) + C'$$

Arbitrarily, set $V_n = 0$, at $x = -x_n$, then

$$C' = \frac{eN_d x_n^2}{2\epsilon_s} \text{ so then}$$

$$V_n(x) = \frac{eN_d}{2\epsilon_s} (x + x_n)^2$$

At $x = 0$, $V_n = \frac{eN_d x_n^2}{2\epsilon_s}$ which is the voltage drop

across the n-region. Because of symmetry,

$V_n = V_p$. Then for zero bias, we have

$$2V_n + V_{ox} = V_{bi}$$

which can be written as

$$\frac{eN_d x_n^2}{\epsilon_s} + \frac{eN_d x_n t_{ox}}{\epsilon_s} = V_{bi}$$

or

$$x_n^2 + x_n t_{ox} - \frac{V_{bi} \epsilon_s}{eN_d} = 0$$

Solving for x_n , we find

$$x_n = -\frac{t_{ox}}{2} + \sqrt{\left(\frac{t_{ox}}{2}\right)^2 + \frac{\epsilon_s V_{bi}}{eN_d}}$$

If we apply a voltage V_G , then replace V_{bi} by $V_{bi} + V_G$, so

$$x_n = x_p = -\frac{t_{ox}}{2} + \sqrt{\left(\frac{t_{ox}}{2}\right)^2 + \frac{\epsilon_s (V_{bi} + V_G)}{eN_d}}$$

We then find

$$x_n = x_p = -\frac{500 \times 10^{-8}}{2} + \sqrt{\left(\frac{500 \times 10^{-8}}{2}\right)^2 + \frac{(11.7)(8.85 \times 10^{-14})(3.695)}{(1.6 \times 10^{-19})(10^{16})}}$$

which yields

$$x_n = x_p = 4.646 \times 10^{-5} \text{ cm}$$

Now

$$V_{ox} = \frac{eN_d x_n t_{ox}}{\epsilon_s} = \frac{(1.6 \times 10^{-19})(10^{16})(4.646 \times 10^{-5})(500 \times 10^{-8})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$V_{ox} = 0.359 \text{ V}$$

We can also find

$$V_n = \frac{eN_d x_n^2}{2\epsilon_s} = \frac{(1.6 \times 10^{-19})(10^{16})(4.646 \times 10^{-5})^2}{2(11.7)(8.85 \times 10^{-14})}$$

or, the voltage drop across each of the semiconductor regions is

$$V_n = V_p = 1.67 \text{ V}$$

11.27

(a) n-type

(b) We have

$$C_{ox} = \frac{200 \times 10^{-12}}{2 \times 10^{-3}} = 1 \times 10^{-7} \text{ F / cm}^2$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\epsilon_{ox}}{C_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{10^{-7}}$$

or

$$t_{ox} = 3.45 \times 10^{-6} \text{ cm} = 345 \text{ \AA}$$

(c)

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

or

$$-0.80 = -0.50 - \frac{Q'_{ss}}{10^{-7}}$$

or

$$Q'_{ss} = 3 \times 10^{-8} \text{ C / cm}^2 = 1.875 \times 10^{11} \text{ cm}^{-2}$$

(d)

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s} \right) \sqrt{\left(\frac{kT}{e} \right) \left(\frac{\epsilon_s}{eN_a} \right)}}$$

$$= \frac{(3.9)(8.85 \times 10^{-14})}{3.45 \times 10^{-6} + \left(\frac{3.9}{11.7} \right) \sqrt{(0.0259) \left[\frac{(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(2 \times 10^{16})} \right]}}$$

or

$$C'_{FB} = 7.82 \times 10^{-8} \text{ F / cm}^2$$

and

$$C_{FB} = 156 \text{ pF}$$

11.28

- (a) Point 1: Inversion
2: Threshold
3: Depletion
4: Flat-band
5: Accumulation

11.29

We have

$$Q'_n = -C_{ox} [(V_{GS} - V_x) - (\phi_{ms} + 2\phi_{fp})]$$

$$-(Q'_{ss} + Q'_{SD}(\text{max}))$$

Now let $V_x = V_{DS}$, so

$$Q'_n = -C_{ox} \left\{ (V_{GS} - V_{DS}) + \left[\frac{Q'_{SD}(\text{max}) + Q'_{ss}}{C_{ox}} - (\phi_{ms} + 2\phi_{fp}) \right] \right\}$$

For a p-type substrate, $Q'_{SD}(\text{max})$ is really a negative value, so we can write

$$Q'_n = -C_{ox} \left\{ (V_{GS} - V_{DS}) - \left[\frac{|Q'_{SD}(\text{max})| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp} \right] \right\}$$

Using the definition of threshold voltage V_T , we have

$$Q'_n = -C_{ox} [(V_{GS} - V_{DS}) - V_T]$$

At saturation,

$$V_{DS} = V_{DS}(\text{sat}) = V_{GS} - V_T$$

which then makes Q'_n equal to zero at the drain terminal.

11.30

$$I_D(\text{sat}) = \frac{W\mu_n C_{ox}}{2L} (V_{GS} - V_T)^2$$

where

$$\frac{W\mu_n C_{ox}}{2L} = \frac{(30 \times 10^{-4})(450)(3.9)(8.85 \times 10^{-14})}{2(2 \times 10^{-4})(350 \times 10^{-8})}$$

$$= 0.333 \times 10^{-3} \text{ A / V}^2 = 0.333 \text{ mA / V}^2$$

We have $V_{DS}(\text{sat}) = V_{GS} - V_T$, then

(a)

V_{GS}	$V_{DS}(\text{sat})$	$I_D(\text{sat}) \text{ (mA)}$
1	0.2	0.0133
2	1.2	0.48
3	2.2	1.61
4	3.2	3.41
5	4.2	5.87

(b)

$$\sqrt{I_D(\text{sat})} = \sqrt{0.333(V_{GS} - V_T)} \text{ (mA)}^{1/2}$$

then

V_{GS}	$\sqrt{I_D(\text{sat})} \text{ (mA)}^{1/2}$
1	0.115
2	0.693
3	1.27
4	1.85
5	2.42

11.31

We have

$$\frac{W\mu_p C_{ox}}{2L} = \frac{(15 \times 10^{-4})(300)(3.9)(8.85 \times 10^{-14})}{2(1.5 \times 10^{-4})(350 \times 10^{-8})}$$

$$= 0.148 \text{ mA / V}^2$$

We can write

$$I_D(\text{sat}) = \frac{W\mu_p C_{ox}}{2L} (V_{SG} + V_T)^2$$

and

$$V_{SD}(\text{sat}) = V_{SG} + V_T$$

Then

V_{SG}	$V_{SD}(sat)$	$I_D(sat) (mA)$
1	0.2	0.00592
2	1.2	0.213
3	2.2	0.716
4	3.2	1.52
5	4.2	2.61

11.32

$$(a) \quad I_D(sat) = \frac{W\mu_n C_{ox}}{2L} (V_{GS} - V_T)^2$$

$$\text{From Problem 11.30, } \frac{W\mu_n C_{ox}}{2L} = 0.333 \text{ mA/V}^2$$

We have

$$V_{DS}(sat) = V_{GS} - V_T$$

Then

V_{GS}	$V_{DS}(sat)$	$I_D(sat) (mA)$
-2	0	0
-1	1	0.333
0	2	1.33
+1	3	3.0
+2	4	5.33
+3	5	8.33

(b)

We have

$$\sqrt{I_D(sat)} = \sqrt{0.333} (V_{GS} - V_T) (mA)^{1/2}$$

Now

V_{GS}	$\sqrt{I_D(sat)} (mA)^{1/2}$
-2	0
-1	0.577
0	1.15
+1	1.73
+2	2.31
+3	2.89

11.33

Sketch

11.34

Plots

11.35

We have

$$V_{DS}(sat) = V_{GS} - V_T = V_{DS} - V_T$$

so that

$$V_{DS} = V_{DS}(sat) + V_T$$

Since $V_{DS} > V_{DS}(sat)$, the transistor is always biased in the saturation region. Then

$$I_D = K_n (V_{GS} - V_T)^2$$

where, from Problem 11.30,

$$K_n = 0.333 \text{ mA/V}^2.$$

Then

$V_{DS} = V_{GS}$	$I_D (mA)$
0	0
1	0.0133
2	0.48
3	1.61
4	3.41
5	5.87

11.36

$$I_D = 0.148 [2(V_{SG} + V_T)V_{SD} - V_{SD}^2] (mA)$$

where $V_T = -0.8 \text{ V}$.

Now

$$g_d = \left. \frac{\partial I_D}{\partial V_{SD}} \right|_{V_{DS}=0} = 0.148 [2(V_{SG} + V_T)] \text{ mS}$$

Then

V_{SG}	$g_d (mS)$
1	0.0592
2	0.355
3	0.651
4	0.947
5	1.24

11.37

We find that $V_T \approx 0.2 \text{ V}$

Now

$$\sqrt{I_D(sat)} = \sqrt{\frac{W\mu_n C_{ox}}{2L}} (V_{GS} - V_T)$$

where

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{425 \times 10^{-8}}$$

or

$$C_{ox} = 8.12 \times 10^{-8} \text{ F/cm}^2$$

We are given $W/L = 10$.

From the graph, for $V_{GS} = 3 \text{ V}$,

$$\sqrt{I_D(sat)} \approx 0.033, \text{ then}$$

$$0.033 = \sqrt{\frac{W\mu_n C_{ox}}{2L}} (3 - 0.2)$$

or

$$\frac{W\mu_n C_{ox}}{2L} = 0.139 \times 10^{-3} = \frac{1}{2} (10) \mu_n (8.12 \times 10^{-8})$$

which yields

$$\mu_n = 342 \text{ cm}^2 / V - s$$

11.38

(a)

$$V_{DS}(sat) = V_{GS} - V_T$$

or

$$4 = V_{GS} - 0.8 \Rightarrow \underline{V_{GS} = 4.8 \text{ V}}$$

(b)

$$I_D(sat) = K_n (V_{GS} - V_T)^2 = K_n V_{DS}^2(sat)$$

so

$$2 \times 10^{-4} = K_n (4)^2$$

or

$$\underline{K_n = 12.5 \mu A / V^2}$$

(c)

$$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2 \text{ V}$$

so $V_{DS} > V_{DS}(sat)$

$$I_D(sat) = (1.25 \times 10^{-5}) (2 - 0.8)^2$$

or

$$\underline{I_D(sat) = 18 \mu A}$$

(d)

$$V_{DS} < V_{DS}(sat)$$

$$\begin{aligned} I_D &= K_n [2(V_{GS} - V_T)V_{DS} - V_{DS}^2] \\ &= (1.25 \times 10^{-5}) [2(3 - 0.8)(1) - (1)^2] \end{aligned}$$

or

$$\underline{I_D = 42.5 \mu A}$$

11.39

(a) We have

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L} (V_{GS} - V_T)^2$$

Now

$$\begin{aligned} 6 \times 10^{-3} &= \left(\frac{W}{L} \right) \left(\frac{525}{2} \right) \frac{(3.9)(8.85 \times 10^{-14})}{(400 \times 10^{-8})} (5 - 0.75)^2 \end{aligned}$$

which yields

$$\underline{\frac{W}{L} = 14.7}$$

(b)

$$I_D(sat) = \frac{W\mu_p C_{ox}}{2L} (V_{SG} + V_T)^2$$

We have

$$\begin{aligned} 6 \times 10^{-3} &= \left(\frac{W}{L} \right) \left(\frac{300}{2} \right) \frac{(3.9)(8.85 \times 10^{-14})}{(400 \times 10^{-8})} (5 - 0.75)^2 \end{aligned}$$

which yields

$$\underline{\frac{W}{L} = 25.7}$$

11.40

From Problem 11.30, we have

(a) In nonsaturation

$$I_D = 0.333 [2(V_{GS} - V_T)V_{DS} - V_{DS}^2]$$

Now

$$g_{mL} = \frac{\partial I_D}{\partial V_{GS}} = (0.333)(2V_{DS})$$

At $V_{DS} = 0.5 \text{ V}$, we find

$$\underline{g_{mL} = 0.333 \text{ mS}}$$

(b) In saturation

$$I_D = 0.333 (V_{GS} - V_T)^2$$

so that

$$g_{mS} = \frac{\partial I_D}{\partial V_{GS}} = 2(0.333)(V_{GS} - V_T)$$

For $V_T = 0.80 \text{ V}$ and at $V_{GS} = 4 \text{ V}$,

We obtain

$$\underline{g_{mS} = 2.13 \text{ mS}}$$

11.41

From Problem 11.31, we have

(a) In nonsaturation,

$$I_D = 0.148 [2(V_{SG} + V_T)V_{SD} - V_{SD}^2] \text{ (mA)}$$

Then

$$g_{mL} = \frac{\partial I_D}{\partial V_{SG}} = (0.148)(2V_{SD})$$

For $V_{SD} = 0.5 \text{ V}$, we obtain

$$\underline{g_{mL} = 0.148 \text{ mS}}$$

(b) In saturation

$$I_D = 0.148 (V_{SG} + V_T)^2$$

so that

$$g_{mS} = \frac{\partial I_D}{\partial V_{SG}} = 2(0.148)(V_{SG} + V_T)$$

For $V_T = -0.8 \text{ V}$ and at $V_{SG} = 4 \text{ V}$,
We obtain

$$g_{mS} = 0.947 \text{ mS}$$

11.42

We can write, for $V_{SB} = 0$,

$$V_{TO} = V_{FB} + \frac{|Q'_{SD}(\text{max})|}{C_{ox}} + 2\phi_{fp}$$

We find

$$\phi_{fp} = (0.0259) \ln \left(\frac{5 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.389 \text{ V}$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.389)}{(1.6 \times 10^{-19})(5 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.142 \text{ } \mu\text{m}$$

Then

$$|Q'_{SD}(\text{max})| = (1.6 \times 10^{-19})(5 \times 10^{16})(0.142 \times 10^{-4})$$

or

$$|Q'_{SD}(\text{max})| = 1.14 \times 10^{-7} \text{ C / cm}^2$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

Now

$$V_{TO} = -0.5 + \frac{1.14 \times 10^{-7}}{8.63 \times 10^{-8}} + 2(0.389)$$

or

$$V_{TO} = +1.60 \text{ V}$$

Then

$$\begin{aligned} I_D(\text{sat}) &= \frac{W\mu_n C_{ox}}{2L} (V_{GS} - V_T)^2 \\ &= \left(\frac{10}{2} \right) \left(\frac{450}{2} \right) (8.63 \times 10^{-8}) (V_{GS} - V_T)^2 \end{aligned}$$

or

$$I_D(\text{sat}) = 0.097 (V_{GS} - V_T)^2 \text{ (mA)}$$

For $I_D(\text{sat}) = 1 \text{ mA}$, $V_{GS} - V_T = 3.21 \text{ V}$

Now with substrate voltage applied,

$$\begin{aligned} \Delta V_T &= \frac{\sqrt{2e\epsilon_s N_a}}{C_{ox}} \left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right] \\ &= \frac{\left[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(5 \times 10^{16}) \right]^{1/2}}{8.63 \times 10^{-8}} \\ &\quad \times \left[\sqrt{2(0.389) + V_{SB}} - \sqrt{2(0.389)} \right] \end{aligned}$$

or

$$\Delta V_T = 1.49 \left[\sqrt{0.778 + V_{SB}} - 0.882 \right]$$

We find that

V_{SB}	ΔV_T	V_T
0	0	1.60
1	0.673	2.27
2	1.17	2.77
4	1.94	3.54

11.43

For a p-channel MOSFET,

$$\Delta V_T = - \frac{\sqrt{2e\epsilon_s N_d}}{C_{ox}} \left[\sqrt{2\phi_{fn} + V_{BS}} - \sqrt{2\phi_{fn}} \right]$$

We find

$$\phi_{fn} = (0.0259) \ln \left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}} \right) = 0.329 \text{ V}$$

and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{600 \times 10^{-8}}$$

or

$$C_{ox} = 5.75 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\Delta V_T = -1.5 \text{ V}$$

$$\begin{aligned} &= \frac{- \left[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(5 \times 10^{15}) \right]^{1/2}}{5.75 \times 10^{-8}} \\ &\quad \times \left[\sqrt{0.658 + V_{BS}} - 0.811 \right] \end{aligned}$$

or

$$1.5 = 0.708 \left[\sqrt{0.658 + V_{BS}} - 0.811 \right]$$

which yields

$$V_{BS} = 7.92 \text{ V}$$

11.44

(a)

n^+ poly-to-p type $\Rightarrow \phi_{ms} \approx -1.0 \text{ V}$

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

also

$$x_{dT} = \left[\frac{4 \epsilon \phi_{fp}}{e N_a} \right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.863 \text{ } \mu\text{m}$$

$$|Q'_{SD}(\text{max})| = (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4})$$

or

$$|Q'_{SD}(\text{max})| = 1.38 \times 10^{-8} \text{ C / cm}^2$$

also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

Now

$$Q'_{SS} = (1.6 \times 10^{-19})(5 \times 10^{10}) = 8 \times 10^{-9} \text{ C / cm}^2$$

Then

$$V_T = \frac{(|Q'_{SD}(\text{max})| - Q'_{SS})}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$

$$= \left(\frac{1.38 \times 10^{-8} - 8 \times 10^{-9}}{8.63 \times 10^{-8}} \right) - 1.0 + 2(0.288)$$

or

$$\underline{V_T = -0.357 \text{ V}}$$

(b)

For NMOS, apply V_{SB} and V_T shifts in a positive direction, so for $V_T = 0$, we want

$$\Delta V_T = +0.357 \text{ V}.$$

So

$$\Delta V_T = \frac{\sqrt{2e \epsilon N_a}}{C_{ox}} \left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$$

$$+0.357 = \frac{\sqrt{2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})}}{8.63 \times 10^{-8}}$$

$$\times \left[\sqrt{2(0.288) + V_{SB}} - \sqrt{2(0.288)} \right]$$

or

$$0.357 = 0.211 \left[\sqrt{0.576 + V_{SB}} - 0.759 \right]$$

which yields

$$\underline{V_{SB} = 5.43 \text{ V}}$$

11.45

Computer plot

11.46

(a)

$$g_{ms} = \frac{W \mu_n C_{ox}}{L} (V_{GS} - V_T)$$

$$= \frac{(10)(400)(3.9)(8.85 \times 10^{-14})}{475 \times 10^{-8}} (5 - 0.65)$$

or

$$g_{ms} = 1.26 \text{ mS}$$

Now

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = 0.8 = \frac{1}{1 + g_m r_s}$$

which yields

$$r_s = \frac{1}{g_m} \left(\frac{1}{0.8} - 1 \right) = \frac{1}{1.26} \left(\frac{1}{0.8} - 1 \right)$$

or

$$\underline{r_s = 0.198 \text{ k}\Omega}$$

(b)

$$\text{For } V_{GS} = 3 \text{ V} \Rightarrow g_{ms} = 0.683 \text{ mS}$$

Then

$$g'_m = \frac{0.683}{1 + (0.683)(0.198)} = 0.602 \text{ mS}$$

So

$$\frac{g'_m}{g_m} = \frac{0.602}{0.683} = 0.88$$

which is a 12% reduction.

