$$V_k(x,t) = D_k \cdot \cos\left(\left(k+\tfrac{1}{2}\right)x\right) \cdot e^{\left(k+\frac{1}{2}\right)^2 t}$$

$$V_k(x,0) = D_k \cdot \cos\left(\left(k+\tfrac{1}{2}\right)x\right) = \operatorname{sen} x - 10$$

$$D_k = \frac{2}{\pi}\int_0^{\pi} (\operatorname{sen} x - 10) \cdot \cos\left(\left(k+\tfrac{1}{2}\right)x\right) dx$$

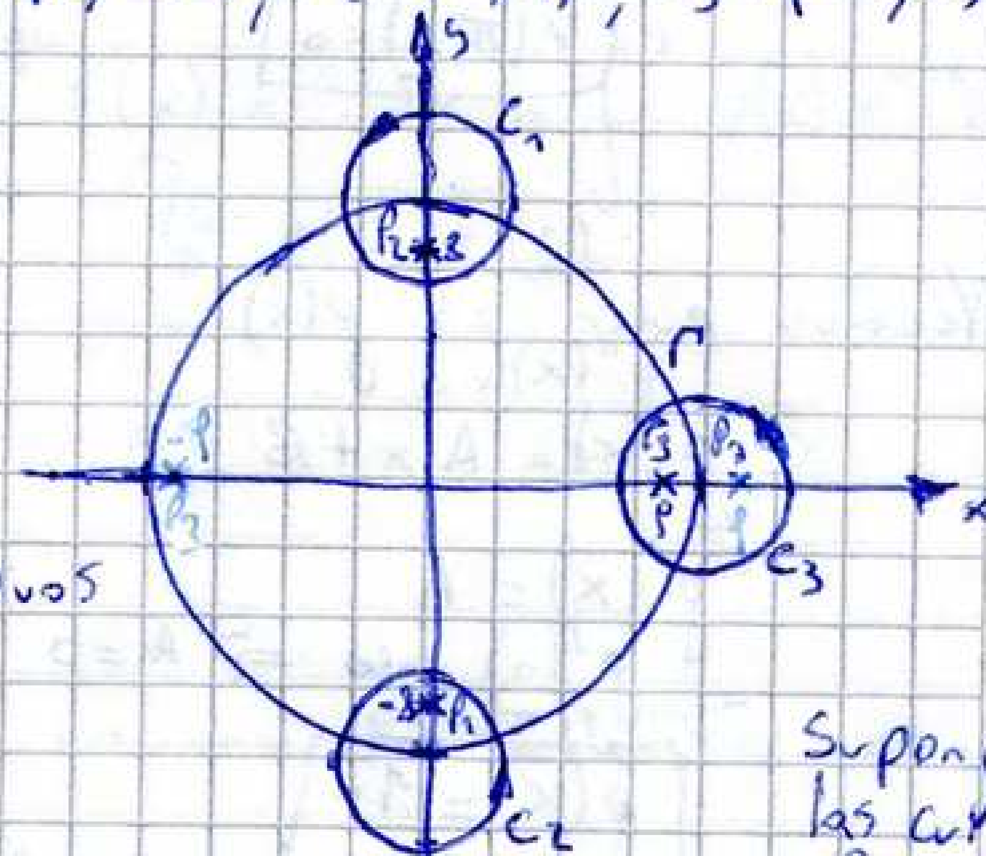
$$\Rightarrow \boxed{U(x,y) = \sum_{k=1}^{\infty} D_k \cdot \cos\left(\left(k+\tfrac{1}{2}\right)x\right) \cdot e^{\left(k+\frac{1}{2}\right)^2 t} + 10}$$

18/2/09

1) a) Singularidades aisladas de $f(z)$ en $\mathbb{C}$: $P_1(0,-8)$, $P_2: (0,8)$, $P_3: (-8,0)$

