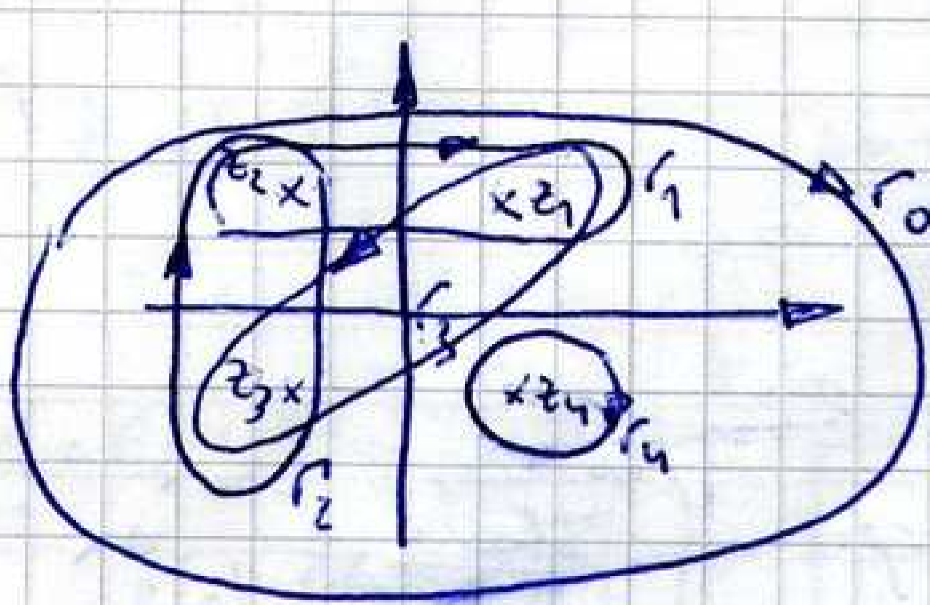
$\boxed{\text{Final } 11/8/09}$

1) f holo en $\mathbb{C} - \{z_1, z_2, z_3, z_4\}$ y $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$ curvas simples cerradas



(*) Por Teorema de los residuos

$$\frac{1}{2\pi i}\oint_{\Gamma_1} f(z)\,dz = -6$$

① $\boxed{6 = \operatorname{Res}(f, z_1) + \operatorname{Res}(f, z_2)}$

$$\frac{1}{2\pi i}\oint_{\Gamma_2} f(z)\,dz = 10$$

Por (*)

② $\boxed{-10 = \operatorname{Res}(f, z_2) + \operatorname{Res}(f, z_3)}$

$$\frac{1}{2\pi i}\oint_{\Gamma_3} f(z)\,dz = 4$$

Por (*)

③ $\boxed{4 = \operatorname{Res}(f, z_1) + \operatorname{Res}(f, z_3)}$

$$\frac{1}{2\pi i}\oint_{\Gamma_0} f(z)\,dz = 8$$

Por (*)

④ $\boxed{-8 = \operatorname{Res}(f, z_1) + \operatorname{Res}(f, z_2) + \operatorname{Res}(f, z_3) + \operatorname{Res}(f, z_4)}$

Nos piden $\underbrace{\frac{1}{2\pi i}\oint_{\Gamma_4} f(z)\,dz}= -\operatorname{Res}(f, z_4)$

De ①, ②, ③, ④ buscamos el valor del $\operatorname{Res}(f, z_4)$.

Sumo ①, ② y ③

$$\overset{-4}{\cancel{0}} = 2\operatorname{Res}(f, z_1) + 2\operatorname{Res}(f, z_2) + 2\operatorname{Res}(f, z_3)$$

$$\Rightarrow \operatorname{Res}(f, z_1) + \operatorname{Res}(f, z_2) + \operatorname{Res}(f, z_3) = 0$$

Reemplazando en 4

$$\operatorname{Res}(f, z_4) = -8$$

$$\Rightarrow \boxed{\frac{1}{2\pi i}\oint_{\Gamma_4} f(z)\,dz = 8}$$

2) 1) ~~Veamos si $\int_{-\infty} \frac{\cos x}{x^2+1} = f(x)$ es absolutamente convergente integrable~~

~~Para ello $\int_{-\infty}^{\infty} \left|\frac{\cos x}{x^2+1}\right| dx$ (c)~~ ~~No tiene puntos de el integrando no esta definido.~~