11.47

(a) The ideal cutoff frequency for no overlap capacitance,

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{\mu_n (V_{GS} - V_T)}{2\pi L^2}$$

$$= \frac{(400)(4 - 0.75)}{2\pi (2 \times 10^{-4})^2}$$

or

$$\underline{f_T = 5.17 \text{ GHz}}$$

(b)

Now

$$f_T = \frac{g_m}{2\pi(C_{gsT} + C_M)}$$

where

$$C_M = C_{gdT}(1 + g_m R_L)$$

We find that

$$\begin{aligned} C_{gdT} &= C_{ox}(0.75 \times 10^{-4})(20 \times 10^{-4}) \\ &= \frac{(3.9)(8.85 \times 10^{-14})(0.75 \times 10^{-4})(20 \times 10^{-4})}{500 \times 10^{-8}} \end{aligned}$$

or

$$C_{gdT} = 1.04 \times 10^{-14} \text{ F}$$

Also

$$\begin{aligned} g_{ms} &= \frac{W\mu_n C_{ox}}{L}(V_{GS} - V_T) \\ &= \frac{(20 \times 10^{-4})(400)(3.9)(8.85 \times 10^{-14})}{(2 \times 10^{-4})(500 \times 10^{-8})}(4 - 0.75) \end{aligned}$$

or

$$g_{ms} = 0.897 \times 10^{-3} \text{ S}$$

Then

$$C_M = (1.04 \times 10^{-14})[1 + (0.897 \times 10^{-3})(10 \times 10^3)]$$

or

$$C_M = 1.04 \times 10^{-13} \text{ F}$$

Now

$$\begin{aligned} C_{gsT} &= C_{ox}(L + 0.75 \times 10^{-4})(W) \\ &= \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}}(2 \times 10^{-4} + 0.75 \times 10^{-4})(20 \times 10^{-4}) \end{aligned}$$

or

$$C_{gsT} = 3.8 \times 10^{-14} \text{ F}$$

We now find

$$\begin{aligned} f_T &= \frac{g_m}{2\pi(C_{gsT} + C_M)} \\ &= \frac{0.897 \times 10^{-3}}{2\pi(3.8 \times 10^{-14} + 1.04 \times 10^{-13})} \end{aligned}$$

or

$$f_T = 1.0 \text{ MHz}$$

11.48

(a) For the ideal case

$$f_T = \frac{v_{ds}}{2\pi L} = \frac{4 \times 10^6}{2\pi(2 \times 10^{-4})}$$

or

$$f_T = 3.18 \text{ GHz}$$

(b)

With overlap capacitance (using the values from Problem 11.47),

$$f_T = \frac{g_m}{2\pi(C_{gsT} + C_M)}$$

We find

$$\begin{aligned} g_m &= WC_{ox} v_{ds} \\ &= \frac{(20 \times 10^{-4})(3.9)(8.85 \times 10^{-14})(4 \times 10^6)}{500 \times 10^{-8}} \end{aligned}$$

or

$$g_m = 0.552 \times 10^{-3} \text{ S}$$

We have

$$\begin{aligned} C_M &= C_{gdT}(1 + g_m R_L) \\ &= (1.04 \times 10^{-14})[1 + (0.552 \times 10^{-3})(10 \times 10^3)] \end{aligned}$$

or

$$C_M = 6.78 \times 10^{-14} \text{ F}$$

Then

$$f_T = \frac{0.552 \times 10^{-3}}{2\pi(3.8 \times 10^{-14} + 6.78 \times 10^{-14})}$$

or

$$f_T = 0.83 \text{ GHz}$$

Chapter 12

Problem Solutions

12.1

(a)

$$I_D = 10^{-15} \exp \left[\frac{V_{GS}}{(2.1)V_t} \right]$$

For $V_{GS} = 0.5 V$,

$$I_D = 10^{-15} \exp \left[\frac{0.5}{(2.1)(0.0259)} \right] \Rightarrow$$

$$I_D = 9.83 \times 10^{-12} A$$

For $V_{GS} = 0.7 V$,

$$I_D = 3.88 \times 10^{-10} A$$

For $V_{GS} = 0.9 V$,

$$I_D = 1.54 \times 10^{-8} A$$

Then the total current is:

$$I_{Total} = I_D (10^6)$$

For $V_{GS} = 0.5 V$, $I_{Total} = 9.83 \mu A$

For $V_{GS} = 0.7 V$, $I_{Total} = 0.388 mA$

For $V_{GS} = 0.9 V$, $I_{Total} = 15.4 mA$

(b)

Power: $P = I_{Total} \cdot V_{DD}$

Then

For $V_{GS} = 0.5 V$, $P = 49.2 \mu W$

For $V_{GS} = 0.7 V$, $P = 1.94 mW$

For $V_{GS} = 0.9 V$, $P = 77 mW$

12.2

We have

$$\Delta L = \sqrt{\frac{2 \epsilon}{e N_a}} \cdot \left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)} \right]$$

where

$$\phi_{fp} = V_t \ln \left(\frac{N_a}{n_i} \right) = (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right)$$

or

$$\phi_{fp} = 0.347 V$$

We find

$$\sqrt{\frac{2 \epsilon}{e N_a}} = \left[\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

$$= 0.360 \mu m / V^{1/2}$$

We have

$$V_{DS}(sat) = V_{GS} - V_T$$

(a)

For $V_{GS} = 5 V \Rightarrow V_{DS}(sat) = 4.25 V$

Then

$$\Delta L = 0.360 \left[\sqrt{0.347 + 5} - \sqrt{0.347 + 4.25} \right]$$

or

$$\Delta L = 0.0606 \mu m$$

If ΔL is 10% of L , then $L = 0.606 \mu m$

(b)

For $V_{DS} = 5 V$, $V_{GS} = 2 V \Rightarrow V_{DS}(sat) = 1.25 V$

Then

$$\Delta L = 0.360 \left[\sqrt{0.347 + 5} - \sqrt{0.347 + 1.25} \right]$$

or

$$\Delta L = 0.377 \mu m$$

Now if ΔL is 10% of L , then $L = 3.77 \mu m$

12.3

$$\Delta L = \sqrt{\frac{2 \epsilon}{e N_a}} \cdot \left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)} \right]$$

where

$$\phi_{fp} = V_t \ln \left(\frac{N_a}{n_i} \right) = (0.0259) \ln \left(\frac{4 \times 10^{16}}{1.5 \times 10^{10}} \right)$$

or

$$\phi_{fp} = 0.383 V$$

and

$$x_{dT} = \left[\frac{4 \epsilon \phi_{fp}}{e N_a} \right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.383)}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.157 \mu m$$

Then

$$|Q'_{SD}(\max)| = e N_a x_{dT}$$

$$= (1.6 \times 10^{-19})(4 \times 10^{16})(0.157 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 10^{-7} \text{ C/cm}^2$$

Now

$$V_T = (|Q'_{SD}(\max)| - Q'_{SS}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp}$$

so that

$$V_T = \frac{[10^{-7} - (1.6 \times 10^{-19})(3 \times 10^{10})](400 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} + 0 + 2(0.383)$$

or

$$V_T = 1.87 \text{ V}$$

Now

$$V_{DS}(\text{sat}) = V_{GS} - V_T = 5 - 1.87 = 3.13 \text{ V}$$

We find

$$\sqrt{\frac{2\epsilon}{eN_a}} = \left[\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2}$$

$$= 1.80 \times 10^{-5}$$

Now

$$\Delta L = 1.80 \times 10^{-5} \cdot \left[\sqrt{0.383 + 3.13 + \Delta V_{DS}} - \sqrt{0.383 + 3.13} \right]$$

or

$$\Delta L = 1.80 \times 10^{-5} \left[\sqrt{3.513 + \Delta V_{DS}} - \sqrt{3.513} \right]$$

We obtain

ΔV_{DS}	$\Delta L(\mu\text{m})$
0	0
1	0.0451
2	0.0853
3	0.122
4	0.156
5	0.188

12.4

Computer plot

12.5

Plot

12.6

Plot

12.7

(a) Assume $V_{DS}(\text{sat}) = 1 \text{ V}$, We have

$$E_{sat} = \frac{V_{DS}(\text{sat})}{L}$$

We find

$L(\mu\text{m})$	$E_{sat}(\text{V/cm})$
3	3.33×10^3
1	10^4
0.5	2×10^4
0.25	4×10^4
0.13	7.69×10^4

(b)

Assume $\mu_n = 500 \text{ cm}^2/\text{V}\cdot\text{s}$, we have

$$v = \mu_n E_{sat}$$

Then

$$\text{For } L = 3 \mu\text{m}, v = 1.67 \times 10^6 \text{ cm/s}$$

$$\text{For } L = 1 \mu\text{m}, v = 5 \times 10^6 \text{ cm/s}$$

$$\text{For } L \leq 0.5 \mu\text{m}, v \approx 10^7 \text{ cm/s}$$

12.8

We have $I'_D = L(L - \Delta L)^{-1} I_D$

We may write

$$g_o = \frac{\partial I'_D}{\partial V_{DS}} = (-1)L(L - \Delta L)^{-2} I_D \left(\frac{-\partial(\Delta L)}{\partial V_{DS}} \right)$$

$$= \frac{L}{(L - \Delta L)^2} \cdot I_D \cdot \frac{\partial(\Delta L)}{\partial V_{DS}}$$

We have

$$\Delta L = \sqrt{\frac{2\epsilon}{eN_a}} \cdot \left[\sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(\text{sat})} \right]$$

We find

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \sqrt{\frac{2\epsilon}{eN_a}} \cdot \frac{1}{2\sqrt{\phi_{fp} + V_{DS}}}$$

(a)

For $V_{GS} = 2 \text{ V}$, $\Delta V_{DS} = 1 \text{ V}$, and

$$V_{DS}(\text{sat}) = V_{GS} - V_T = 2 - 0.8 = 1.2 \text{ V}$$

Also

$$V_{DS} = V_{DS}(\text{sat}) + \Delta V_{DS} = 1.2 + 1 = 2.2 \text{ V}$$

and

$$\phi_{fp} = (0.0259) \ln \left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.376 \text{ V}$$

Now

$$\sqrt{\frac{2\epsilon}{eN_a}} = \left[\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

$$= 0.2077 \mu\text{m} / V^{1/2}$$

We find

$$\Delta L = 0.2077 \left[\sqrt{0.376 + 2.2} - \sqrt{0.376 + 1.2} \right]$$

$$= 0.0726 \mu\text{m}$$

Then

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \frac{0.2077}{2} \cdot \frac{1}{\sqrt{0.376 + 2.2}}$$

$$= 0.0647 \mu\text{m} / V$$

From the previous problem,

$$I_D = 0.48 \text{ mA}, L = 2 \mu\text{m}$$

Then

$$g_o = \frac{2}{(2 - 0.0726)^2} (0.48 \times 10^{-3}) (0.0647)$$

or

$$g_o = 1.67 \times 10^{-5} \text{ S}$$

so that

$$r_o = \frac{1}{g_o} = 59.8 \text{ k}\Omega$$

(b)

If $L = 1 \mu\text{m}$, then from the previous problem,

we would have $I_D = 0.96 \text{ mA}$, so that

$$g_o = \frac{1}{(1 - 0.0726)^2} (0.96 \times 10^{-3}) (0.0647)$$

or

$$g_o = 7.22 \times 10^{-5} \text{ S}$$

so that

$$r_o = \frac{1}{g_o} = 13.8 \text{ k}\Omega$$

12.9

(a)

$$I_D(\text{sat}) = \frac{W\mu_n C_{ox}}{2L} (V_{GS} - V_T)^2$$

$$= \left(\frac{10}{2} \right) (500) (6.9 \times 10^{-8}) (V_{GS} - 1)^2$$

or

$$I_D(\text{sat}) = 0.173 (V_{GS} - 1)^2 \text{ (mA)}$$

and

$$\sqrt{I_D(\text{sat})} = \sqrt{0.173} (V_{GS} - 1) \text{ (mA)}^{1/2}$$

(b)

$$\text{Let } \mu_{eff} = \mu_o \left(\frac{E_{eff}}{E_c} \right)^{-1/3}$$

Where $\mu_o = 1000 \text{ cm}^2 / V - s$ and

$$E_c = 2.5 \times 10^4 \text{ V} / \text{cm}.$$

$$\text{Let } E_{eff} = \frac{V_{GS}}{t_{ox}}$$

We find

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\epsilon_{ox}}{C_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{6.9 \times 10^{-8}}$$

or

$$t_{ox} = 500 \text{ \AA}$$

Then

$\frac{V_{GS}}{V_T}$	$\frac{E_{eff}}{E_c}$	$\frac{\mu_{eff}}{\mu_o}$	$\sqrt{I_D(\text{sat})}$
1	--	--	0
2	4E5	397	0.370
3	6E5	347	0.692
4	8E5	315	0.989
5	10E5	292	1.27

(c)

The slope of the variable mobility curve is not constant, but is continually decreasing.

12.10

Plot

12.11

$$V_T = V_{FB} + \frac{|Q'_{SD}(\text{max})|}{C_{ox}} + 2\phi_{fp}$$

We find

$$\phi_{fp} = V_T \ln \left(\frac{N_a}{n_i} \right) = (0.0259) \ln \left(\frac{5 \times 10^{16}}{1.5 \times 10^{10}} \right)$$

or

$$\phi_{fp} = 0.389 \text{ V}$$

and

$$x_{dT} = \left[\frac{4\epsilon\phi_{fp}}{eN_a} \right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.389)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.142 \mu\text{m}$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT} \\ = (1.6 \times 10^{-19})(5 \times 10^{16})(0.142 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.14 \times 10^{-7} \text{ C / cm}^2$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

Then

$$V_T = -1.12 + \frac{1.14 \times 10^{-7}}{8.63 \times 10^{-8}} + 2(0.389)$$

or

$$V_T = +0.90 \text{ V}$$

(a)

$$I_D = \frac{W \mu_n C_{ox}}{2L} [2(V_{GS} - V_T)V_{DS} - V_{DS}^2]$$

and

$$V_{DS}(\text{sat}) = V_{GS} - V_T$$

We have

$$I_D = \left(\frac{20}{2}\right) \left(\frac{1}{2}\right) (400)(8.63 \times 10^{-8}) \\ \times [2(V_{GS} - V_T)V_{DS} - V_{DS}^2]$$

or

$$I_D = 0.173 [2(V_{GS} - V_T)V_{DS} - V_{DS}^2] \text{ (mA)}$$

For $V_{DS} = V_{DS}(\text{sat}) = V_{GS} - V_T = 1 \text{ V}$,

$$I_D(\text{sat}) = 0.173 \text{ mA}$$

For $V_{DS} = V_{DS}(\text{sat}) = V_{GS} - V_T = 2 \text{ V}$,

$$I_D(\text{sat}) = 0.692 \text{ mA}$$

(b)

For $V_{DS} \leq 1.25 \text{ V}$, $\mu = \mu_n = 400 \text{ cm}^2 / \text{V} \cdot \text{s}$.

The curve for $V_{GS} - V_T = 1 \text{ V}$ is unchanged. For

$V_{GS} - V_T = 2 \text{ V}$ and $0 \leq V_{DS} \leq 1.25 \text{ V}$, the curve is unchanged. For $V_{DS} \geq 1.25 \text{ V}$, the current is constant at

$$I_D = 0.173 [2(2)(1.25) - (1.25)^2] = 0.595 \text{ mA}$$

When velocity saturation occurs,

$V_{DS}(\text{sat}) = 1.25 \text{ V}$ for the case of

$V_{GS} - V_T = 2 \text{ V}$.