① $\int_{C_1} f(z)\,dz = 2$ ② $\int_{C_3} f(z)\,dz = 3$

③ $\int_{C_2} f(z)\,dz = 3$



Por T. Residuos

Supongo todas las curvas orientadas ⊕

De ①

$2\pi i \operatorname{Res}(f, 8i) = 2$

$\operatorname{Res}(f, 8i) = \frac{1}{\pi i} \cdot \frac{i}{i} = -\frac{1}{\pi} i$

De ② $2\pi i \operatorname{Res}(f, 8) = 3$

$\operatorname{Res}(f, 8) = \frac{-3i}{2\pi}$

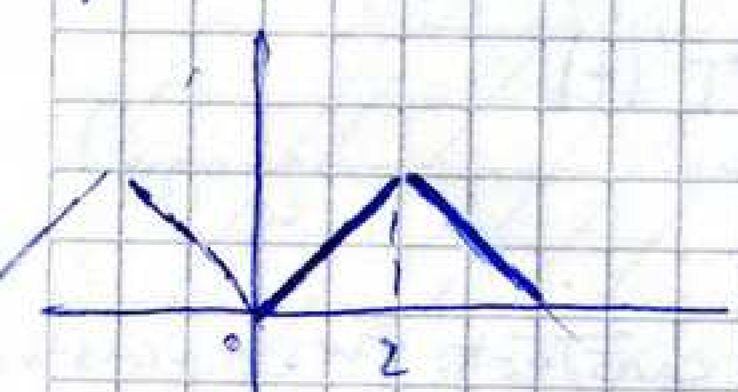
De ③ $2\pi i \operatorname{Res}(f, -8i) = 3$

$\operatorname{Res}(f, -8i) = \frac{-3i}{2\pi}$

$$\int_\Gamma f(z)\,dz = \operatorname{Res}(f, 8i) + \operatorname{Res}(f, 8) + \operatorname{Res}(f, -8i) = -\frac{1}{\pi}i - \frac{3i}{2\pi} - \frac{3i}{2\pi}$$

$$\boxed{\int_\Gamma f(z)\,dz = \frac{-2i - 3i - 3i}{2\pi} = \frac{-8i}{2\pi} = -\frac{4}{\pi} i}$$

b) $F(x) = x$



$\mathcal{F}_c(f(x)) = \frac{1}{\cdot}\int_0^2$

$$a_n = 1\int_0^2 x \cdot \cos\left(\frac{n\pi x}{2}\right) dx$$

$$a_n = \frac{x \cdot \operatorname{sen}\left(\frac{n\pi x}{2}\right)}{\frac{n\pi}{2}}\Big|_0^2 - \int_0^2 \frac{\operatorname{sen}\left(\frac{n\pi x}{2}\right)}{\frac{n\pi}{2}} dx$$

$$a_n = + \frac{\cos\left(\frac{n\pi}{2}x\right)}{\frac{(n\pi)^2}{4}}\Big|_0^2 = \frac{1}{\pi}$$

$$F(x) = x = \sum_{n=1}^{\infty} 4 \cdot \frac{((-1)^n - 1)}{(n\pi)^2} \cdot \cos\left(\frac{n\pi x}{2}\right)$$

$$\boxed{a_n = 4 \cdot \left(\frac{(-1)^n - 1}{(n\pi)^2}\right)}$$

$a_0 = 0$

Por ser f continua en $[0,2]$ converge uniformemente

$\Rightarrow$ En $[0,2]$ la serie converge puntualmente en todo punto y también hay convergencia en media cuadrática

NOTA

$$F(x) = \sum^{\infty} \frac{-8}{\quad} \cdot \cos(2k-1)\pi x$$

1) Usamos la identidad de Parseval para mostrar lo pedido.