~~$= \int_{-\infty}^{\infty} \frac{|\cos x|}{x^2+1}\,dx$~~ ~~Utilicemos el criterio de Abel para mostrar que converge en $\int_{+\infty}$~~

~~$\int_0^{\infty} |\cos x| \leq 2 = M$~~ ~~$\left(\frac{1}{x^2+1}\right)' = -\frac{1}{(x^2+1)^2} 2x$~~ ~~cre. en $(0,\infty)$ $x \geq 0 \Rightarrow \frac{1}{(x^2+1)}$ decrece~~

~~$\lim_{x\to\infty} \frac{1}{(x^2+1)^2} = 0$~~ ~~y $\frac{1}{\ldots}$ es C^1 en $[0,\infty)$~~

$\Rightarrow$ ~~Por Abel~~ $\cancel{\int_{0}^{\infty} \frac{|\cos x|}{x^2+1}\,dx \ (c)}$

~~Para~~ ~~hacemos cambio de variable $x=-t$~~

$$\cancel{\int_{-\infty}^{0} \frac{|\cos x|}{x^2+1}\,dx = -\int_{\infty}^{0} \frac{|\cos(-t)|}{\cos(-t)^2+1}\,dt = \int_{0}^{\infty} \frac{|\cos t|}{t^2+1}\,dt \ (c)}$$

~~el la que hemos ya por resolverla mostrado que converge~~

$$\cancel{\Rightarrow \int_{-\infty}^{0} \frac{|\cos x|}{x^2+1}\,dx \ (c)}$$

$$\cancel{\boxed{\Rightarrow \int_{-\infty}^{\infty} \left|\frac{\cos x}{x^2+1}\right|\,dx \ (c)}}$$

2) 1) Para poder afirmar que tiene ~~TF~~ debe converger $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+1}\cdot e^{-iwx}\,dx$

$$= \underbrace{\int_{-\infty}^{\infty} \frac{\cos x\cdot\cos wx}{\underbrace{x^2+1}_{\text{par}}}\,dx}_{2\int_{0}^{\infty} \frac{\cos x\cdot\cos wx}{x^2+1}\,dx} - i\underbrace{\int_{-\infty}^{\infty} \frac{\cos x\cdot\text{sen}(wx)}{x^2+1}\,dx}_{0 \text{ pues es impar en un intervalo simétrico Continuo}}$$

$$\left|\int_{0}^{\infty} \cos x\cdot\cos wx\,dx\right| = \left|\int_{0}^{\infty} \frac{\cos(x+wx)+\cos(x-wx)}{2}\,dx\right| = \left|\frac{\text{sen}(x+wx)}{2\cdot(1+w)} + \frac{\text{sen}(x-wx)}{2\cdot(1-w)}\right|_{0}^{\infty}$$

$$= \left|\underbrace{\frac{\text{sen}[x(1+w)]}{2(1+w)}}_{\text{en } -1 \to 1} + \underbrace{\frac{\text{sen}[x(1-w)]}{2(1-w)}}_{\text{en } 1 \to 1 \text{, es}}\right|_{0}^{\infty} \leq 1 + 1 = 2$$

$\cos(x+wx) = \cos x\cdot\cos(wx) - \text{sen}\,x\cdot\text{sen}(wx)$

$\cos(x-wx) = \cos x\cdot\cos(wx) + \text{sen}\,x\cdot\text{sen}(wx)$ Sumo

$\frac{\cos(x+wx)+\cos(x-wx)}{2} = \cos(x)\cos(wx)$

$\left(\frac{1}{x^2+1}\right)' = \frac{-2x}{(x^2+1)^2}$ en $(0,\infty)$ decrece

$\lim_{x\to\infty} \frac{1}{x^2+1} = 0$ y $\frac{1}{x^2+1} \in C^1$ en $(0,\infty)$ $\Rightarrow$ Por Abel $\int_{0}^{\infty} \frac{\cos x\cdot\cos wx}{x^2+1}\,dx$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{\cos x}{x^2+1}\cdot e^{-iwx}\,dx \ (c)$$

2) Veamos si $\frac{1}{x^2+1}$ ~~converge~~ es absolutamente integrable

$$\int_{-\infty}^{\infty} \left|\frac{1}{x^2+1}\right|dx = \int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx \overset{\text{por ser par}}{=} 2\int_{0}^{\infty} \frac{1}{x^2+1}\,dx = 2\left(\int_{0}^{1} \frac{1}{x^2+1}\,dx + \int_{1}^{\infty} \frac{1}{x^2+1}\,dx\right)$$