12.12

Plot

12.13

(a) Non-saturation region

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) [2(V_{GS} - V_T)V_{DS} - V_{DS}^2]$$

We have

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \Rightarrow \frac{C_{ox}}{k}$$

and

$$W \Rightarrow kW, L \Rightarrow kL$$

also

$$V_{GS} \Rightarrow kV_{GS}, V_{DS} \Rightarrow kV_{DS}$$

So

$$I_D = \frac{1}{2} \mu_n \left(\frac{C_{ox}}{k}\right) \left(\frac{kW}{kL}\right) [2(kV_{GS} - V_T)kV_{DS} - (kV_{DS})^2]$$

Then

$$I_D \Rightarrow \approx kI_D$$

In the saturation region

$$I_D = \frac{1}{2} \mu_n \left(\frac{C_{ox}}{k}\right) \left(\frac{kW}{kL}\right) [kV_{GS} - V_T]^2$$

Then

$$I_D \Rightarrow \approx kI_D$$

(b)

$$P = I_D V_{DD} \Rightarrow (kI_D)(kV_{DD}) \Rightarrow k^2 P$$

12.14

$$I_D(\text{sat}) = WC_{ox}(V_{GS} - V_T)v_{sat}$$

$$\Rightarrow (kW) \left(\frac{C_{ox}}{k}\right) (kV_{GS} - V_T)v_{sat}$$

or

$$I_D(\text{sat}) \approx kI_D(\text{sat})$$

12.15

(a)

$$(i) I_D = K_n (V_{GS} - V_T)^2 = (0.1)(5 - 0.8)^2$$

or

$$I_D = 1.764 \text{ mA}$$

(ii)

$$I_D = \left(\frac{0.1}{0.6}\right) [(0.6)(5) - 0.8]^2$$

or

$$I_D = 0.807 \text{ mA}$$

(b)

$$(i) P = (1.764)(5) \Rightarrow P = 8.82 \text{ mW}$$

$$(ii) P = (0.807)(0.6)(5) \Rightarrow P = 2.42 \text{ mW}$$

(c)

$$\text{Current: Ratio} = \frac{0.807}{1.764} = 0.457$$

$$\text{Power: Ratio} = \frac{2.42}{8.82} = 0.274$$

12.16

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}} \left\{ \frac{r_j}{L} \left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1 \right] \right\}$$

Now

$$\phi_{fp} = V_i \ln \left(\frac{N_a}{n_i} \right) = (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right)$$

or

$$\phi_{fp} = 0.347 \text{ V}$$

and

$$x_{dT} = \left[\frac{4 \epsilon \phi_{fp}}{eN_a} \right]^{1/2} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.30 \text{ } \mu\text{m}$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}} = 7.67 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\Delta V_T = -\frac{(1.6 \times 10^{-19})(10^{16})(0.3 \times 10^{-4})}{7.67 \times 10^{-8}} \times \left\{ \frac{0.3}{1} \left[\sqrt{1 + \frac{2(0.3)}{0.3}} - 1 \right] \right\}$$

or

$$\Delta V_T = -0.137 \text{ V}$$

12.17

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}} \left\{ \frac{r_j}{L} \left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1 \right] \right\}$$

Now

$$\phi_{fp} = (0.0259) \ln \left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.376 \text{ V}$$

and

$$x_{dT} = \left[\frac{4 \epsilon \phi_{fp}}{eN_a} \right]^{1/2} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.376)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.180 \text{ } \mu\text{m}$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{800 \times 10^{-8}}$$

or

$$C_{ox} = 4.31 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\Delta V_T = -0.20 = -\frac{(1.6 \times 10^{-19})(3 \times 10^{16})(0.18 \times 10^{-4})}{4.31 \times 10^{-8}} \times \left\{ \frac{0.6}{L} \left[\sqrt{1 + \frac{2(0.18)}{0.6}} - 1 \right] \right\}$$

or

$$= -0.20 = -\frac{0.319}{L}$$

which yields

$$L = 1.59 \text{ } \mu\text{m}$$

12.18

We have

$$L' = L - (a + b)$$

and from the geometry

$$(1) \quad (a + r_j)^2 + x_{dT}^2 = (r_j + x_{ds})^2$$

and

$$(2) \quad (b + r_j)^2 + x_{dT}^2 = (r_j + x_{db})^2$$

From (1),

$$(a + r_j)^2 = (r_j + x_{ds})^2 - x_{dT}^2$$

so that

$$a = \sqrt{(r_j + x_{ds})^2 - x_{dT}^2} - r_j$$

which can be written as

$$a = r_j \left[\sqrt{\left(1 + \frac{x_{ds}}{r_j} \right)^2 - \left(\frac{x_{dT}}{r_j} \right)^2} - 1 \right]$$

or

$$a = r_j \left[\sqrt{1 + \frac{2x_{ds}}{r_j} + \left(\frac{x_{ds}}{r_j} \right)^2} - \left(\frac{x_{dT}}{r_j} \right)^2 - 1 \right]$$

Define

$$\alpha^2 = \frac{x_{ds}^2 - x_{dT}^2}{r_j^2}$$

We can then write

$$a = r_j \left[\sqrt{1 + \frac{2x_{ds}}{r_j} + \alpha^2} - 1 \right]$$

Similarly from (2), we will have

$$b = r_j \left[\sqrt{1 + \frac{2x_{dD}}{r_j} + \beta^2} - 1 \right]$$

where

$$\beta^2 = \frac{x_{dD}^2 - x_{dT}^2}{r_j^2}$$

The average bulk charge in the trapezoid (per unit area) is

$$|Q'_B| \cdot L = eN_a x_{dT} \left(\frac{L + L'}{2} \right)$$

or

$$|Q'_B| = eN_a x_{dT} \left(\frac{L + L'}{2L} \right)$$

We can write

$$\frac{L + L'}{2L} = \frac{1}{2} + \frac{L'}{2L} = \frac{1}{2} + \frac{1}{2L} [L - (a + b)]$$

which is

$$= 1 - \frac{(a + b)}{2L}$$

Then

$$|Q'_B| = eN_a x_{dT} \left[1 - \frac{(a + b)}{2L} \right]$$

Now $|Q'_B|$ replaces $|Q'_{SD}(\max)|$ in the threshold equation. Then

$$\begin{aligned} \Delta V_T &= \frac{|Q'_B|}{C_{ox}} - \frac{|Q'_{SD}(\max)|}{C_{ox}} \\ &= \frac{eN_a x_{dT}}{C_{ox}} \left[1 - \frac{(a + b)}{2L} \right] - \frac{eN_a x_{dT}}{C_{ox}} \end{aligned}$$

or

$$\Delta V_T = - \frac{eN_a x_{dT}}{C_{ox}} \cdot \frac{(a + b)}{2L}$$

Then substituting, we obtain

$$\Delta V_T = - \frac{eN_a x_{dT}}{C_{ox}} \cdot \frac{r_j}{2L} \left\{ \left[\sqrt{1 + \frac{2x_{ds}}{r_j} + \alpha^2} - 1 \right] + \left[\sqrt{1 + \frac{2x_{dD}}{r_j} + \beta^2} - 1 \right] \right\}$$

Note that if $x_{ds} = x_{dD} = x_{dT}$, then $\alpha = \beta = 0$ and the expression for ΔV_T reduces to that given in the text.

12.19

We have $L' = 0$, so Equation (12.25) becomes

$$\frac{L + L'}{2L} \Rightarrow \frac{L}{2L} = \frac{1}{2} = \left\{ 1 - \frac{r_j}{L} \left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1 \right] \right\}$$

or

$$\frac{r_j}{L} \left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1 \right] = \frac{1}{2}$$

Then Equation (12.26) is

$$|Q'_B| = eN_a x_{dT} \left(\frac{1}{2} \right)$$

The change in the threshold voltage is

$$\Delta V_T = \frac{|Q'_B|}{C_{ox}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

or

$$\Delta V_T = \frac{(1/2)(eN_a x_{dT})}{C_{ox}} - \frac{(eN_a x_{dT})}{C_{ox}}$$

or

$$\Delta V_T = - \left(\frac{1}{2} \right) \frac{(eN_a x_{dT})}{C_{ox}}$$

12.20

Computer plot

12.21

Computer plot

12.22

$$\Delta V_T = - \frac{eN_a x_{dT}}{C_{ox}} \left\{ \frac{r_j}{L} \left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1 \right] \right\}$$

$$\Rightarrow -\frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)} \left\{ \frac{kr_j}{kL} \left[\sqrt{1 + \frac{2kx_{dT}}{kr_j}} - 1 \right] \right\}$$

or

$$\Delta V_T = k\Delta V_T$$

12.23

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}} \left(\frac{\xi x_{dT}}{W} \right)$$

We find

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.347 \text{ V}$$

and

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a} \right]^{1/2} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.30 \text{ } \mu\text{m}$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}}$$

or

$$C_{ox} = 7.67 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\Delta V_T = \frac{(1.6 \times 10^{-19})(10^{16})(0.3 \times 10^{-4})}{7.67 \times 10^{-8}} \times \left[\frac{(\pi/2)(0.3 \times 10^{-4})}{2.5 \times 10^{-4}} \right]$$

or

$$\Delta V_T = +0.118 \text{ V}$$

12.24

Additional bulk charge due to the ends:

$$\Delta Q_B = eN_a L \left(\frac{1}{2} x_{dT}^2 \right) \cdot 2 = eN_a L x_{dT} (\xi x_{dT})$$

where $\xi = 1$.

Then

$$\Delta V_T = \frac{eN_a x_{dT}^2}{C_{ox} W}$$

We find

$$\phi_{fp} = (0.0259) \ln \left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.376 \text{ V}$$

and

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a} \right]^{1/2} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.376)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dT} = 0.180 \text{ } \mu\text{m}$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{800 \times 10^{-8}}$$

or

$$C_{ox} = 4.31 \times 10^{-8} \text{ F / cm}^2$$

Now, we can write

$$W = \frac{eN_a x_{dT}^2}{C_{ox} (\Delta V_T)} = \frac{(1.6 \times 10^{-19})(3 \times 10^{16})(0.18 \times 10^{-4})^2}{(4.31 \times 10^{-8})(0.25)}$$

or

$$W = 1.44 \text{ } \mu\text{m}$$

12.25

Computer plot

12.26

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}} \left(\frac{\xi x_{dT}}{W} \right)$$

Assume that ξ is a constant

$$\Rightarrow \frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)} \left(\frac{\xi \cdot kx_{dT}}{kW} \right)$$

or

$$\Delta V_T = k\Delta V_T$$

12.27

(a)

$$V_{BD} = (6 \times 10^6) t_{ox} = (6 \times 10^6)(250 \times 10^{-8})$$

or

$$\underline{V_{BD} = 15 \text{ V}}$$

(b)

With a safety factor of 3,

$$V_{BD} = \frac{1}{3} \cdot 15 \Rightarrow \underline{V_{BD} = 5 \text{ V}}$$

12.28

We want $V_G = 20 \text{ V}$. With a safety factor of 3, then $V_{BD} = 60 \text{ V}$, so that

$$60 = (6 \times 10^6) t_{ox} \Rightarrow \underline{t_{ox} = 1000 \text{ \AA}}$$

12.29

Snapback breakdown means $\alpha M = 1$, where

$$\alpha = (0.18) \log_{10} \left(\frac{I_D}{3 \times 10^{-9}} \right)$$

and

$$M = \frac{1}{1 - \left(\frac{V_{CE}}{V_{BD}} \right)^m}$$

Let $V_{BD} = 15 \text{ V}$, $m = 3$. Now when

$$\alpha M = 1 = \frac{\alpha}{1 - \left(\frac{V_{CE}}{15} \right)^3}$$

we can write this as

$$1 - \left(\frac{V_{CE}}{15} \right)^3 = \alpha \Rightarrow V_{CE} = 15 \sqrt[3]{1 - \alpha}$$

Now

I_D	α	V_{CE}
E-8	0.0941	14.5
E-7	0.274	13.5
E-6	0.454	12.3
E-5	0.634	10.7
E-4	0.814	8.6
E-3	0.994	2.7

12.30

One Debye length is

$$L_D = \left[\frac{\epsilon (kT/e)}{eN_a} \right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85 \times 10^{-14})(0.0259)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$L_D = 4.09 \times 10^{-6} \text{ cm}$$

Six Debye lengths:

$$6(4.09 \times 10^{-6}) = 0.246 \text{ } \mu\text{m}$$

From Example 12.4, we have $x_{dO} = 0.336 \text{ } \mu\text{m}$, which is the zero-biased source-substrate junction width.

At near punch-through, we will have

$$x_{dO} + 6L_D + x_d = L$$

where x_d is the reverse-biased drain-substrate junction width. Now

$$0.336 + 0.246 + x_d = 1.2 \Rightarrow x_d = 0.618 \text{ } \mu\text{m} \text{ at near punch-through.}$$

We have

$$x_d = \left[\frac{2 \epsilon (V_{bi} + V_{DS})}{eN_a} \right]^{1/2}$$

or

$$V_{bi} + V_{DS} = \frac{x_d^2 e N_a}{2 \epsilon}$$

$$= \frac{(0.618 \times 10^{-4})^2 (1.6 \times 10^{-19})(10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

which yields

$$V_{bi} + V_{DS} = 2.95 \text{ V}$$

From Example 12.4, we have $V_{bi} = 0.874 \text{ V}$, so that

$$\underline{V_{DS} = 2.08 \text{ V}}$$

which is the near punch-through voltage. The ideal punch-through voltage was

$$\underline{V_{DS} = 4.9 \text{ V}}$$

12.31

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{19})(3 \times 10^{16})}{(1.5 \times 10^{10})^2} \right] = 0.902 \text{ V}$$

The zero-biased source-substrate junction width:

$$x_{dO} = \left[\frac{2 \epsilon V_{bi}}{eN_a} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.902)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dO} = 0.197 \text{ } \mu\text{m}$$

The Debye length is

$$L_D = \left[\frac{\epsilon (kT/e)}{eN_a} \right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85 \times 10^{-14})(0.0259)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$L_D = 2.36 \times 10^{-6} \text{ cm}$$

so that

$$6L_D = 6(2.36 \times 10^{-6}) = 0.142 \text{ } \mu\text{m}$$

Now

$$x_{dO} + 6L_D + x_d = L$$

We have for $V_{DS} = 5 \text{ V}$,

$$x_d = \left[\frac{2 \epsilon (V_{bi} + V_{DS})}{eN_a} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.902 + 5)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_d = 0.505 \text{ } \mu\text{m}$$

Then

$$L = 0.197 + 0.142 + 0.505$$

or

$$L = 0.844 \text{ } \mu\text{m}$$

12.32

With a source-to-substrate voltage of 2 volts,

$$x_{dO} = \left[\frac{2 \epsilon (V_{bi} + V_{SB})}{eN_a} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.902 + 2)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_{dO} = 0.354 \text{ } \mu\text{m}$$

We have $6L_D = 0.142 \text{ } \mu\text{m}$ from the previous problem.

Now

$$x_d = \left[\frac{2 \epsilon (V_{bi} + V_{DS} + V_{SB})}{eN_a} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.902 + 5 + 2)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_d = 0.584 \text{ } \mu\text{m}$$

Then

$$L = x_{dO} + 6L_D + x_d$$

$$= 0.354 + 0.142 + 0.584$$

or

$$L = 1.08 \text{ } \mu\text{m}$$

12.33

$$(a) \quad \phi_{fp} = (0.0259) \ln \left(\frac{2 \times 10^{15}}{1.5 \times 10^{10}} \right) = 0.306 \text{ V}$$

and

$$\phi_{ms} = - \left(\frac{E_g}{2e} + \phi_{fp} \right) = - \left(\frac{1.12}{2} + 0.306 \right)$$

or

$$\phi_{ms} = -0.866 \text{ V}$$

Also

$$x_{dT} = \left[\frac{4 \epsilon \phi_{fp}}{eN_a} \right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.306)}{(1.6 \times 10^{-19})(2 \times 10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.629 \text{ } \mu\text{m}$$

Now

$$|Q'_{SD}(\text{max})| = eN_a x_{dT}$$

$$= (1.6 \times 10^{-19})(2 \times 10^{15})(0.629 \times 10^{-4})$$

or

$$|Q'_{SD}(\text{max})| = 2.01 \times 10^{-8} \text{ C / cm}^2$$

We have

$$Q'_{SS} = (2 \times 10^{11})(1.6 \times 10^{-19}) = 3.2 \times 10^{-8} \text{ C / cm}^2$$

Then

$$V_T = (|Q'_{SD}(\text{max})| - Q'_{SS}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{(2.01 \times 10^{-8} - 3.2 \times 10^{-8})(650 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})}$$

$$-0.866 + 2(0.306)$$

which yields

$$V_T = -0.478 \text{ V}$$

(b) We need a shift in threshold voltage in the positive direction, which means we must add acceptor atoms. We need

$$\Delta V_T = +0.80 - (-0.478) = 1.28 \text{ V}$$

Then

$$D_i = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.28)(3.9)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(650 \times 10^{-8})}$$

or

$$D_i = 4.25 \times 10^{11} \text{ cm}^{-2}$$

12.34

$$(a) \quad \phi_{fn} = (0.0259) \ln \left(\frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.347 \text{ V}$$

and

$$\begin{aligned} \phi_{ms} &= \phi'_{ms} - \left(\chi' + \frac{E_g}{2e} - \phi_{fn} \right) \\ &= 3.2 - (3.25 + 0.56 - 0.347) \end{aligned}$$

or

$$\phi_{ms} = -0.263 \text{ V}$$

Also

$$\begin{aligned} x_{dT} &= \left[\frac{4 \epsilon \phi_{fn}}{e N_d} \right]^{1/2} \\ &= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2} \end{aligned}$$

or

$$x_{dT} = 0.30 \text{ } \mu\text{m}$$

Now

$$\begin{aligned} |Q'_{SD}(\text{max})| &= e N_d x_{dT} \\ &= (1.6 \times 10^{-19})(10^{16})(0.30 \times 10^{-4}) \end{aligned}$$

or

$$|Q'_{SD}(\text{max})| = 4.8 \times 10^{-8} \text{ C / cm}^2$$

We have

$$Q'_{SS} = (5 \times 10^{11})(1.6 \times 10^{-19}) = 8 \times 10^{-8} \text{ C / cm}^2$$

Now

$$\begin{aligned} V_T &= -(|Q'_{SD}(\text{max})| + Q'_{SS}) \left(\frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} - 2\phi_{fn} \\ &= \frac{-(4.8 \times 10^{-8} + 8 \times 10^{-8})(750 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} \\ &\quad -0.263 - 2(0.347) \end{aligned}$$

which becomes

$$V_T = -3.74 \text{ V}$$

(b)

We want $V_T = -0.50 \text{ V}$. Need to shift V_T in the positive direction which means we need to add acceptor atoms.