$$\sum_{n=1}^{\infty} |a_n|^2 = \frac{1}{2}\int_{-2}^{2} |f_{(x)}|^2 \, dx \qquad \frac{1}{2} x \Big|_{-2}^{2} = \frac{1}{2}(2+2) = 2$$

$$\sum_{n=1}^{\infty} |a_n|^2 = \sum_{n=1}^{\infty} \left| \frac{4((-1)^n - 1)}{(n\pi)^2} \right|^2 = \sum_{k=1}^{\infty} \left| \frac{-16}{(2k+1)^2\pi^2} \right|^2$$

$$\sum_{n=1}^{\infty} 2.$$

$$\frac{-16}{2^2 n^2 \pi^2}$$

Para mi hay un error

90	2
45	5
9	3
3	3
1	

2)

a) $W = \text{Potencial Complejo} = \underbrace{U(x,y)}_{\text{funcion potencial}} + i \underbrace{V(x,y)}_{\text{funcion de Corriente}}$

$$\overline{W'} = V = \text{Campo de Velocidades} = \nabla U = \left(\frac{6(x-1)}{(x-1)^2 + (y-1)^2} , \frac{6(y-1)}{(x-1)^2 + (y-1)^2} \right)$$

$\overline{W'}$ $\overline{W'}$ $8 \operatorname{Sen}(2k-1)\pi$

$$U_x = \frac{6(x-1)}{(x-1)^2 + (y-1)^2}$$

$w = x-1$

$$\cancel{U(x,y) = \int \frac{6(x-1)}{(x-1)^2 (y-1)^2} dx = \int \frac{6w}{w^2 + (y-1)^2} = \frac{6 \ln(x-1)}{(y-1)^2} + cte}$$

$$U_y(x,y) = 6 \cdot \ln(x-1).$$

$$U(x,y) = \int \frac{6(x-1)}{(x-1)^2 + (y-1)^2} = \int \frac{6\cancel{(x-1)}\, dp}{p \;\; 2\cancel{(x-1)}} \qquad \begin{array}{|l} (x-1)^2 + (y-1)^2 = p \\ 2(x-1)\,dx = dp \end{array}$$

$$U(x,y) = 3 \cdot \ln|(x-1)^2 + (y-1)^2| + cte(y)$$

$$U_y(x,y) = 3 \cdot \frac{1}{(x-1)^2 + (y-1)^2} \cdot 2(y-1) + \underbrace{cte'(y)}_{0} = \frac{6(y-1)}{(x-1)^2 + (y-1)^2}$$

$$U(x,y) = 3 \cdot \ln|(x-1)^2 + (y-1)^2| + c$$

$$cte(y) = cte$$

$$\overline{W'} = \frac{6(x-1)}{x^2 - 2x + 1 + y^2 - 2y + 1} + i\,\frac{6(y-1)}{}$$

$$\cancel{(z-1-i)^2 = ((x-1) + i(y-1))^2 = (x-1)^2 + 2i(x-1)(y-1) - (y-1)^2}$$

$$\overline{\left(\frac{6}{(z-i-1)}\right)} = \overline{W'} = \frac{6}{(x-1) + i(y-1)} \cdot \frac{(x-1) - i(y-1)}{(x-1) - i(y-1)} \left(\frac{6(x-1) - i(y-1)6}{(x-1)^2 + (y-1)^2} \right)$$

$$W' = \frac{6}{(z-i-1)} \qquad \boxed{W = 6 \log(z-i-1)} \; !$$

$$z \neq (1+i)$$

NOTA

$$W = 6\log(z-i-1) = 6 . \log((x-1)+i(y-1)) = 6(\ln|z-1-i| + i\arg(z-1-i))$$

b) $$V(x,y) = 6\arg(z-1-i) = 6\,\mathrm{arc\,Tg}\left(\frac{y-1}{x-1}\right).$$

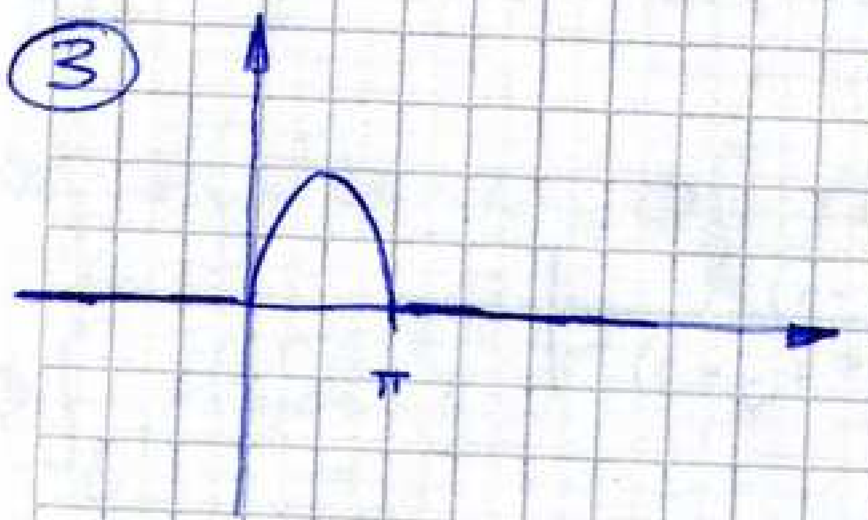
La partícula seguirá la trayectoria dada por

$$6\,\mathrm{arc\,Tg}\left(\frac{y-1}{x-1}\right) = 6\,\mathrm{arc\,Tg}\left(\frac{8-1}{5-1}\right) = 6\,\mathrm{arc\,Tg}\left(\frac{7}{4}\right)$$

$$\frac{y-1}{x-1} = \frac{7}{4}$$

$$y-1 = \frac{7}{4}x - \frac{7}{4} \Rightarrow \boxed{y = \frac{7}{4}x - \frac{3}{4}}$$

③



F coseno

$$F_c(f)_{(w)} = \int_0^\infty f(t) . \cos(wt)\,dt$$
$$= \int_0^\pi \mathrm{sen}\,t . \cos(wt)\,dt =$$

$$\int \mathrm{sen}\,t . \cos(wt)\,dt = \mathrm{sen}\,t . \frac{\mathrm{sen}(wt)}{w} - \underbrace{\int \cos t . \frac{\mathrm{sen}(wt)}{w}\,dt}_{-\cos t . \frac{\cos wt}{w^2} + \int \mathrm{sen}\,t . \frac{\cos wt}{w^2}\,dt}$$

$$\underbrace{\left(1-\frac{1}{w^2}\right)}_{\frac{w^2-1}{w^2}} \int \mathrm{sen}\,t . \cos(wt)\,dt = \mathrm{sen}\,t . \frac{\mathrm{sen}(wt)}{w} + \frac{\cos t . \cos(wt)}{w^2}$$

$$\int_0^\pi \mathrm{sen}\,t . \cos(wt)\,dt = \left.\frac{w . \mathrm{sen}\,t . \mathrm{sen}(wt) + \cos t . \cos(wt)}{w^2-1}\right|_0^\pi = \boxed{\frac{-\cos(w\pi) - 1}{w^2-1}}$$

Para $w=0$ $\int_0^\pi \mathrm{sen}\,t . 1\,dt = -\cos(t)\Big|_0^\pi = 1+1 = 2$ Vale para $w=1$ También

Para sen t.

$$\boxed{F_c(f)_{(w)}} = \frac{-\cos(w\pi)-1}{w^2-1} = \boxed{\frac{\cos(w\pi)+1}{1-w^2}}$$

$$f(t) = \frac{2}{\pi}\int_0^\infty \frac{\cos(w\pi)+1}{1-w^2} . \cos(wt)\,dw$$

$$0 = f(0) = \frac{2}{\pi}\int_0^\infty \frac{\cos(w\pi)+1}{1-w^2}\,dw$$

$$\Rightarrow \boxed{\int_0^\infty \frac{1+\cos(\alpha\pi)}{1-\alpha^2}\,d\alpha = 0}$$