So

$$\Delta V_T = -0.50 - (-3.74) = 3.24 \text{ V}$$

Now

$$D_i = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(3.24)(3.9)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(750 \times 10^{-8})}$$

or

$$D_i = 9.32 \times 10^{11} \text{ cm}^{-2}$$

12.35

$$(a) \quad \phi_{fp} = (0.0259) \ln \left(\frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

and

$$\begin{aligned} x_{dT} &= \left[\frac{4 \epsilon \phi_{fp}}{e N_a} \right]^{1/2} \\ &= \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2} \end{aligned}$$

or

$$x_{dT} = 0.863 \text{ } \mu\text{m}$$

Now

$$\begin{aligned} |Q'_{SD}(\text{max})| &= e N_a x_{dT} \\ &= (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4}) \end{aligned}$$

or

$$|Q'_{SD}(\text{max})| = 1.38 \times 10^{-8} \text{ C / cm}^2$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{750 \times 10^{-8}}$$

or

$$C_{ox} = 4.6 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\begin{aligned} V_T &= V_{FB} + 2\phi_{fp} + \frac{|Q'_{SD}(\text{max})|}{C_{ox}} \\ &= -1.50 + 2(0.288) + \frac{1.38 \times 10^{-8}}{4.6 \times 10^{-8}} \end{aligned}$$

or

$$V_T = -0.624 \text{ V}$$

(b)

Want $V_T = +0.90 \text{ V}$, which is a positive shift and we must add acceptor atoms.

$$\Delta V_T = 0.90 - (-0.624) = 1.52 \text{ V}$$

Then

$$D_i = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.52)(4.6 \times 10^{-8})}{1.6 \times 10^{-19}}$$

or

$$D_i = 4.37 \times 10^{11} \text{ cm}^{-2}$$

(c)

With an applied substrate voltage,

$$\begin{aligned} \Delta V_T &= \frac{\sqrt{2e \epsilon N_a}}{C_{ox}} \left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right] \\ &= \frac{[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})]^{1/2}}{4.6 \times 10^{-8}} \\ &\quad \times [\sqrt{2(0.288) + 2} - \sqrt{2(0.288)}] \end{aligned}$$

or

$$\Delta V_T = +0.335 \text{ V}$$

Then the threshold voltage is

$$V_T = +0.90 + 0.335$$

or

$$V_T = 1.24 \text{ V}$$

12.36

The total space charge width is greater than x_i , so from chapter 11,

$$\Delta V_T = \frac{\sqrt{2e \epsilon N_a}}{C_{ox}} \left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$$

Now

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{14}}{1.5 \times 10^{10}} \right) = 0.228 \text{ V}$$

and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}}$$

or

$$C_{ox} = 6.90 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\begin{aligned} \Delta V_T &= \frac{[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{14})]^{1/2}}{6.90 \times 10^{-8}} \\ &\quad \times [\sqrt{2(0.228) + V_{SB}} - \sqrt{2(0.228)}] \end{aligned}$$

or

$$\Delta V_T = 0.0834 [\sqrt{0.456 + V_{SB}} - \sqrt{0.456}]$$

Then

$V_{SB} (V)$	$\Delta V_T (V)$
1	0.0443
3	0.0987
5	0.399

11.37

$$(a) \quad \phi_{fn} = (0.0259) \ln \left(\frac{10^{17}}{1.5 \times 10^{10}} \right) = 0.407 \text{ V}$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.407)}{(1.6 \times 10^{-19})(10^{17})} \right]^{1/2}$$

or

$$x_{dT} = 1.026 \times 10^{-5} \text{ cm}$$

$$n^+ \text{ poly on } n \Rightarrow \phi_{ms} = -0.32 \text{ V}$$

We have

$$|Q'_{SD}(\text{max})| = (1.6 \times 10^{-19})(10^{17})(1.026 \times 10^{-5})$$

or

$$|Q'_{SD}(\text{max})| = 1.64 \times 10^{-7} \text{ C / cm}^2$$

Now

$$\begin{aligned} V_{TP} &= [-1.64 \times 10^{-7} - (1.6 \times 10^{-19})(5 \times 10^{10})] \\ &\quad \times \frac{(80 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} - 0.32 - 2(0.407) \end{aligned}$$

or

$$V_{TP} = -1.53 \text{ V, Enhancement PMOS}$$

(b)

For $V_T = 0$, shift threshold voltage in positive direction, so implant acceptor ions

$$\Delta V_T = \frac{eD_i}{C_{ox}} \Rightarrow D_i = \frac{(\Delta V_T)C_{ox}}{e}$$

so

$$D_i = \frac{(1.53)(3.9)(8.85 \times 10^{-14})}{(80 \times 10^{-8})(1.6 \times 10^{-19})}$$

or

$$D_i = 4.13 \times 10^{12} \text{ cm}^{-2}$$

12.38

Shift in negative direction means implanting donor ions. We have

$$\Delta V_T = \frac{eD_i}{C_{ox}}$$

where

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F / cm}^2$$

Now

$$D_i = \frac{C_{ox}(\Delta V_T)}{e} = \frac{(8.63 \times 10^{-8})(1.4)}{1.6 \times 10^{-19}}$$

or

$$D_i = 7.55 \times 10^{11} \text{ cm}^{-2}$$

12.39

The areal density of generated holes is

$$= (8 \times 10^{12})(10^5)(750 \times 10^{-8}) = 6 \times 10^{12} \text{ cm}^{-2}$$

The equivalent surface charge trapped is

$$= (0.10)(6 \times 10^{12}) = 6 \times 10^{11} \text{ cm}^{-2}$$

Then

$$\Delta V_T = -\frac{Q'_{ss}}{C_{ox}} = -\frac{(6 \times 10^{11})(1.6 \times 10^{-19})}{(3.9)(8.85 \times 10^{-14})}(750 \times 10^{-8})$$

or

$$\Delta V_T = -2.09 \text{ V}$$

12.40

The areal density of generated holes is

$6 \times 10^{12} \text{ cm}^{-2}$. Now

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{750 \times 10^{-8}}$$

or

$$C_{ox} = 4.6 \times 10^{-8} \text{ F / cm}^2$$

Then

$$\Delta V_T = -\frac{Q'_{ss}}{C_{ox}} = -\frac{(6 \times 10^{12})(x)(1.6 \times 10^{-19})}{4.6 \times 10^{-8}}$$

For $\Delta V_T = -0.50 \text{ V}$

Where the parameter x is the maximum fraction of holes that can be trapped. Then we find

$$x = 0.024 \Rightarrow 2.4\%$$

12.41

We have the areal density of generated holes as

$= (g)(\gamma)(t_{ox})$ where g is the generation rate

and γ is the dose. The equivalent charge

trapped is $= xg\gamma t_{ox}$.

Then

$$\Delta V_T = -\frac{Q'_{ss}}{C_{ox}} = -\frac{exg\gamma t_{ox}}{(\epsilon_{ox}/t_{ox})} = -exg\gamma(t_{ox})^2$$

so that

$$\Delta V_T \propto -(t_{ox})^2$$

Chapter 13

Problem Solutions

13.1

Sketch

13.2

Sketch

13.3

p-channel JFET – Silicon

(a)

$$V_{PO} = \frac{ea^2 N_a}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2 (3 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 5.79 \text{ V}$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(3 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.884 \text{ V}$$

so

$$V_P = V_{PO} - V_{bi} = 5.79 - 0.884$$

or

$$V_P = 4.91 \text{ V}$$

(b)

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} - V_{DS} + V_{GS})}{e N_a} \right]^{1/2}$$

(i)

For $V_{GS} = 1 \text{ V}$, $V_{DS} = 0$

Then

$$a - h = 0.5 \times 10^{-4} - \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.884 + 1 - V_{DS})}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$a - h = 0.5 \times 10^{-4} - \left[(4.31 \times 10^{-10})(1.884 - V_{DS}) \right]^{1/2}$$

or

$$a - h = 0.215 \text{ } \mu\text{m}$$

(ii) For $V_{GS} = 1 \text{ V}$, $V_{DS} = -2.5 \text{ V}$

$$a - h = 0.0653 \text{ } \mu\text{m}$$

(iii) For $V_{GS} = 1 \text{ V}$, $V_{DS} = -5 \text{ V}$

$$a - h = -0.045 \text{ } \mu\text{m}$$

which implies no undepleted region.

13.4

p-channel JFET – GaAs

(a)

$$V_{PO} = \frac{2a^2 N_a}{2 \epsilon} = \frac{2(0.5 \times 10^{-4})^2 (3 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 5.18 \text{ V}$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(3 \times 10^{16})}{(1.8 \times 10^6)^2} \right]$$

or

$$V_{bi} = 1.35 \text{ V}$$

so

$$V_P = V_{PO} - V_{bi} = 5.18 - 1.35$$

or

$$V_P = 3.83 \text{ V}$$

(b)

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} - V_{DS} + V_{GS})}{e N_a} \right]^{1/2}$$

(i) For $V_{GS} = 1 \text{ V}$, $V_{DS} = 0$

Then

$$a - h = 0.5 \times 10^{-4} - \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.35 + 1 - V_{DS})}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$a - h = 0.5 \times 10^{-4} - \left[(4.83 \times 10^{-10})(2.35 - V_{DS}) \right]^{1/2}$$

which yields

$$a - h = 0.163 \text{ } \mu\text{m}$$

(ii) For $V_{GS} = 1 \text{ V}$, $V_{DS} = -2.5 \text{ V}$

$$a - h = 0.016 \text{ } \mu\text{m}$$

(iii) For $V_{GS} = 1 \text{ V}$, $V_{DS} = -5 \text{ V}$

$$a - h = -0.096 \text{ } \mu\text{m}$$

which implies no undepleted region.

13.5

$$(a) \quad V_{PO} = \frac{ea^2 N_d}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2 (8 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

or

$$\underline{V_{PO} = 15.5 \text{ V}}$$

(b)

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} - V_{GS})}{e N_d} \right]^{1/2}$$

so

$$0.2 \times 10^{-4} = 0.5 \times 10^{-4} - \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} - V_{GS})}{(1.6 \times 10^{-19})(8 \times 10^{16})} \right]^{1/2}$$

or

$$9 \times 10^{-10} = 1.618 \times 10^{-10} (V_{bi} - V_{GS})$$

which yields

$$V_{bi} - V_{GS} = 5.56 \text{ V}$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(3 \times 10^{18})(8 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.896 \text{ V}$$

Then

$$V_{GS} = 0.896 - 5.56 \Rightarrow \underline{V_{GS} = -4.66 \text{ V}}$$

13.6

For GaAs:

(a)

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2 (8 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$\underline{V_{PO} = 13.8 \text{ V}}$$

(b)

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} - V_{GS})}{e N_d} \right]^{1/2}$$

$$0.2 \times 10^{-4} = 0.5 \times 10^{-4}$$

$$- \left[\frac{2(13.1)(8.85 \times 10^{-14})(V_{bi} - V_{GS})}{(1.6 \times 10^{-19})(8 \times 10^{16})} \right]^{1/2}$$

which can be written as

$$9 \times 10^{-10} = 1.811 \times 10^{-10} (V_{bi} - V_{GS})$$

or

$$V_{bi} - V_{GS} = 4.97 \text{ V}$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(3 \times 10^{18})(8 \times 10^{16})}{(1.8 \times 10^6)^2} \right]$$

or

$$V_{bi} = 1.36 \text{ V}$$

Then

$$V_{GS} = V_{bi} - 4.97 = 1.36 - 4.97$$

or

$$\underline{V_{GS} = -3.61 \text{ V}}$$

13.7

$$(a) \quad V_{PO} = \frac{ea^2 N_a}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.3 \times 10^{-4})^2 (3 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$\underline{V_{PO} = 1.863 \text{ V}}$$

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(3 \times 10^{16})}{(1.8 \times 10^6)^2} \right]$$

or

$$V_{bi} = 1.352 \text{ V}$$

Then

$$V_P = V_{PO} - V_{bi} = 1.863 - 1.352$$

or

$$\underline{V_P = 0.511 \text{ V}}$$

(b) (i)

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} + V_{GS})}{e N_a} \right]^{1/2}$$

or

$$a - h = (0.3 \times 10^{-4})$$

$$- \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.352)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

which yields

$$\begin{aligned} a - h &= 4.45 \times 10^{-6} \text{ cm} \\ \text{(ii)} \quad a - h &= (0.3 \times 10^{-4}) \\ &\quad - \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.351 + 1)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2} \end{aligned}$$

which yields

$$\begin{aligned} a - h &= -3.7 \times 10^{-6} \text{ cm} \\ \text{which implies no undepleted region.} \end{aligned}$$

13.8

(a) n-channel JFET – Silicon

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.35 \times 10^{-4})^2 (4 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 3.79 \text{ V}$$

Now

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(4 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.892 \text{ V}$$

so that

$$V_P = V_{bi} - V_{PO} = 0.892 - 3.79$$

or

$$V_P = -2.90 \text{ V}$$

(b)

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} + V_{DS} - V_{GS})}{e N_d} \right]^{1/2}$$

We have

$$\begin{aligned} a - h &= 0.35 \times 10^{-4} \\ &\quad - \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.892 + V_{DS} - V_{GS})}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2} \end{aligned}$$

or

$$\begin{aligned} a - h &= 0.35 \times 10^{-4} \\ &\quad - \left[(3.24 \times 10^{-10})(0.892 + V_{DS} - V_{GS}) \right]^{1/2} \end{aligned}$$

(i) For $V_{GS} = 0$, $V_{DS} = 1 \text{ V}$,

$$a - h = 0.102 \text{ } \mu\text{m}$$

(ii) For $V_{GS} = -1 \text{ V}$, $V_{DS} = 1 \text{ V}$,

$$a - h = 0.044 \text{ } \mu\text{m}$$

(iii) For $V_{GS} = -1 \text{ V}$, $V_{DS} = 2 \text{ V}$,
 $a - h = -0.0051 \text{ } \mu\text{m}$

which implies no undepleted region

13.9

$$V_{bi} = (0.0259) \ln \left(\frac{(5 \times 10^{18})(4 \times 10^{16})}{(1.8 \times 10^6)^2} \right)$$

or

$$V_{bi} = 1.359 \text{ V}$$

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} + V_{DS} - V_{GS})}{e N_D} \right]^{1/2}$$

or

$$\begin{aligned} a - h &= 0.35 \times 10^{-4} \\ &\quad - \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.359 + V_{DS} - V_{GS})}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2} \end{aligned}$$

We want $a - h = 0.05 \times 10^{-4} \text{ cm}$,

Then

$$\begin{aligned} 0.05 \times 10^{-4} &= 0.35 \times 10^{-4} \\ &\quad - \left[(3.623 \times 10^{-10})(1.359 + V_{DS} - V_{GS}) \right]^{1/2} \end{aligned}$$

(a)

For $V_{DS} = 0$, we find

$$V_{GS} = -1.125 \text{ V}$$

(b)

For $V_{DS} = 1 \text{ V}$, we find

$$V_{GS} = -0.125 \text{ V}$$

13.10

(a)

$$\begin{aligned} I_{P1} &= \frac{\mu_n (e N_d)^2 W a^3}{6 \epsilon L} \\ &= \frac{(1000) [(1.6 \times 10^{-19})(10^{16})]^2}{6(11.7)(8.85 \times 10^{-14})} \\ &\quad \times \frac{(400 \times 10^{-4})(0.5 \times 10^{-4})^3}{(20 \times 10^{-4})} \end{aligned}$$

or

$$I_{P1} = 1.03 \text{ mA}$$

(b)

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon}$$

$$= \left[\frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2 (10^{16})}{2(11.7)(8.85 \times 10^{-14})} \right]$$

or

$$\underline{V_{PO} = 1.93 \text{ V}}$$

Also

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{19})(10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.874 \text{ V}$$

Now

$$\begin{aligned} V_{DS}(sat) &= V_{PO} - (V_{bi} - V_{GS}) \\ &= 1.93 - 0.874 + V_{GS} \end{aligned}$$

or

$$V_{DS}(sat) = 1.06 + V_{GS}$$

We have

$$V_P = V_{bi} - V_{PO} = 0.874 - 1.93$$

or

$$\underline{V_P = -1.06 \text{ V}}$$

Then

$$(i) \quad V_{GS} = 0 \Rightarrow \underline{V_{DS}(sat) = 1.06 \text{ V}}$$

$$(ii) \quad V_{GS} = \frac{1}{4} V_P = -0.265 \text{ V} \Rightarrow$$

$$\underline{V_{DS}(sat) = 0.795 \text{ V}}$$

$$(iii) \quad V_{GS} = \frac{1}{2} V_P = -0.53 \text{ V} \Rightarrow$$

$$\underline{V_{DS}(sat) = 0.53 \text{ V}}$$

$$(iv) \quad V_{GS} = \frac{3}{4} V_P = -0.795 \text{ V} \Rightarrow$$

$$\underline{V_{DS}(sat) = 0.265 \text{ V}}$$

(c)

$$\begin{aligned} I_{D1}(sat) &= I_{P1} \left[1 - 3 \left(\frac{V_{bi} - V_{GS}}{V_{PO}} \right) \left(1 - \frac{2}{3} \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}} \right) \right] \\ &= 1.03 \left[1 - 3 \left(\frac{0.874 - V_{GS}}{1.93} \right) \right. \\ &\quad \left. \times \left(1 - \frac{2}{3} \sqrt{\frac{0.874 - V_{GS}}{1.93}} \right) \right] \end{aligned}$$

$$(i) \quad \text{For } V_{GS} = 0 \Rightarrow \underline{I_{D1}(sat) = 0.258 \text{ mA}}$$

$$(ii) \quad \text{For } V_{GS} = -0.265 \text{ V} \Rightarrow$$

$$\underline{I_{D1}(sat) = 0.140 \text{ mA}}$$

$$(iii) \quad \text{For } V_{GS} = -0.53 \text{ V} \Rightarrow$$

$$\underline{I_{D1}(sat) = 0.061 \text{ mA}}$$

$$(iv) \quad \text{For } V_{GS} = -0.795 \text{ V} \Rightarrow$$

$$\underline{I_{D1}(sat) = 0.0145 \text{ mA}}$$

13.11

$$g_d = G_{O1} \left[1 - \left(\frac{V_{bi} - V_{GS}}{V_{PO}} \right)^{1/2} \right]$$

where

$$G_{O1} = \frac{3I_{P1}}{V_{PO}} = \frac{3(1.03 \times 10^{-3})}{1.93} = 1.60 \times 10^{-3}$$

or

$$G_{O1} = 1.60 \text{ mS}$$

Then

$\underline{V_{GS}}$	$\underline{[(V_{bi} - V_{GS}) / V_{PO}]}$	$\underline{g_d (mS)}$
0	0.453	0.523
-0.265	0.590	0.371
-0.53	0.727	0.236
-0.795	0.945	0.112
-1.06	1.0	0

13.12

n-channel JFET – GaAs

(a)

$$\begin{aligned} G_{O1} &= \frac{e\mu_n N_d W a}{L} \\ &= \frac{(1.6 \times 10^{-19})(8000)(2 \times 10^{16})(30 \times 10^{-4})(0.35 \times 10^{-4})}{10 \times 10^{-4}} \end{aligned}$$

or

$$\underline{G_{O1} = 2.69 \times 10^{-3} \text{ S}}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$

We have

$$\begin{aligned} V_{PO} &= \frac{ea^2 N_d}{2\epsilon} \\ &= \frac{(1.6 \times 10^{-19})(0.35 \times 10^{-4})^2 (2 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$\underline{V_{PO} = 1.69 \text{ V}}$$

We find

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(2 \times 10^{16})}{(1.8 \times 10^6)^2} \right]$$

or

$$V_{bi} = 1.34 \text{ V}$$

Then

$$V_P = V_{bi} - V_{PO} = 1.34 - 1.69$$

or

$$V_P = -0.35 \text{ V}$$

We then obtain

$$V_{DS}(\text{sat}) = 1.69 - (1.34 - V_{GS}) = 0.35 + V_{GS}$$

$$\text{For } V_{GS} = 0 \Rightarrow V_{DS}(\text{sat}) = 0.35 \text{ V}$$

$$\text{For } V_{GS} = \frac{1}{2} V_P = -0.175 \text{ V} \Rightarrow$$

$$V_{DS}(\text{sat}) = 0.175 \text{ V}$$

(c)

$$\begin{aligned} I_{D1}(\text{sat}) &= I_{P1} \left[1 - 3 \left(\frac{V_{bi} - V_{GS}}{V_{PO}} \right) \left(1 - \frac{2}{3} \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}} \right) \right] \end{aligned}$$

where

$$\begin{aligned} I_{P1} &= \frac{\mu_n (eN_d)^2 W a^3}{6 \epsilon L} \\ &= \frac{(8000) [(1.6 \times 10^{-19})(2 \times 10^{16})]^2}{6(13.1)(8.85 \times 10^{-14})} \\ &\quad \times \frac{(30 \times 10^{-4})(0.35 \times 10^{-4})^3}{(10 \times 10^{-4})} \end{aligned}$$

or

$$I_{P1} = 1.51 \text{ mA}$$

Then

$$\begin{aligned} I_{D1}(\text{sat}) &= 1.51 \left[1 - 3 \left(\frac{1.34 - V_{GS}}{1.69} \right) \right. \\ &\quad \left. \times \left(1 - \frac{2}{3} \sqrt{\frac{1.34 - V_{GS}}{1.69}} \right) \right] (\text{mA}) \end{aligned}$$

For

$$V_{GS} = 0 \Rightarrow I_{D1}(\text{sat}) = 0.0504 \text{ mA}$$

and for

$$V_{GS} = -0.175 \text{ V} \Rightarrow I_{D1}(\text{sat}) = 0.0123 \text{ mA}$$

13.13

$$g_{mS} = \frac{3I_{P1}}{V_{PO}} \left(1 - \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}} \right)$$

We have

$$I_{P1} = 1.03 \text{ mA}, V_{PO} = 1.93 \text{ V}, V_{bi} = 0.874 \text{ V}$$

The maximum transconductance occurs when

$$V_{GS} = 0$$

Then

$$g_{mS}(\text{max}) = \frac{3(1.03)}{1.93} \left(1 - \sqrt{\frac{0.874}{1.93}} \right)$$

or

$$g_{mS} = 0.524 \text{ mS}$$

For $W = 400 \mu\text{m}$,

We have

$$g_{mS}(\text{max}) = \frac{0.524 \text{ mS}}{400 \times 10^{-4} \text{ cm}}$$

or

$$g_{mS} = 13.1 \text{ mS/cm} = 1.31 \text{ mS/mm}$$

13.14

The maximum transconductance occurs for

$V_{GS} = 0$, so we have

(a)

$$\begin{aligned} g_{mS}(\text{max}) &= \frac{3I_{P1}}{V_{PO}} \left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}} \right) \\ &= G_{O1} \left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}} \right) \end{aligned}$$

We found

$$G_{O1} = 2.69 \text{ mS}, V_{bi} = 1.34 \text{ V}, V_{PO} = 1.69 \text{ V}$$

Then

$$g_{mS}(\text{max}) = (2.69) \left(1 - \sqrt{\frac{1.34}{1.69}} \right)$$

or

$$g_{mS}(\text{max}) = 0.295 \text{ mS}$$

This is for a channel length of $L = 10 \mu\text{m}$.