NOTA

HOJA N°

FECHA

b) f y f' continuas, f'' seccionalmente continua, $\lim_{x\to\infty} f(x) = 0$

$\mathcal{F}_s(f)_{(w)} = \mathcal{F}_s(w)$

por partes

$\cancel{\mathcal{F}_s(f'(x)) = \int_0^\infty f'(x)\cdot \operatorname{sen}(wx)\,dx = f(x)\cdot \operatorname{sen} wx\Big|_0^\infty - \int_0^\infty f\cdot\cos(wx)\cdot w\,dx}$

$$\mathcal{F}_s(f''(x)) = \int_0^\infty f''(x)\cdot \operatorname{sen}(wx)\,dx = \underbrace{f'(x)\cdot \operatorname{sen} wx\Big|_0^\infty}_{0} - \underbrace{\int_0^\infty f'(x)\cdot\cos(wx)\,w\,dx}_{-w\,\mathcal{F}_c(f'(x))_{(w)}}$$

$$\mathcal{F}_c(f'(x)) = \int_0^\infty f'(x)\cdot\cos(wx)\,dx = \underbrace{f(x)\cdot\cos(wx)\Big|_0^\infty}_{-f(0^+)} + \underbrace{\int_0^\infty f(x)\cdot \operatorname{sen}(wx)\,w\,dx}_{\mathcal{F}_s(w)\,w}$$

$$\boxed{\mathcal{F}_s(f''(x)) = -w\,\mathcal{F}_c(f'(x))_{(w)} = -w\left(-f(0^+) + \mathcal{F}_s(w)\cdot w\right) = -w^2\mathcal{F}_s(w) + w\,f(0^+)}$$

4 b) $u_t = u_{xx} - \alpha u \qquad 0 < x < \pi \qquad t > 0$

$u(x,0) = \operatorname{sen} x$

$u(0,t) = u(\pi,t) = 0$

Transformo Laplace respecto de t

$L(u(x,t)) = U_{(x,s)}$

$L(u_t(x,t)) = s\cdot U_{(x,s)} + \underbrace{u_{(x,0^+)}}_{\operatorname{sen} x}$

$L(u(0,t)) = U(0,s) = 0$

$U(\pi,s) = 0$

$s\cdot U(x,s) - \operatorname{sen} x - U_{xx}(x,s) + \alpha\,U(x,s) = 0$

$(s+\alpha)\,U(x,s) - U_{xx}(x,s) = \operatorname{sen} x$

$\cancel{U_p(x,s) = \operatorname{sen}(x)\cdot\frac{1}{(s+\alpha)}}$

$\cancel{U_p(x,s) = \cos x\,\frac{\alpha}{2(s+\alpha)}}$

$U_p = A\operatorname{sen} x$

$U_p' = A\cos x$

$U_p'' = -A\operatorname{sen} x$

$(s+\alpha)\cdot A\cdot\operatorname{sen} x + A\cdot\operatorname{sen} x = \operatorname{sen} x$

$(s+\alpha+1)\,A\operatorname{sen} x = \operatorname{sen} x$

$A = \dfrac{1}{s+\alpha+1}$

$$\boxed{\hat{U}_p(x,s) = \frac{1}{(s+\alpha+1)}\cdot\operatorname{sen} x}$$

$\hat{U}_h(x,s) = A\cdot e^{\sqrt{s+\alpha}} + B\cdot e^{-\sqrt{s+\alpha}}$

$\hat{U}(x,s) = \dfrac{\operatorname{sen} x}{(s+\alpha+1)} + A\cdot e^{\sqrt{s+\alpha}} + B\cdot e^{-\sqrt{s+\alpha}}$

$A\cdot e^{\sqrt{s+\alpha}} + B\cdot e^{-\sqrt{s+\alpha}} = 0$

$\dfrac{A\,e^{\sqrt{s+\alpha}}}{B\,e^{-\sqrt{s+\alpha}}} = -B$

$A\cdot e^{2\sqrt{s+\alpha}}/B = -B$

$+A\cdot e^{\sqrt{s+\alpha}} - A\cdot e^{2\sqrt{s+\alpha}}\cdot e^{-\sqrt{s+\alpha}}$

$\cancel{A\cdot e^{\sqrt{s+\alpha}}} - \cancel{A\cdot e^{\sqrt{s+\alpha}}}$

A y B = 0

NOTA

$\hat{U}(x,s) = \dfrac{\operatorname{sen}\,x}{s+\alpha+1}$

$$\Rightarrow \boxed{U(x,t) = \operatorname{Sen} x \cdot e^{(-1-\alpha)t}} \qquad t>0$$

Satisface la ecuacion inicial?

$$\cancel{\operatorname{Sen} x} \cdot (-1-\alpha) \cdot e^{(-1-\alpha)t} = -e^{(-1-\alpha)t} \cdot \cancel{\operatorname{Sen} x} - \alpha \cdot \cancel{\operatorname{Sen} x} \cdot e^{(-1-\alpha)t}$$

$$\cancel{-e^{(-1-\alpha)t}} \cancel{- \alpha \cdot e^{(-1-\alpha)t}} = \cancel{-e^{(-1-\alpha)t}} \cancel{- \alpha \cdot e^{(-1-\alpha)t}} \quad \checkmark$$

a) $U\,U_{t(x,t)} - U_{xx}(x,t) + \alpha \cdot U_{(x,t)} = 0$

$$U_{(x,t)} = X(x) \cdot T(t)$$

$$X(x) \cdot T'(t) - X''(x) \cdot T(t) + \alpha \cdot X(x) \cdot T(t) = 0$$

divido por X y por T

$$\frac{T'(t)}{T(t)} - \frac{X''(x)}{X(x)} + \alpha = 0$$

$$\alpha = \frac{X''(x)}{X(x)} = -\frac{T'(t)}{T(t)} = K$$

$k - \alpha = \pm r^2$

$k = r = (k-\alpha)$

$r = \pm\sqrt{k-\alpha}$

Resuelvo por separado

$$T'(t) + kT(t) = 0$$

$$T(t) = A \cdot e^{-kt}$$

$$X''(x) + (k-\alpha)\,X(x) = 0$$

$$X''(x) \pm r^2 \, X(x) = 0$$

No $\cancel{-k\alpha = 0}$ $\quad X(x) = A + Bx$

Por CI

$X(0) = 0 = A$

$X(\pi) = 0 = B\pi \to B = 0$

Sol

Este es ok $\quad k - \alpha > 0 \quad X(x) = A.\cos(rx) + B\operatorname{Sen}(rx)$

$X(0) = 0 = A.\cos(rx)$

$X(\pi) = 0 = B.\operatorname{Sen}(r\pi)$

$$\boxed{X(x) = B.\operatorname{Sen}(\sqrt{k-\alpha}\,x)}$$

$X(x) = B.\operatorname{Sen}$ $r\pi = n\pi$

$\boxed{r = n}$

$\sqrt{k-\alpha} =$

$\cancel{k - \alpha < 0}$

$$X(x) = A.e^{(k-\alpha)x} + B e^{-(k-\alpha)x}$$

$$X(0) = A + B = 0 \to A = -B$$

$$X(\pi) = Ae^{(k-\alpha)\pi} + A.e^{-(k-\alpha)\pi} = 0$$

$$2A.\operatorname{Senh}((k-\alpha)\pi) = 0$$

$k - \alpha = 0$ lo cual es absurdo. No me sirve.

$$U(x,t) = C.\operatorname{Sen}\left(\sqrt{k-\alpha}\,x\right) \cdot e^{-kt}$$

$$U(x,0) = C.\operatorname{Sen}\left(\sqrt{k-\alpha}\,x\right) = \operatorname{Sen} x$$

$$\sqrt{k-\alpha} = 1$$

$$k - \alpha = 1$$

$$k = 1 + \alpha$$

$$\boxed{U(x,t) = \operatorname{Sen} x \cdot e^{-(1+\alpha)t} = \operatorname{Sen} x \cdot e^{(-1-\alpha)t}}$$

NOTA