(b)

If the channel length is reduced to $L = 2 \mu\text{m}$, then

$$g_{mS}(\text{max}) = (0.295) \left(\frac{10}{2} \right) \Rightarrow$$

$$g_{mS}(\text{max}) = 1.48 \text{ mS}$$

13.15

n-channel MESFET – GaAs

(a)

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2 (1.5 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 2.59 \text{ V}$$

Now

$$V_{bi} = \phi_{Bn} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right) = (0.0259) \ln\left(\frac{4.7 \times 10^{17}}{1.5 \times 10^{16}}\right)$$

or

$$\phi_n = 0.0892 \text{ V}$$

so that

$$V_{bi} = 0.90 - 0.0892 = 0.811 \text{ V}$$

Then

$$V_T = V_{bi} - V_{PO} = 0.811 - 2.59$$

or

$$V_T = -1.78 \text{ V}$$

(b)

If $V_T < 0$ for an n-channel device, the device is a depletion mode MESFET.

13.16

n-channel MESFET – GaAs

(a)

We want $V_T = +0.10 \text{ V}$

Then

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

so

$$V_T = 0.10 = 0.89 - V_t \ln\left(\frac{N_c}{N_d}\right) - \frac{ea^2 N_d}{2 \epsilon}$$

which can be written as

$$(0.0259) \ln\left(\frac{4.7 \times 10^{17}}{N_d}\right) + \frac{(1.6 \times 10^{-19})(0.35 \times 10^{-4})^2 N_d}{2(13.1)(8.85 \times 10^{-14})} = 0.89 - 0.10$$

or

$$(0.0259) \ln\left(\frac{4.7 \times 10^{17}}{N_d}\right) + (8.45 \times 10^{17}) N_d = 0.79$$

By trial and error

$$N_d = 8.1 \times 10^{15} \text{ cm}^{-3}$$

(b)

At $T = 400 \text{ K}$,

$$N_c(400) = N_c(300) \cdot \left(\frac{400}{300}\right)^{3/2} = (4.7 \times 10^{17})(1.54)$$

or

$$N_c(400) = 7.24 \times 10^{17} \text{ cm}^{-3}$$

Also

$$V_t = (0.0259) \left(\frac{400}{300}\right) = 0.03453$$

Then

$$V_T = 0.89 - (0.03453) \ln\left(\frac{7.24 \times 10^{17}}{8.1 \times 10^{15}}\right) - (8.45 \times 10^{-17})(8.1 \times 10^{15})$$

which becomes

$$V_T = +0.051 \text{ V}$$

13.17

We have

$$a - h = a - \left[\frac{2 \epsilon (V_{bi} + V_{DS} - V_{GS})}{e N_d} \right]^{1/2}$$

where

$$V_{bi} = \phi_{Bn} - \phi_n$$

Now

$$\phi_n = (0.0259) \ln\left(\frac{4.7 \times 10^{17}}{5 \times 10^{16}}\right) = 0.058 \text{ V}$$

Then

$$V_{bi} = 0.80 - 0.058 = 0.742 \text{ V}$$

For $V_{GS} = 0.5 \text{ V}$,

$$a - h = (0.8 \times 10^{-4})$$

$$- \left[\frac{2(13.1)(8.85 \times 10^{-14})(0.742 + V_{DS} - 0.5)}{(1.6 \times 10^{-19})(5 \times 10^{16})} \right]^{1/2}$$

or

$$a - h = (0.80 \times 10^{-4}) - \left[(2.898 \times 10^{-10})(0.242 + V_{DS}) \right]^{1/2}$$

Then

$V_{DS} (V)$	$a - h \ (\mu m)$
0	0.716
1	0.610
2	0.545
5	0.410

13.18

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

We want

$$V_T = 0 \Rightarrow \phi_n + V_{PO} = \phi_{Bn}$$

$$\text{Device 1: } N_d = 3 \times 10^{16} \text{ cm}^{-3}$$

Then

$$\phi_n = (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{3 \times 10^{16}} \right) = 0.0713 \text{ V}$$

so that

$$V_{PO} = 0.89 - 0.0713 = 0.8187 \text{ V}$$

Now

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon} \Rightarrow a = \left[\frac{2 \epsilon V_{PO}}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(0.8187)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$a = 0.199 \ \mu m$$

$$\text{Device 2: } N_d = 3 \times 10^{17} \text{ cm}^{-3}$$

Then

$$\phi_n = (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{3 \times 10^{17}} \right) = 0.0116 \text{ V}$$

so that

$$V_{PO} = 0.89 - 0.0116 = 0.8784 \text{ V}$$

Now

$$a = \left[\frac{2 \epsilon V_{PO}}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(0.8784)}{(1.6 \times 10^{-19})(3 \times 10^{17})} \right]^{1/2}$$

or

$$a = 0.0651 \ \mu m$$

13.19

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

We want $V_T = 0.5 \text{ V}$, so

$$0.5 = 0.85 - \phi_n - V_{PO}$$

Now

$$\phi_n = (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{N_d} \right)$$

and

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(0.25 \times 10^{-4})^2 N_d}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = (4.31 \times 10^{-17}) N_d$$

Then

$$0.5 = 0.85 - (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{N_d} \right)$$

$$- (4.31 \times 10^{-17}) N_d$$

By trial and error, we find

$$N_d = 5.45 \times 10^{15} \text{ cm}^{-3}$$

13.20

n-channel MESFET – silicon

(a) For a gold contact, $\phi_{Bn} = 0.82 \text{ V}$.

We find

$$\phi_n = (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{10^{16}} \right) = 0.206 \text{ V}$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.82 - 0.206 = 0.614 \text{ V}$$

With $V_{DS} = 0$, $V_{GS} = 0.35 \text{ V}$

We find

$$a - h = 0.075 \times 10^{-4}$$

$$= a - \left[\frac{2 \epsilon (V_{bi} - V_{GS})}{e N_d} \right]^{1/2}$$

so that

$$a = 0.075 \times 10^{-4}$$

$$+ \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.614 - 0.35)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

or

$$a = 0.26 \ \mu m$$

Now

$$V_T = V_{bi} - V_{PO} = 0.614 - \frac{ea^2 N_d}{2 \epsilon}$$

or

$$V_T = 0.614 - \frac{(1.6 \times 10^{-19})(0.26 \times 10^{-4})^2 (10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

We obtain

$$\underline{V_T = 0.092 \text{ V}}$$

(b)

$$\begin{aligned} V_{DS}(sat) &= V_{PO} - (V_{bi} - V_{GS}) \\ &= (V_{bi} - V_T) - (V_{bi} - V_{GS}) = V_{GS} - V_T \end{aligned}$$

Now

$$V_{DS}(sat) = 0.35 - 0.092$$

or

$$\underline{V_{DS}(sat) = 0.258 \text{ V}}$$

13.21

(a) n-channel MESFET - silicon

$$V_{bi} = \phi_{Bn} - \phi_n$$

and

$$\phi_n = (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{2 \times 10^{16}} \right) = 0.188 \text{ V}$$

so

$$V_{bi} = 0.80 - 0.188 \Rightarrow \underline{V_{bi} = 0.612 \text{ V}}$$

Now

$$\begin{aligned} V_{PO} &= \frac{ea^2 N_d}{2 \epsilon} \\ &= \frac{(1.6 \times 10^{-19})(0.4 \times 10^{-4})^2 (2 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$\underline{V_{PO} = 2.47 \text{ V}}$$

We find

$$V_T = V_{bi} - V_{PO} = 0.612 - 2.47$$

or

$$\underline{V_T = -1.86 \text{ V}}$$

and

$$\begin{aligned} V_{DS}(sat) &= V_{PO} - (V_{bi} - V_{GS}) \\ &= 2.47 - (0.612 - (-1)) \end{aligned}$$

or

$$\underline{V_{DS}(sat) = 0.858 \text{ V}}$$

(b)

For $V_{PO} = 4.5 \text{ V}$, additional donor atoms must be added.

We have

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon} \Rightarrow N_d = \frac{2 \epsilon V_{PO}}{ea^2}$$

so that

$$N_d = \frac{2(11.7)(8.85 \times 10^{-14})(4.5)}{(1.6 \times 10^{-19})(0.4 \times 10^{-4})^2}$$

or

$$\underline{N_d = 3.64 \times 10^{16} \text{ cm}^{-3}}$$

which means that

$$\Delta N_d = 3.64 \times 10^{16} - 2 \times 10^{16}$$

or

$$\underline{\Delta N_d = 1.64 \times 10^{16} \text{ cm}^{-3}}$$

Donors must be added

Then

$$\phi_n = (0.0259) \ln \left(\frac{2.8 \times 10^{19}}{3.64 \times 10^{16}} \right) = 0.172 \text{ V}$$

so that

$$V_{bi} = 0.80 - 0.172 = 0.628 \text{ V}$$

We find

$$V_T = V_{bi} - V_{PO} = 0.628 - 4.5$$

or

$$\underline{V_T = -3.87 \text{ V}}$$

Also

$$\begin{aligned} V_{DS}(sat) &= V_{PO} - (V_{bi} - V_{GS}) \\ &= 4.5 - (0.628 - (-1)) \end{aligned}$$

or

$$\underline{V_{DS}(sat) = 2.87 \text{ V}}$$

13.22

$$\begin{aligned} \text{(a)} \quad k_n &= \frac{\mu_n \epsilon W}{2aL} \\ &= \frac{(7800)(13.1)(8.85 \times 10^{-14})(20 \times 10^{-4})}{2(0.30 \times 10^{-4})(1.2 \times 10^{-4})} \end{aligned}$$

or

$$\underline{k_n = 2.51 \text{ mA/V}^2}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS}) = V_{GS} - V_T$$

So for $V_{GS} = 1.5V_T \Rightarrow V_{DS}(sat) = (0.5)(0.12)$

Or

$$\underline{V_{DS}(sat) = 0.06 \text{ V}}$$

and for $V_{GS} = 2V_T \Rightarrow V_{DS}(sat) = (1)(0.12)$

or

$$\underline{V_{DS}(sat) = 0.12 \text{ V}}$$

(c)

$$I_{D1}(sat) = k_n (V_{GS} - V_T)^2$$

For $V_{GS} = 1.5V_T \Rightarrow I_{D1}(sat) = (2.51)(0.06)^2$

Or

$$\underline{I_{D1}(sat) = 9.04 \mu A}$$

and for $V_{GS} = 2V_T \Rightarrow I_{D1}(sat) = (2.51)(0.12)^2$

or

$$\underline{I_{D1}(sat) = 36.1 \mu A}$$

13.23

(a) We have

$$g_m = 2k_n(V_{GS} - V_T)$$

so that

$$1.75 \times 10^{-3} = 2k_n(0.50 - 0.25)$$

which gives

$$k_n = 3.5 \times 10^{-3} \text{ A/V}^2 = \frac{\mu_n \epsilon W}{2aL}$$

We obtain

$$W = \frac{(3.5 \times 10^{-3})(2)(0.35 \times 10^{-4})(10^{-4})}{(8000)(13.1)(8.85 \times 10^{-14})}$$

or

$$\underline{W = 26.4 \mu m}$$

(b)

$$I_{D1}(sat) = k_n(V_{GS} - V_T)^2$$

For $V_{GS} = 0.4 V$,

$$I_{D1}(sat) = (3.5 \times 10^{-3})(0.4 - 0.25)^2$$

or

$$\underline{I_{D1}(sat) = 78.8 \mu A}$$

For $V_{GS} = 0.65 V$,

$$I_{D1}(sat) = (3.5 \times 10^{-3})(0.65 - 0.25)^2$$

or

$$\underline{I_{D1}(sat) = 0.56 mA}$$

13.24

Computer plot

13.25

Computer plot

13.26

We have $L' = L - \frac{1}{2} \Delta L$

Or

$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2} \cdot \frac{\Delta L}{L}$$

We have

$$\Delta L = \left[\frac{2 \epsilon (V_{DS} - V_{DS}(sat))}{eN_d} \right]^{1/2}$$

and

For $V_{GS} = 0$, $V_{DS}(sat) = V_{PO} - V_{bi}$

We find

$$\begin{aligned} V_{PO} &= \frac{ea^2 N_d}{2 \epsilon} \\ &= \frac{(1.6 \times 10^{-19})(0.4 \times 10^{-4})^2 (3 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})} \end{aligned}$$

or

$$V_{PO} = 3.71 V$$

and

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{19})(3 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.902 V$$

so that

$$V_{DS}(sat) = 3.71 - 0.902 = 2.81 V$$

Then

$$\Delta L = \left[\frac{2(11.7)(8.85 \times 10^{-14})(5 - 2.81)}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$\Delta L = 0.307 \mu m$$

Now

$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2} \cdot \frac{\Delta L}{L}$$

or

$$\frac{1}{2} \cdot \frac{\Delta L}{L} = 1 - 0.9 = 0.10$$

so

$$L = \frac{\Delta L}{2(0.10)} = \frac{0.307 \times 10^{-4}}{2(0.10)}$$

or

$$\underline{L = 1.54 \mu m}$$

13.27

We have that $I'_{D1} = I_{D1} \left(\frac{L}{L - (1/2)\Delta L} \right)$

Assuming that we are in the saturation region, then $I'_{D1} = I'_{D1}(sat)$ and $I_{D1} = I_{D1}(sat)$. We can write

$$I'_{D1}(sat) = I_{D1}(sat) \cdot \frac{1}{1 - \frac{1}{2} \cdot \frac{\Delta L}{L}}$$

If $\Delta L \ll L$, then

$$I'_{D1}(sat) = I_{D1}(sat) \left[1 + \frac{1}{2} \cdot \frac{\Delta L}{L} \right]$$

We have that

$$\begin{aligned} \Delta L &= \left[\frac{2 \in (V_{DS} - V_{DS}(sat))}{eN_d} \right]^{1/2} \\ &= \left[\frac{2 \in V_{DS} \left(1 - \frac{V_{DS}(sat)}{V_{DS}} \right)}{eN_d} \right]^{1/2} \end{aligned}$$

which can be written as

$$\Delta L = V_{DS} \left[\frac{2 \in}{eN_d V_{DS}} \left(1 - \frac{V_{DS}(sat)}{V_{DS}} \right) \right]^{1/2}$$

If we write

$$I'_{D1}(sat) = I_{D1}(sat) (1 + \lambda V_{DS})$$

then by comparing equations, we have

$$\lambda = \frac{1}{2L} \left[\frac{2 \in}{eN_d V_{DS}} \left(1 - \frac{V_{DS}(sat)}{V_{DS}} \right) \right]^{1/2}$$

The parameter is not independent of V_{DS} . Define

$$x = \frac{V_{DS}}{V_{DS}(sat)} \text{ and consider the function}$$

$$f = \frac{1}{x} \left(1 - \frac{1}{x} \right) \text{ which is directly proportional to}$$

λ . We find that

x	$f(x)$
1.5	0.222
1.75	0.245
2.0	0.250
2.25	0.247
2.50	0.240
2.75	0.231
3.0	0.222

So that λ is nearly a constant.

13.28

(a) Saturation occurs when $E = 1 \times 10^4 \text{ V/cm}$

As a first approximation, let

$$E = \frac{V_{DS}}{L}$$

Then

$$V_{DS} = E \cdot L = (1 \times 10^4)(2 \times 10^{-4})$$

or

$$V_{DS} = 2 \text{ V}$$

(b)

We have that

$$h_2 = h_{sat} = \left[\frac{2 \in (V_{bi} + V_{DS} - V_{GS})}{eN_d} \right]^{1/2}$$

and

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(4 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.892 \text{ V}$$

For $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.892 + 2)}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2}$$

or

$$h_{sat} = 0.306 \text{ } \mu\text{m}$$

(c)

We then find

$$\begin{aligned} I_{D1}(sat) &= eN_d v_{sat} (a - h_{sat}) W \\ &= (1.6 \times 10^{-19})(4 \times 10^{16})(10^7)(0.50 - 0.306) \\ &\quad \times (10^{-4})(30 \times 10^{-4}) \end{aligned}$$

or

$$I_{D1}(sat) = 3.72 \text{ mA}$$

(d)

For $V_{GS} = 0$, we have

$$I_{D1}(sat) = I_{P1} \left[1 - 3 \left(\frac{V_{bi}}{V_{PO}} \right) \right] \left(1 - \frac{2}{3} \sqrt{\frac{V_{bi}}{V_{PO}}} \right)$$

Now

$$I_{P1} = \frac{\mu_n (eN_d)^2 W a^3}{6 \in L}$$

$$= \frac{(1000) \left[(1.6 \times 10^{-19}) (4 \times 10^{16}) \right]^2}{6(11.7)(8.85 \times 10^{-14})} \times \frac{(30 \times 10^{-4})(0.5 \times 10^{-4})^3}{(2 \times 10^{-4})}$$

or

$$I_{P1} = 12.4 \text{ mA}$$

Also

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon} = \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2 (4 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 7.73 \text{ V}$$

Then

$$I_{D1}(\text{sat}) = 12.4 \left[1 - 3 \left(\frac{0.892}{7.73} \right) \left(1 - \frac{2}{3} \sqrt{\frac{0.892}{7.73}} \right) \right]$$

or

$$I_{D1}(\text{sat}) = 9.08 \text{ mA}$$

13.29

(a) If $L = 1 \mu\text{m}$, then saturation will occur when

$$V_{DS} = E \cdot L = (10^4)(1 \times 10^{-4}) = 1 \text{ V}$$

We find

$$h_2 = h_{sat} = \left[\frac{2 \epsilon (V_{bi} + V_{DS} - V_{GS})}{e N_d} \right]^{1/2}$$

We have $V_{bi} = 0.892 \text{ V}$ and for $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.892 + 1)}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2}$$

or

$$h_{sat} = 0.247 \mu\text{m}$$

Then

$$\begin{aligned} I_{D1}(\text{sat}) &= e N_d v_{sat} (a - h_{sat}) W \\ &= (1.6 \times 10^{-19})(4 \times 10^{16})(10^7)(0.50 - 0.247) \\ &\quad \times (10^{-4})(30 \times 10^{-4}) \end{aligned}$$

or

$$I_{D1}(\text{sat}) = 4.86 \text{ mA}$$

If velocity saturation did not occur, then from the previous problem, we would have

$$I_{D1}(\text{sat}) = 9.08 \left(\frac{2}{1} \right) \Rightarrow I_{D1}(\text{sat}) = 18.2 \text{ mA}$$

(b)

If velocity saturation occurs, then the relation $I_{D1}(\text{sat}) \propto (1/L)$ does not apply.

13.30

(a)

$$v = \mu_n E = (8000)(5 \times 10^3) = 4 \times 10^7 \text{ cm/s}$$

Then

$$t_d = \frac{L}{v} = \frac{2 \times 10^{-4}}{4 \times 10^7} \Rightarrow$$

or

$$t_d = 5 \text{ ps}$$

(b)

Assume $v_{sat} = 10^7 \text{ cm/s}$

Then

$$t_d = \frac{L}{v_{sat}} = \frac{2 \times 10^{-4}}{10^7} \Rightarrow$$

$$t_d = 20 \text{ ps}$$

13.31

(a) $v = \mu_n E = (1000)(10^4) = 10^7 \text{ cm/s}$

$$t_d = \frac{L}{v} = \frac{2 \times 10^{-4}}{10^7} \Rightarrow t_d = 20 \text{ ps}$$

(b)

For $v_{sat} = 10^7 \text{ cm/s}$,

$$t_d = \frac{L}{v_{sat}} = \frac{2 \times 10^{-4}}{10^7} \Rightarrow t_d = 20 \text{ ps}$$

13.32

The reverse-bias current is dominated by the generation current. We have

$$V_P = V_{bi} - V_{PO}$$

We find

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{18})(3 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

or

$$V_{bi} = 0.884 \text{ V}$$

and

$$V_{PO} = \frac{ea^2 N_d}{2 \epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(0.3 \times 10^{-4})^2 (3 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 2.09 \text{ V}$$

Then

$$V_p = 0.884 - 2.09 = -1.21 = V_{GS}$$

Now

$$x_n = \left[\frac{2 \epsilon (V_{bi} + V_{DS} - V_{GS})}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.884 + V_{DS} - (-1.21))}{(1.6 \times 10^{-19})(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_n = \left[(4.31 \times 10^{-10})(2.09 + V_{DS}) \right]^{1/2}$$

(a)

$$\text{For } V_{DS} = 0 \Rightarrow x_n = 0.30 \text{ } \mu\text{m}$$

(b)

$$\text{For } V_{DS} = 1 \text{ V} \Rightarrow x_n = 0.365 \text{ } \mu\text{m}$$

(c)

$$\text{For } V_{DS} = 5 \text{ V} \Rightarrow x_n = 0.553 \text{ } \mu\text{m}$$

The depletion region volume is

$$Vol = (a) \left(\frac{L}{2} \right) (W) + (x_n)(2a)(W)$$

$$= (0.3 \times 10^{-4}) \left(\frac{2.4 \times 10^{-4}}{2} \right) (30 \times 10^{-4})$$

$$+ (x_n)(0.6 \times 10^{-4})(30 \times 10^{-4})$$

or

$$Vol = 10.8 \times 10^{-12} + x_n (18 \times 10^{-8})$$

(a)

$$\text{For } V_{DS} = 0 \Rightarrow Vol = 1.62 \times 10^{-11} \text{ cm}^3$$

(b)

$$\text{For } V_{DS} = 1 \text{ V} \Rightarrow Vol = 1.74 \times 10^{-11} \text{ cm}^3$$

(c)

$$\text{For } V_{DS} = 5 \text{ V} \Rightarrow Vol = 2.08 \times 10^{-11} \text{ cm}^3$$

The generation current is

$$I_{DG} = e \left(\frac{n_i}{2\tau_o} \right) \cdot Vol = \frac{(1.6 \times 10^{-19})(1.5 \times 10^{10})}{2(5 \times 10^{-8})} \cdot Vol$$

or

$$I_{DG} = (2.4 \times 10^{-2}) \cdot Vol$$

(a)

$$\text{For } V_{DS} = 0 \Rightarrow I_{DG} = 0.39 \text{ pA}$$

(b)

$$\text{For } V_{DS} = 1 \text{ V} \Rightarrow I_{DG} = 0.42 \text{ pA}$$

(c)

$$\text{For } V_{DS} = 5 \text{ V} \Rightarrow I_{DG} = 0.50 \text{ pA}$$

13.33

(a) The ideal transconductance for $V_{GS} = 0$ is

$$g_{mS} = G_{O1} \left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}} \right)$$

where

$$G_{O1} = \frac{e\mu_n N_d W a}{L}$$

$$= \frac{(1.6 \times 10^{-19})(4500)(7 \times 10^{16})}{1.5 \times 10^{-4}}$$

$$\times (5 \times 10^{-4})(0.3 \times 10^{-4})$$

or

$$G_{O1} = 5.04 \text{ mS}$$

We find

$$V_{PO} = \frac{e a^2 N_d}{2 \epsilon}$$

$$= \frac{(1.6 \times 10^{-19})(0.3 \times 10^{-4})^2 (7 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 4.35 \text{ V}$$

We have

$$\phi_n = (0.0259) \ln \left(\frac{4.7 \times 10^{17}}{7 \times 10^{16}} \right) = 0.049 \text{ V}$$

so that

$$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.049 = 0.841 \text{ V}$$

Then

$$g_{mS} = 5.04 \left(1 - \sqrt{\frac{0.841}{4.35}} \right)$$

or

$$g_{mS} = 2.82 \text{ mS}$$

(b)

With a source resistance

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = \frac{1}{1 + g_m r_s}$$

For

$$\frac{g'_m}{g_m} = 0.80 = \frac{1}{1 + (2.82) r_s}$$

which yields

$$\underline{r_s = 88.7 \, \Omega}$$

(c)

$$r_s = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e\mu_n n)(0.3 \times 10^{-4})(5 \times 10^{-4})}$$

so

$$L = (88.7)(1.6 \times 10^{-19})(4500)(7 \times 10^{16}) \times (0.3 \times 10^{-4})(5 \times 10^{-4})$$

or

$$\underline{L = 0.67 \, \mu m}$$

13.34

$$f_T = \frac{g_m}{2\pi C_G}$$

where

$$C_G = \frac{\epsilon WL}{a} = \frac{(13.1)(8.85 \times 10^{-14})(5 \times 10^{-4})(1.5 \times 10^{-4})}{0.3 \times 10^{-4}}$$

or

$$C_G = 2.9 \times 10^{-15} \, F$$

We must use g'_m , so we obtain

$$f_T = \frac{(2.82 \times 10^{-3})(0.80)}{2\pi(2.9 \times 10^{-15})} = 124 \, GHz$$

We have

$$f_T = \frac{1}{2\pi\tau_c} \Rightarrow \tau_c = \frac{1}{2\pi f_T} = \frac{1}{2\pi(124 \times 10^9)}$$

or

$$\tau_c = 1.28 \times 10^{-12} \, s$$

The channel transit time is

$$t_i = \frac{1.5 \times 10^{-4}}{10^7} = 1.5 \times 10^{-11} \, s$$

The total time constant is

$$\tau = 1.5 \times 10^{-11} + 1.28 \times 10^{-12} = 1.63 \times 10^{-11} \, s$$

so that

$$f_T = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.63 \times 10^{-11})}$$

or

$$\underline{f_T = 9.76 \, GHz}$$

13.35

(a) For a constant mobility

$$f_T = \frac{e\mu_n N_d a^2}{2\pi \epsilon L^2} = \frac{(1.6 \times 10^{-19})(5500)(10^{17})(0.25 \times 10^{-4})^2}{2\pi(13.1)(8.85 \times 10^{-14})(10^{-4})^2}$$

or

$$\underline{f_T = 755 \, GHz}$$

(b)

Saturation velocity model:

$$f_T = \frac{v_{sat}}{2\pi L}$$

Assuming $v_{sat} = 10^7 \, cm/s$, we find

$$f_T = \frac{10^7}{2\pi(10^{-4})}$$

or

$$\underline{f_T = 15.9 \, GHz}$$

13.36

$$(a) \, V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$$

where

$$V_{P2} = \frac{eN_d d_d^2}{2\epsilon_N} = \frac{(1.6 \times 10^{-19})(3 \times 10^{18})(350 \times 10^{-8})^2}{2(12.2)(8.85 \times 10^{-14})}$$

or

$$V_{P2} = 2.72 \, V$$

Then

$$V_{off} = 0.89 - 0.24 - 2.72$$

or

$$\underline{V_{off} = -2.07 \, V}$$

(b)

$$n_s = \frac{\epsilon_N}{e(d + \Delta d)}(V_g - V_{off})$$

For $V_g = 0$, we have

$$n_s = \frac{(12.2)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(350 + 80) \cdot 10^{-8}} (2.07)$$

or

$$\underline{n_s = 3.25 \times 10^{12} \, cm^{-2}}$$

13.37

(a) We have

$$I_D(sat) = \frac{\epsilon_N W}{(d + \Delta d)} (V_g - V_{off} - V_o) v_s$$

We find

$$\begin{aligned} \left(\frac{g_{mS}}{W} \right) &= \frac{\partial}{\partial V_g} \left[\frac{I_D(sat)}{W} \right] = \frac{\epsilon_N v_s}{(d + \Delta d)} \\ &= \frac{(12.2)(8.85 \times 10^{-14})(2 \times 10^7)}{(350 + 80) \cdot 10^{-8}} = 5.02 \frac{S}{cm} \end{aligned}$$

or

$$\frac{g_{mS}}{W} = 502 \frac{mS}{mm}$$

(b)

At $V_g = 0$, we obtain

$$\begin{aligned} \frac{I_D(sat)}{W} &= \frac{\epsilon_N}{(d + \Delta d)} (-V_{off} - V_o) v_s \\ &= \frac{(12.2)(8.85 \times 10^{-14})}{(350 + 80) \cdot 10^{-8}} (2.07 - 1)(2 \times 10^7) \end{aligned}$$

or

$$\frac{I_D(sat)}{W} = 5.37 \text{ A/cm} = 537 \text{ mA/mm}$$

13.38

$$V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$$

We want $V_{off} = -0.3 \text{ V}$, so

$$-0.30 = 0.85 - 0.22 - V_{P2}$$

or

$$V_{P2} = 0.93 \text{ V} = \frac{e N_d d_d^2}{2 \epsilon_N}$$

We can then write

$$\begin{aligned} d_d^2 &= \frac{2 \epsilon_N V_{P2}}{e N_d} \\ &= \frac{2(12.2)(8.85 \times 10^{-14})(0.93)}{(1.6 \times 10^{-19})(2 \times 10^{18})} \end{aligned}$$

We then obtain

$$d_d = 2.51 \times 10^{-6} \text{ cm} = 251 \text{ \AA}$$

Chapter 14

Problem Solutions

14.1

(a) $\lambda = \frac{1.24}{E} \mu m$

Then

Ge: $E_g = 0.66 \text{ eV} \Rightarrow \lambda = 1.88 \mu m$

Si: $E_g = 1.12 \text{ eV} \Rightarrow \lambda = 1.11 \mu m$

GaAs: $E_g = 1.42 \text{ eV} \Rightarrow \lambda = 0.873 \mu m$

(b)

$$E = \frac{1.24}{\lambda}$$

For $\lambda = 570 \text{ nm} \Rightarrow E = 2.18 \text{ eV}$

For $\lambda = 700 \text{ nm} \Rightarrow E = 1.77 \text{ eV}$

14.2

(a) GaAs

$h\nu = 2 \text{ eV} \Rightarrow \lambda = 0.62 \mu m$

so

$$\alpha \approx 1.5 \times 10^4 \text{ cm}^{-1}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp[-(1.5 \times 10^4)(0.35 \times 10^{-4})]$$

or

$$\frac{I(x)}{I_o} = 0.59$$

so the percent absorbed is (1-0.59), or
41%

(b) Silicon

Again $h\nu = 2 \text{ eV} \Rightarrow \lambda = 0.62 \mu m$

So

$$\alpha \approx 4 \times 10^3 \text{ cm}^{-1}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp[-(4 \times 10^3)(0.35 \times 10^{-4})]$$

or

$$\frac{I(x)}{I_o} = 0.87$$

so the percent absorbed is (1-0.87), or
13%

14.3

$$g' = \frac{\alpha I(x)}{h\nu}$$

For $h\nu = 1.3 \text{ eV} \Rightarrow \lambda = \frac{1.24}{1.3} = 0.95 \mu m$

For silicon, $\alpha \approx 3 \times 10^2 \text{ cm}^{-1}$,

Then for

$$I(x) = 10^{-2} \text{ W / cm}^2$$

we obtain

$$g' = \frac{(3 \times 10^2)(10^{-2})}{(1.6 \times 10^{-19})(1.3)} \Rightarrow$$

$$g' = 1.44 \times 10^{19} \text{ cm}^{-3} \text{ s}^{-1}$$

The excess concentration is

$$\delta n = g' \tau = (1.44 \times 10^{19})(10^{-6}) \Rightarrow$$

$$\delta n = 1.44 \times 10^{13} \text{ cm}^{-3}$$

14.4

n-type GaAs, $\tau = 10^{-7} \text{ s}$

(a)

We want

$$\delta n = \delta p = 10^{15} \text{ cm}^{-3} = g' \tau = g'(10^{-7})$$

or

$$g' = \frac{10^{15}}{10^{-7}} = 10^{22} \text{ cm}^{-3} \text{ s}^{-1}$$

We have

$$h\nu = 1.9 \text{ eV} \Rightarrow \lambda = \frac{1.24}{1.9} = 0.65 \mu m$$

so that

$$\alpha \approx 1.3 \times 10^4 \text{ cm}^{-1}$$

Then

$$g' = \frac{\alpha I(x)}{h\nu} \Rightarrow I(x) = \frac{(g')(h\nu)}{\alpha}$$

$$= \frac{(10^{22})(1.6 \times 10^{-19})(1.9)}{1.3 \times 10^4}$$

or

$$I(0) = 0.234 \text{ W / cm}^2 = I_o$$

(b)

$$\frac{I(x)}{I_o} = 0.20 = \exp[-(1.3 \times 10^4)x]$$

We obtain $x = 1.24 \mu m$

14.5

GaAs

(a)

For $h\nu = 1.65 \text{ eV} \Rightarrow \lambda = 0.75 \mu\text{m}$

So

$$\alpha \approx 0.7 \times 10^4 \text{ cm}^{-1}$$

For 75% absorbed,

$$\frac{I(x)}{I_o} = 0.25 = \exp(-\alpha x)$$

Then

$$\alpha x = \ln\left(\frac{1}{0.25}\right) \Rightarrow x = \frac{1}{0.7 \times 10^4} \ln\left(\frac{1}{0.25}\right)$$

or

$$x = 1.98 \mu\text{m}$$

(b)

For 75% transmitted,

$$\frac{I(x)}{I_o} = 0.75 = \exp[-(0.7 \times 10^4)x]$$

we obtain

$$x = 0.41 \mu\text{m}$$

14.6

GaAs

For $x = 1 \mu\text{m} = 10^{-4} \text{ cm}$, we have 50% absorbed or 50% transmitted, then

$$\frac{I(x)}{I_o} = 0.50 = \exp(-\alpha x)$$

We can write

$$\alpha = \left(\frac{1}{x}\right) \cdot \ln\left(\frac{1}{0.5}\right) = \left(\frac{1}{10^{-4}}\right) \cdot \ln(2)$$

or

$$\alpha = 0.69 \times 10^4 \text{ cm}^{-1}$$

This value corresponds to

$$\lambda = 0.75 \mu\text{m}, E = 1.65 \text{ eV}$$

14.7

The ambipolar transport equation for minority carrier holes in steady state is

$$D_p \frac{d^2(\delta p_n)}{dx^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p^2 = D_p \tau_p$

The photon flux in the semiconductor is

$$\Phi(x) = \Phi_o \exp(-\alpha x)$$

and the generation rate is

$$G_L = \alpha \Phi(x) = \alpha \Phi_o \exp(-\alpha x)$$

so we have

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{\alpha \Phi_o}{D_p} \exp(-\alpha x)$$

The general solution is of the form

$$\delta p_n = A \exp\left(\frac{-x}{L_p}\right) + B \exp\left(\frac{+x}{L_p}\right) - \frac{\alpha \Phi_o \tau_p}{\alpha^2 L_p^2 - 1} \exp(-\alpha x)$$

At $x \rightarrow \infty$, $\delta p_n = 0$

So that $B = 0$, then

$$\delta p_n = A \exp\left(\frac{-x}{L_p}\right) - \frac{\alpha \Phi_o \tau_p}{\alpha^2 L_p^2 - 1} \exp(-\alpha x)$$

At $x = 0$, we have

$$D_p \frac{d(\delta p_n)}{dx} \Big|_{x=0} = s \delta p_n \Big|_{x=0}$$

so we can write

$$\delta p_n \Big|_{x=0} = A - \frac{\alpha \Phi_o \tau_p}{\alpha^2 L_p^2 - 1}$$

and

$$\frac{d(\delta p_n)}{dx} \Big|_{x=0} = -\frac{A}{L_p} + \frac{\alpha^2 \Phi_o \tau_p}{\alpha^2 L_p^2 - 1}$$

Then we have

$$-\frac{AD_p}{L_p} + \frac{\alpha^2 \Phi_o \tau_p D_p}{\alpha^2 L_p^2 - 1} = sA - \frac{s \alpha \Phi_o \tau_p}{\alpha^2 L_p^2 - 1}$$

Solving for A , we find

$$A = \frac{\alpha \Phi_o \tau_p}{\alpha^2 L_p^2 - 1} \cdot \left[\frac{s + \alpha D_p}{s + (D_p/L_p)} \right]$$

The solution can now be written as

$$\delta p_n = \frac{\alpha \Phi_o \tau_p}{\alpha^2 L_p^2 - 1} \cdot \left\{ \frac{s + \alpha D_p}{s + (D_p/L_p)} \cdot \exp\left(\frac{-x}{L_p}\right) - \exp(-\alpha x) \right\}$$

14.8

We have

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where $L_n^2 = D_n \tau_n$

The general solution can be written in the form

$$\delta n_p = A \cosh\left(\frac{x}{L_n}\right) + B \sinh\left(\frac{x}{L_n}\right) + G_L \tau_n$$

For $s = \infty$ at $x = 0$ means that $\delta n_p(0) = 0$,

Then

$$0 = A + G_L \tau_n \Rightarrow A = -G_L \tau_n$$

At $x = W$,

$$-D_n \left. \frac{d(\delta n_p)}{dx} \right|_{x=W} = s_o \delta n_p \Big|_{x=W}$$

Now

$$\delta n_p(W) = -G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B \sinh\left(\frac{W}{L_n}\right) + G_L \tau_n$$

and

$$\left. \frac{d(\delta n_p)}{dx} \right|_{x=W} = -\frac{G_L \tau_n}{L_n} \sinh\left(\frac{W}{L_n}\right) + \frac{B}{L_n} \cosh\left(\frac{W}{L_n}\right)$$

so we can write

$$\begin{aligned} & \frac{G_L \tau_n D_n}{L_n} \sinh\left(\frac{W}{L_n}\right) - \frac{B D_n}{L_n} \cosh\left(\frac{W}{L_n}\right) \\ &= s_o \left[-G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B \sinh\left(\frac{W}{L_n}\right) + G_L \tau_n \right] \end{aligned}$$

Solving for B , we find

$$B = \frac{G_L \left[L_n \sinh\left(\frac{W}{L_n}\right) + s_o \tau_n \cosh\left(\frac{W}{L_n}\right) - s_o \tau_n \right]}{\frac{D_n}{L_n} \cosh\left(\frac{W}{L_n}\right) + s_o \sinh\left(\frac{W}{L_n}\right)}$$

The solution is then

$$\delta n_p = G_L \tau_n \left[1 - \cosh\left(\frac{x}{L_n}\right) \right] + B \sinh\left(\frac{x}{L_n}\right)$$

where B was just given.

14.9

$$\begin{aligned} V_{oc} &= V_t \ln \left(1 + \frac{J_L}{J_s} \right) \\ &= (0.0259) \ln \left(1 + \frac{30 \times 10^{-3}}{J_s} \right) \end{aligned}$$

where

$$J_s = e n_i^2 \left[\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_p}} \right]$$

which becomes

$$\begin{aligned} J_s &= (1.6 \times 10^{-19}) (1.8 \times 10^6)^2 \\ &\times \left[\frac{1}{N_a} \cdot \sqrt{\frac{225}{5 \times 10^{-8}}} + \frac{1}{10^{19}} \cdot \sqrt{\frac{7}{5 \times 10^{-8}}} \right] \end{aligned}$$

or

$$J_s = (5.18 \times 10^{-7}) \left[\frac{6.7 \times 10^4}{N_a} + 1.18 \times 10^{-15} \right]$$

Then

$\frac{N_a}{\text{cm}^{-3}}$	$J_s (A / \text{cm}^2)$	$V_{oc} (V)$
1E15	3.47E-17	0.891
1E16	3.47E-18	0.950
1E17	3.48E-19	1.01
1E18	3.53E-20	1.07

14.10

(a)

$$I_L = J_L \cdot A = (25 \times 10^{-3})(2) = 50 \text{ mA}$$

We have

$$J_s = e n_i^2 \left[\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_p}} \right]$$

or

$$\begin{aligned} J_s &= (1.6 \times 10^{-19}) (1.5 \times 10^{10})^2 \\ &\times \left[\frac{1}{3 \times 10^{16}} \cdot \sqrt{\frac{18}{5 \times 10^{-6}}} + \frac{1}{10^{19}} \cdot \sqrt{\frac{6}{5 \times 10^{-7}}} \right] \end{aligned}$$

which becomes

$$J_s = 2.29 \times 10^{-12} A / \text{cm}^2$$

or

$$I_s = 4.58 \times 10^{-12} A$$

We have

$$I = I_L - I_s \left[\exp\left(\frac{V}{V_t}\right) - 1 \right]$$

or

$$I = 50 \times 10^{-3} - 4.58 \times 10^{-12} \left[\exp\left(\frac{V}{V_t}\right) - 1 \right]$$

We see that when $I = 0$, $V = V_{oc} = 0.599 V$.

We find

$V(V)$	$I(mA)$
0	50
0.1	50
0.2	50
0.3	50
0.4	49.9
0.45	49.8
0.50	48.9
0.55	42.4
0.57	33.5
0.59	14.2

(b)

The voltage at the maximum power point is found from

$$\left[1 + \frac{V_m}{V_t} \right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_s}$$

$$= 1 + \frac{50 \times 10^{-3}}{4.58 \times 10^{-12}} = 1.092 \times 10^{10}$$

By trial and error,

$$V_m = 0.520 V$$

At this point, we find

$$I_m = 47.6 mA$$

so the maximum power is

$$P_m = I_m V_m = (47.6)(0.520)$$

or

$$P_m = 24.8 mW$$

(c)

We have

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{V_m}{I_m} = \frac{0.520}{47.6 \times 10^{-3}}$$

or

$$R = 10.9 \Omega$$

14.11

If the solar intensity increases by a factor of 10, then I_L increases by a factor of 10 so that

$I_L = 500 mA$. Then

$$I = 500 \times 10^{-3} - 4.58 \times 10^{-12} \left[\exp\left(\frac{V}{V_t}\right) - 1 \right]$$

At the maximum power point

$$\left[1 + \frac{V_m}{V_t} \right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_s}$$

$$= 1 + \frac{500 \times 10^{-3}}{4.58 \times 10^{-12}} = 1.092 \times 10^{11}$$

By trial and error, we find

$$V_m = 0.577 V$$

and the current at the maximum power point is

$$I_m = 478.3 mA$$

The maximum power is then

$$P_m = I_m V_m = 276 mW$$

The maximum power has increased by a factor of 11.1 compared to the previous problem, which means that the efficiency has increased slightly.

14.12

Let $x = 0$ correspond to the edge of the space charge region in the p-type material. Then

$$D_n \frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{\tau_n} = -G_L$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where

$$G_L = \alpha \Phi(x) = \alpha \Phi_o \exp(-\alpha x)$$

Then we have

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{\alpha \Phi_o}{D_n} \exp(-\alpha x)$$

The general solution is of the form

$$\delta n_p = A \exp\left(\frac{-x}{L_n}\right) + B \exp\left(\frac{+x}{L_p}\right)$$

$$- \frac{\alpha \Phi_o \tau_n}{\alpha^2 L_n^2 - 1} \exp(-\alpha x)$$

At $x \rightarrow \infty$, $\delta n_p = 0$ so that $B = 0$, then

$$\delta n_p = A \exp\left(\frac{-x}{L_n}\right) - \frac{\alpha \Phi_o \tau_n}{\alpha^2 L_n^2 - 1} \exp(-\alpha x)$$

We also have $\delta n_p(0) = 0 = A - \frac{\alpha \Phi_o \tau_n}{\alpha^2 L_n^2 - 1}$,

which yields

$$A = \frac{\alpha \Phi_o \tau_n}{\alpha^2 L_n^2 - 1}$$

We then obtain

$$\delta n_p = \frac{\alpha \Phi_o \tau_n}{\alpha^2 L_n^2 - 1} \left[\exp\left(\frac{-x}{L_n}\right) - \exp(-\alpha x) \right]$$

where Φ_o is the incident flux at $x = 0$.

14.13

For 90% absorption, we have

$$\frac{\Phi(x)}{\Phi_o} = \exp(-\alpha x) = 0.10$$

Then

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

or

$$x = \left(\frac{1}{\alpha}\right) \cdot \ln(10)$$

For $h\nu = 1.7 \text{ eV}$, $\alpha \approx 10^4 \text{ cm}^{-1}$

Then

$$x = \left(\frac{1}{10^4}\right) \cdot \ln(10) \Rightarrow x = 2.3 \text{ } \mu\text{m}$$

and for $h\nu = 2.0 \text{ eV}$, $\alpha \approx 10^5 \text{ cm}^{-1}$, so that

$$x = 0.23 \text{ } \mu\text{m}$$

14.14

$G_L = 10^{20} \text{ cm}^{-3} \text{ s}^{-1}$ and $N_d > N_a$ so holes are the minority carrier.

(a)

$$\delta p = g' \tau = G_L \tau_p$$

so that

$$\delta p = \delta n = (10^{20})(10^{-7})$$

or

$$\delta p = \delta n = 10^{13} \text{ cm}^{-3}$$

(b)

$$\begin{aligned} \Delta \sigma &= e(\delta p)(\mu_n + \mu_p) \\ &= (1.6 \times 10^{-19})(10^{13})(1000 + 430) \end{aligned}$$

or

$$\Delta \sigma = 2.29 \times 10^{-3} (\Omega - \text{cm})^{-1}$$

(c)

$$\begin{aligned} I_L &= J_L \cdot A = \frac{(\Delta \sigma)AV}{L} \\ &= \frac{(2.29 \times 10^{-3})(10^{-3})(5)}{100 \times 10^{-4}} \end{aligned}$$

or

$$I_L = 1.15 \text{ mA}$$

(d)

The photoconductor gain is

$$\Gamma_{ph} = \frac{\tau_p}{t_n} \left(1 + \frac{\mu_p}{\mu_n} \right)$$

where

$$t_n = \frac{L}{\mu_n E} = \frac{L^2}{\mu_n V}$$

Then

$$\Gamma_{ph} = \frac{\tau_p \mu_n V}{L^2} \left(1 + \frac{\mu_p}{\mu_n} \right) = \frac{\tau_p V}{L^2} (\mu_n + \mu_p)$$

or

$$\Gamma_{ph} = \frac{(10^{-7})(5)}{(100 \times 10^{-4})^2} (1000 + 430)$$

or

$$\Gamma_{ph} = 7.15$$

14.15

n-type, so holes are the minority carrier

(a)

$$\delta p = G_L \tau_p = (10^{21})(10^{-8})$$

so that

$$\delta p = \delta n = 10^{13} \text{ cm}^{-3}$$

(b)

$$\begin{aligned} \Delta \sigma &= e(\delta p)(\mu_n + \mu_p) \\ &= (1.6 \times 10^{-19})(10^{13})(8000 + 250) \end{aligned}$$

or

$$\Delta \sigma = 1.32 \times 10^{-2} (\Omega - \text{cm})^{-1}$$

(c)

$$\begin{aligned} I_L &= J_L \cdot A = (\Delta \sigma)AE = \frac{(\Delta \sigma)AV}{L} \\ &= \frac{(1.32 \times 10^{-2})(10^{-4})(5)}{100 \times 10^{-4}} \end{aligned}$$

$$\text{or } I_L = 0.66 \text{ mA}$$

(d)

$$\begin{aligned} \Gamma_{ph} &= \frac{\tau_p}{t_n} \left(1 + \frac{\mu_p}{\mu_n} \right) = \frac{\tau_p V}{L^2} (\mu_n + \mu_p) \\ &= \frac{(10^{-8})(5)}{(100 \times 10^{-4})^2} (8000 + 250) \end{aligned}$$

$$\text{or } \Gamma_{ph} = 4.13$$

14.16

$$\Phi(x) = \Phi_o \exp(-\alpha x)$$

The electron-hole generation rate is

$$g' = \alpha \Phi(x) = \alpha \Phi_o \exp(-\alpha x)$$

and the excess carrier concentration is

$$\delta p = \tau_p \alpha \Phi(x)$$

Now

$$\Delta \sigma = e(\delta p)(\mu_n + \mu_p)$$

and

$$J_L = \Delta \sigma E$$

The photocurrent is now found from

$$\begin{aligned} I_L &= \iint \Delta \sigma E \cdot dA = \int_0^W dy \int_0^{x_0} \Delta \sigma E \cdot dx \\ &= We(\mu_n + \mu_p)E \int_0^{x_0} \delta p \cdot dx \end{aligned}$$

Then

$$\begin{aligned} I_L &= We(\mu_n + \mu_p)E \alpha \Phi_o \tau_p \int_0^{x_0} \exp(-\alpha x) dx \\ &= We(\mu_n + \mu_p)E \alpha \Phi_o \tau_p \left[-\frac{1}{\alpha} \exp(-\alpha x) \right]_0^{x_0} \end{aligned}$$

which becomes

$$I_L = We(\mu_n + \mu_p)E \Phi_o \tau_p [1 - \exp(-\alpha x_0)]$$

Now

$$\begin{aligned} I_L &= (50 \times 10^{-4})(1.6 \times 10^{-19})(1200 + 450)(50) \\ &\quad \times (10^{16})(2 \times 10^{-7}) [1 - \exp(-(5 \times 10^4)(10^{-4}))] \end{aligned}$$

or

$$I_L = 0.131 \mu A$$

14.17

(a)

$$V_{bi} = (0.0259) \ln \left[\frac{(2 \times 10^{16})(10^{18})}{(1.5 \times 10^{10})^2} \right] = 0.832 V$$

The space charge width is

$$\begin{aligned} W &= \left[\frac{2 \in (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.832 + 5)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{2 \times 10^{16} + 10^{18}}{(2 \times 10^{16})(10^{18})} \right) \right]^{1/2} \end{aligned}$$

or

$$W = 0.620 \mu m$$

The prompt photocurrent density is

$$J_{L1} = e G_L W = (1.6 \times 10^{-19})(10^{21})(0.620 \times 10^{-4})$$

or

$$J_{L1} = 9.92 mA / cm^2$$

(b)

The total steady-state photocurrent density is

$$J_L = e(W + L_n + L_p)G_L$$

We find

$$L_n = \sqrt{D_n \tau_n} = \sqrt{(25)(2 \times 10^{-7})} = 22.4 \mu m$$

and

$$L_p = \sqrt{D_p \tau_p} = \sqrt{(10)(10^{-7})} = 10.0 \mu m$$

Then

$$J_L = (1.6 \times 10^{-19})(0.62 + 22.4 + 10.0)(10^{-4})(10^{21})$$

or

$$J_L = 0.528 A / cm^2$$

14.18

In the n-region under steady state and for $E = 0$, we have

$$D_p \frac{d^2(\delta p_n)}{dx'^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx'^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p^2 = D_p \tau_p$ and where x' is positive in the negative x direction. The homogeneous solution is found from

$$\frac{d^2(\delta p_{nh})}{dx'^2} - \frac{\delta p_{nh}}{L_p^2} = 0$$

The general solution is found to be

$$\delta p_{nh} = A \exp\left(\frac{-x'}{L_p}\right) + B \exp\left(\frac{+x'}{L_p}\right)$$

The particular solution is found from

$$\frac{-\delta p_{np}}{L_p^2} = \frac{-G_L}{D_p}$$

which yields

$$\delta p_{np} = \frac{G_L L_p^2}{D_p} = G_L \tau_p$$

The total solution is the sum of the homogeneous and particular solutions, so we have

$$\delta p_n = A \exp\left(\frac{-x'}{L_p}\right) + B \exp\left(\frac{+x'}{L_p}\right) + G_L \tau_p$$

One boundary condition is that δp_n remains finite as $x' \rightarrow \infty$ which means that $B = 0$. Then At $x' = 0$, $p_n(0) = 0 = \delta p_n(0) + p_{n0}$, so that

$$\delta p_n(0) = -p_{n0}$$

We find that

$$A = -(p_{n0} + G_L \tau_p)$$

The solution is then written as

$$\delta p_n = G_L \tau_p - (G_L \tau_p + p_{n0}) \exp\left(\frac{-x'}{L_p}\right)$$

The diffusion current density is found as

$$J_p = -eD_p \frac{d(\delta p_n)}{dx} \Big|_{x'=0}$$

But

$$\frac{d(\delta p_n)}{dx} = - \frac{d(\delta p_n)}{dx'}$$

since x and x' are in opposite directions.

So

$$\begin{aligned} J_p &= +eD_p \frac{d(\delta p_n)}{dx'} \Big|_{x'=0} \\ &= eD_p \left[-(G_L \tau_p + p_{n0}) \right] \left[\left(\frac{-1}{L_p} \right) \exp\left(\frac{-x'}{L_p}\right) \right] \Big|_{x'=0} \end{aligned}$$

Then

$$J_p = eG_L L_p + \frac{eD_p p_{n0}}{L_p}$$

14.19

We have

$$\begin{aligned} J_L &= e\Phi_o [1 - \exp(-\alpha W)] \\ &= (1.6 \times 10^{-19})(10^{17}) [1 - \exp(-(3 \times 10^3)W)] \end{aligned}$$

or

$$J_L = 16 [1 - \exp(-(3 \times 10^3)W)] \text{ (mA)}$$

Then for $W = 1 \mu\text{m} = 10^{-4} \text{ cm}$, we find

$$J_L = 4.15 \text{ mA}$$

$$\text{For } W = 10 \mu\text{m} \Rightarrow J_L = 15.2 \text{ mA}$$

$$\text{For } W = 100 \mu\text{m} \Rightarrow J_L = 16 \text{ mA}$$

14.20

The minimum α occurs when $\lambda = 1 \mu\text{m}$ which gives $\alpha = 10^2 \text{ cm}^{-1}$. We want

$$\frac{\Phi(x')}{\Phi_o} = \exp(-\alpha x) = 0.10$$

which can be written as

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

Then

$$x = \frac{1}{\alpha} \ln(10) = \frac{1}{10^2} \ln(10)$$

or

$$x = 230 \mu\text{m}$$

14.21

For the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ system, a direct bandgap for $0 \leq x \leq 0.45$, we have

$$E_g = 1.424 + 1.247x$$

At $x = 0.45$, $E_g = 1.985 \text{ eV}$, so for the direct

bandgap

$$1.424 \leq E_g \leq 1.985 \text{ eV}$$

which yields

$$0.625 \leq \lambda \leq 0.871 \mu\text{m}$$

14.22

For $x = 0.35$ in $\text{GaAs}_{1-x}\text{P}_x$, we find

$$(a) E_g = 1.85 \text{ eV} \quad \text{and} \quad (b) \lambda = 0.670 \mu\text{m}$$

14.23

(a)

For GaAs, $\bar{n}_2 = 3.66$ and for air, $\bar{n}_1 = 1.0$.

The critical angle is

$$\theta_c = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.9^\circ$$

The fraction of photons that will not experience total internal reflection is

$$\frac{2\theta_c}{360} = \frac{2(15.9)}{360} \Rightarrow \underline{8.83\%}$$

(b)

Fresnel loss:

$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1} \right)^2 = \left(\frac{3.66 - 1}{3.66 + 1} \right)^2 = 0.326$$

The fraction of photons emitted is then

$$(0.0883)(1 - 0.326) = 0.0595 \Rightarrow \underline{5.95\%}$$

14.24

We can write the external quantum efficiency as

$$\eta_{ext} = T_1 \cdot T_2$$

where $T_1 = 1 - R_1$ with R_1 is the reflection

coefficient (Fresnel loss), and the factor T_2 is the fraction of photons that do not experience total internal reflection. We have

$$R_1 = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1} \right)^2$$

so that

$$T_1 = 1 - R_1 = 1 - \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1} \right)^2$$

which reduces to

$$T_1 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}$$

Now consider a solid angle from the source point. The surface area described by the solid angle is πp^2 . The factor T_1 is given by

$$T_1 = \frac{\pi p^2}{4\pi R^2}$$

From the geometry, we have

$$\sin\left(\frac{\theta_c}{2}\right) = \frac{p/2}{R} \Rightarrow p = 2R \sin\left(\frac{\theta_c}{2}\right)$$

Then the area is

$$A = \pi p^2 = 4R^2 \pi \sin^2\left(\frac{\theta_c}{2}\right)$$

Now

$$T_1 = \frac{\pi p^2}{4\pi R^2} = \sin^2\left(\frac{\theta_c}{2}\right)$$

From a trig identity, we have

$$\sin^2\left(\frac{\theta_c}{2}\right) = \frac{1}{2}(1 - \cos\theta_c)$$

Then

$$T_1 = \frac{1}{2}(1 - \cos\theta_c)$$

The external quantum efficiency is now

$$\eta_{ext} = T_1 \cdot T_2 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2} \cdot \frac{1}{2}(1 - \cos\theta_c)$$

or

$$\eta_{ext} = \frac{2\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}(1 - \cos\theta_c)$$

14.25

For an optical cavity, we have

$$N\left(\frac{\lambda}{2}\right) = L$$

If λ changes slightly, then N changes slightly also. We can write

$$\frac{N_1\lambda_1}{2} = \frac{(N_1 + 1)\lambda_2}{2}$$

Rearranging terms, we find

$$\frac{N_1\lambda_1}{2} - \frac{(N_1 + 1)\lambda_2}{2} = \frac{N_1\lambda_1}{2} - \frac{N_1\lambda_2}{2} - \frac{\lambda_2}{2} = 0$$

If we define $\Delta\lambda = \lambda_1 - \lambda_2$, then we have

$$\frac{N_1}{2} \Delta\lambda = \frac{\lambda_2}{2}$$

We can approximate $\lambda_2 = \lambda$, then

$$\frac{N_1\lambda}{2} = L \Rightarrow N_1 = \frac{2L}{\lambda}$$

Then

$$\frac{1}{2} \cdot \frac{2L}{\lambda} \Delta\lambda = \frac{\lambda}{2}$$

which yields

$$\Delta\lambda = \frac{\lambda^2}{2L}$$

14.26

For GaAs,

$$h\nu = 1.42 \text{ eV} \Rightarrow \lambda = \frac{1.24}{E} = \frac{1.24}{1.42}$$

or

$$\lambda = 0.873 \mu m$$

Then

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{(0.873 \times 10^{-4})^2}{2(0.75 \times 10^{-4})} = 5.08 \times 10^{-7} \text{ cm}$$

or

$$\Delta\lambda = 5.08 \times 10^{-3} \mu m$$

Chapter 15

Problem Solutions

15.1

The limit of low injection means that

$$n_B(0) = (0.1)N_B = (0.1)(10^{16}) = 10^{15} \text{ cm}^{-3}$$

Now

$$I_C = \frac{AeD_B n_B(0)}{x_B} = \frac{(0.5)(1.6 \times 10^{-19})(20)(10^{15})}{3 \times 10^{-4}}$$

or

$$I_C = 5.33 \text{ A}$$

15.2

From the junction breakdown curve, for $BV_{CBO} = 1000 \text{ V}$, we need the collector doping concentration to be $N_C \approx 2 \times 10^{14} \text{ cm}^{-3}$

Depletion width into the base (neglect V_{bi})

$$x_p = \left[\frac{2 \epsilon V_{BC}}{e} \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_C + N_B} \right) \right]^{1/2} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(1000)}{1.6 \times 10^{-19}} \times \left(\frac{2 \times 10^{14}}{5 \times 10^{15}} \right) \left(\frac{1}{5 \times 10^{15} + 2 \times 10^{14}} \right) \right]^{1/2}$$

or

$$x_p = 3.16 \text{ } \mu\text{m} \text{ (Minimum base width)}$$

Depletion width into the collector

$$x_n = \left[\frac{2(11.7)(8.85 \times 10^{-14})(1000)}{1.6 \times 10^{-19}} \times \left(\frac{5 \times 10^{15}}{2 \times 10^{14}} \right) \left(\frac{1}{5 \times 10^{15} + 2 \times 10^{14}} \right) \right]^{1/2}$$

or

$$x_n = 78.9 \text{ } \mu\text{m} \text{ (Minimum collector width)}$$

15.3

Compute plot

15.4

(a)

We have $\beta_{eff} = \beta_A \beta_B + \beta_A + \beta_B$

Then

$$180 = 25\beta_B + 25 + \beta_B$$

or

$$155 = 26\beta_B$$

which yields

$$\beta_B = 5.96$$

(b)

We have

$$\beta_B i_{EA} = i_{CB}$$

or

$$\beta_B \left(\frac{1 + \beta_A}{\beta_A} \right) i_{CA} = i_{CB}$$

so

$$(5.96) \left(\frac{1 + 25}{25} \right) i_{CA} = 20$$

which yields

$$i_{CA} = 3.23 \text{ A}$$

15.5

Sketch

15.6

We want

$$P_T = \frac{1}{2} I_{C,rated} \cdot \frac{V_{CC}}{2} \Rightarrow \frac{1}{2} I_{C,rated} \left(\frac{24}{2} \right) = 20$$

which yields

$$I_{C,rated} = 3.33 \text{ A}$$

Then

$$R_L = \frac{V_{CC}}{I_{C,rated}} = \frac{24}{3.33}$$

or

$$R_L = 7.2 \text{ } \Omega$$

15.7

If $V_{CC} = 25 \text{ V}$, then

$$I_C(\text{max}) = \frac{V_{CC}}{R_L} = \frac{25}{100} = 0.25 \text{ A} < I_{C,rated}$$

The power

$$P = I_C V_{CE} = I_C (V_{CC} - I_C R_L)$$

To find the maximum power point, set

$$\frac{dP}{dI_C} = 0 = V_{CC} - 2I_C R_L = 25 - I_C(2)(100)$$

which yields $I_C = 0.125 \text{ A}$

So

$$P(\text{max}) = (0.125)[25 - (0.125)(100)]$$

or

$$P(\text{max}) = 1.56 \text{ W} < P_T$$

So, maximum V_{CC} is $V_{CC} = 25 \text{ V}$

15.8

$$\text{Now } R_{on} = \frac{V_{DS}}{I_D}$$

Power dissipated in transistor

$$P = I_D V_{DS} = \frac{V_{DS}^2}{R_{on}}$$

We have

$$I_D = \frac{200 - V_{DS}}{100}$$

so we can write

$$P = \left(\frac{200 - V_{DS}}{100} \right) \cdot V_{DS} = \frac{V_{DS}^2}{R_{on}}$$

For $T = 25^\circ\text{C}$, $R_{on} = 2 \Omega$,

Then

$$\left(\frac{200 - V_{DS}}{100} \right) \cdot V_{DS} = \frac{V_{DS}^2}{2}$$

which yields

$$V_{DS} = 3.92 \text{ V and}$$

$$P = \left(\frac{200 - 3.92}{100} \right) (3.92) = 7.69 \text{ W}$$

We then have

T	R_{on}	V_{DS}	P
25	2.0	3.92	7.69
50	2.33	4.55	8.89
75	2.67	5.20	10.1
100	3	5.83	11.3

15.9

(a)

We have, for three devices in parallel,

$$\frac{V}{1.8} + \frac{V}{2} + \frac{V}{2.2} = 5 \Rightarrow V(1.51) = 5$$

or

$$V = 3.31 \text{ V}$$

Then, $I = \frac{V}{R}$, so that

$$I_1 = 1.839 \text{ A}$$

$$I_2 = 1.655 \text{ A}$$

$$I_3 = 1.505 \text{ A}$$

Now, $P = IV$, so

$$P_1 = 6.09 \text{ W}$$

$$P_2 = 5.48 \text{ W}$$

$$P_3 = 4.98 \text{ W}$$

(b)

Now

$$V \left(\frac{1}{1.8} + \frac{1}{3.6} + \frac{1}{2.2} \right) = 5 = V(1.288)$$

or

$$V = 3.88 \text{ V}$$

Then

$$I_1 = 2.16 \text{ A}, P_1 = 8.38 \text{ W}$$

$$I_2 = 1.08 \text{ A}, P_2 = 4.19 \text{ W}$$

$$I_3 = 1.77 \text{ A}, P_3 = 6.85 \text{ W}$$

15.10

For $BV = 200 \text{ V}$, from the junction breakdown curve, we need the drain doping concentration to be $N_D \approx 1.5 \times 10^{15} \text{ cm}^{-3}$

For the channel length (neglect V_{bi})

$$\begin{aligned} L(\text{min}) &= \left[\frac{2 \epsilon (V_D)}{e} \left(\frac{N_D}{N_B} \right) \left(\frac{1}{N_D + N_B} \right) \right]^{1/2} \\ &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(200)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{1.5 \times 10^{15}}{10^{16}} \right) \left(\frac{1}{1.5 \times 10^{15} + 10^{16}} \right) \right]^{1/2} \end{aligned}$$

$$\text{or } L(\text{min}) = 1.84 \mu\text{m}$$

For the drift region

$$\begin{aligned} W(\text{min}) &= \left[\frac{2(11.7)(8.85 \times 10^{-14})(200)}{1.6 \times 10^{-19}} \right. \\ &\quad \left. \times \left(\frac{10^{16}}{1.5 \times 10^{15}} \right) \left(\frac{1}{1.5 \times 10^{15} + 10^{16}} \right) \right]^{1/2} \end{aligned}$$

$$\text{or } W(\text{min}) = 12.3 \mu\text{m}$$

15.11

(b) In saturation region,

$$I_D = K_n (V_{GS} - V_T)^2 = 0.25(V_{GS} - 4)^2 \text{ and}$$

$$V_{DS} = V_{DD} - I_D R = 40 - I_D (10)$$

We find

$$V_{GS} = 5 \text{ V}, I_D = 0.25 \text{ A}, V_{DS} = 37.5 > V_{DS}(\text{sat})$$

$$V_{GS} = 6 \text{ V}, I_D = 1 \text{ A}, V_{DS} = 30 > V_{DS}(\text{sat})$$

$$V_{GS} = 7 \text{ V}, I_D = 2.25 \text{ A}, V_{DS} = 17.5 > V_{DS}(\text{sat})$$

For $V_{GS} = 8 \text{ V}$ and $V_{GS} = 9 \text{ V}$, transistor is biased in the nonsaturation region. For

$$V_{GS} = 8 \text{ V}.$$

$$I_D = \frac{40 - V_{DS}}{10} = 0.25[2(8 - 4)V_{DS} - V_{DS}^2]$$

We find

$$V_{DS} = 2.92 \text{ V}, I_D = 3.71 \text{ A}$$

For $V_{GS} = 9 \text{ V}$,

$$I_D = \frac{40 - V_{DS}}{10} = 0.25[2(9 - 4)V_{DS} - V_{DS}^2]$$

and we find

$$V_{DS} = 1.88 \text{ V}, I_D = 3.81 \text{ A}$$

Power dissipated in the transistor is $P_T = I_D V_{DS}$.

We find

$$V_{GS} = 5 \text{ V}, P_T = 9.375 \text{ W}$$

$$V_{GS} = 6 \text{ V}, P_T = 30 \text{ W}$$

$$V_{GS} = 7 \text{ V}, P_T = 39.4 \text{ W}$$

$$V_{GS} = 8 \text{ V}, P_T = 10.8 \text{ W}$$

$$V_{GS} = 9 \text{ V}, P_T = 7.16 \text{ W}$$

15.12

$$T_{dev} - T_{amb} = P_D (\theta_{dev-case} - \theta_{case-amb})$$

which can be written as

$$\begin{aligned} \theta_{dev-case} &= \frac{T_{dev} - T_{amb}}{P_D} - \theta_{case-amb} \\ &= \frac{175 - 25}{10} - 6 = 9^\circ \text{ C/W} \end{aligned}$$

Now

$$P_{D,rated} = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case}} = \frac{175 - 25}{9}$$

or

$$P_{D,rated} = 16.7 \text{ W}$$

15.13

$$P_{D,rated} = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case}}$$

or

$$\begin{aligned} \theta_{dev-case} &= \frac{T_{j,max} - T_{amb}}{P_{D,rated}} \\ &= \frac{150 - 25}{50} = 2.5^\circ \text{ C/W} \end{aligned}$$

Then

$$T_{dev} - T_{amb} = P_D (\theta_{dev-case} + \theta_{case-amb})$$

so

$$150 - 25 = P_D (2.5 + \theta_{case-amb})$$

or

$$125 = P_D (2.5 + \theta_{case-amb})$$

15.14

We have

$$P_D = I_D \cdot V_{DS} = (4)(5) = 20 \text{ W}$$

Now

$$T_{dev} - T_{amb} = P_D (\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb})$$

or

$$T_{dev} - 25 = 20(1.75 + 0.8 + 3) = 111$$

which yields

$$T_{dev} = 136^\circ \text{ C}$$

Also

$$T_{dev} - T_{case} = P_D \cdot \theta_{dev-case} = (20)(1.75) = 35$$

so

$$T_{case} = T_{dev} - 35 = 136 - 35$$

or

$$T_{case} = 101^\circ \text{ C}$$

And

$$T_{case} - T_{snk} = P_D \cdot \theta_{case-snk} = (20)(0.8) = 16^\circ \text{ C}$$

so

$$T_{snk} = T_{case} - 16 = 101 - 16$$

or

$$T_{snk} = 85^\circ \text{ C}$$

15.15

We have

$$T_{dev} - T_{amb} = P_D (\theta_{dev-case} + \theta_{case-amb})$$

so

$$200 - 25 = 25(3 + \theta_{case-amb})$$

or

$$\theta_{case-amb} = 4^\circ \text{ C/W}$$

15.16

We have

$$\theta_{dev-case} = \frac{T_{j,max} - T_{amb}}{P_{D,rated}} = \frac{175 - 25}{15} = 10^\circ C / W$$

Now

$$P_D = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb}}$$

$$= \frac{175 - 25}{10 + 1 + 4}$$

or

$$P_D = 10 W$$

15.17

We have $\alpha_1 + \alpha_2 = 1$. Now

$$\alpha_1 = \frac{\beta_1}{1 + \beta_1} \quad \text{and} \quad \alpha_2 = \frac{\beta_2}{1 + \beta_2}$$

so

$$\alpha_1 + \alpha_2 = \frac{\beta_1}{1 + \beta_1} + \frac{\beta_2}{1 + \beta_2} = 1$$

which can be written as

$$1 = \frac{\beta_1(1 + \beta_2) + \beta_2(1 + \beta_1)}{(1 + \beta_1)(1 + \beta_2)}$$

or

$$(1 + \beta_1)(1 + \beta_2) = \beta_1(1 + \beta_2) + \beta_2(1 + \beta_1)$$

Expanding, we find

$$1 + \beta_1 + \beta_2 + \beta_1\beta_2 = \beta_1 + \beta_1\beta_2 + \beta_2 + \beta_1\beta_2$$

which yields

$$\beta_1\beta_2 = 1$$

15.18

The reverse-biased p-well to substrate junction corresponds to the J_2 junction in an SCR. The photocurrent generated in this junction will be similar to the avalanche generated current in an SCR, which can trigger the device.

15.19

Case 1: Terminal 1(+), terminal 2(-), and I_G negative. This triggering was discussed in the text.

Case 2: Terminal 1(+), terminal 2(-), and I_G positive. Gate current enters the P2 region directly so that J3 becomes forward biased. Electrons are injected from N2 and diffuse into N1, lowering the potential of N1. The junction J2 becomes more forward biased, and the increased current triggers the SCR so that P2N1P1N4 turns on.

Case 3: Terminal 1(-), terminal 2(+), and I_G positive. Gate current enters the P2 region directly so that the J3 junction becomes more forward biased. More electrons are injected from N2 into N1 so that J1 also becomes more forward biased. The increased current triggers the P1N1P2N2 device into its conducting state.

Case 4: Terminal 1(-), terminal 2(+), and I_G negative. In this case, the J4 junction becomes forward biased. Electrons are injected from N3 and diffuse into N1. The potential of N1 is lowered which increases the forward biased potential of J1. This increased current then triggers the P1N1P2N2 device into its conducting state.