(c) pues es una integral en un intervalo acotado con fcs continuas

El único punto conflictivo es el ∞. Usamos el criterio de comparación en el límite

Comparo con $\frac{1}{x^2}$

$$\lim_{x\to\infty} \frac{x^2}{x^2+1} = \lim_{x\to\infty} \frac{x^2}{x^2\left(1+\frac{1}{x^2}\right)} = 1 = L > 0 \Rightarrow \text{Como } \int_{1}^{\infty} \frac{1}{x^2}\,dx \ (c) \text{ pues } p>1$$

$$\Rightarrow \boxed{\int_{1}^{\infty} \frac{1}{x^2+1}\,dx \ (c)}$$

NOTA

Por lo Tanto, $\int_{-\infty}^{\infty} \left|\frac{1}{x^2+1}\right| dx$ (C)

$\Rightarrow$ existe la Transformada de Fourier de $\frac{1}{x^2+1}$.

3) Ambas funciones son continuas en $(-\infty,\infty)$. Por lo Tanto

si $f(x)=\frac{\cos x}{x^2}$

$\frac{\cos x}{x^2+1}=f(x)$ $\mathcal{F}(w)=\widehat{\left(\frac{\cos x}{x^2+1}\right)}$

$\frac{1}{x^2+1}=g(x)$ $G(w)=\widehat{\left(\frac{1}{x^2+1}\right)}$

$$\boxed{\int_{-\infty}^{\infty} \mathcal{F}(w)\, e^{iwt}\, dw = 2\pi \cdot \frac{\cos x}{x^2+1}}$$

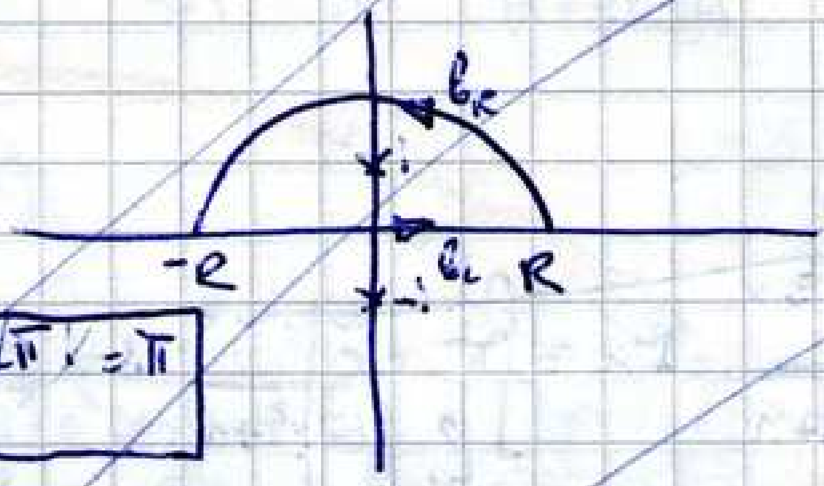
$$\boxed{\int_{-\infty}^{\infty} G(w)\, dw = 2\pi \cdot \frac{1}{x^2+1}}$$

Convergen a las mismas funciones multiplicadas por 2π.

4) $\int_{-\infty}^{\infty} \frac{1}{x^2+1}\, dx$

Lo Llevo al plano complejo para usar las herramientas que este me proporciona

$$\int_{\ell} \frac{1}{z^2+1}\, dz = 2\pi i \cdot \operatorname{Res}\left(\frac{1}{z^2+1}, i\right)$$



$$2\pi i \cdot \operatorname{Res}\left(\frac{1}{z^2+1}, i\right) = 2\pi i \lim_{z\to i} \cancel{(z-i)} \frac{1}{\cancel{(z-i)}(z+i)} = \frac{1}{2i} \boxed{2\pi i = \pi}$$

$$\int_{\ell_L} \frac{1}{z^2+1}\, dz + \underbrace{\int_{\ell_R} \frac{1}{z^2+1}\, dz}_{\text{Por lema de Jordan 1}} = \pi$$

Como

$$\lim_{\substack{|z|\to\infty \\ \operatorname{Im}(z)>0}} \left| z \frac{1}{z^2+1} \right| = \lim_{z\to\infty} \frac{|z|}{|z|\left(|z| + \frac{1}{|z|}\right)} = 0 \Rightarrow \int_{\ell_R} \frac{1}{z^2+1}\, dz \xrightarrow{R\to\infty} 0.$$

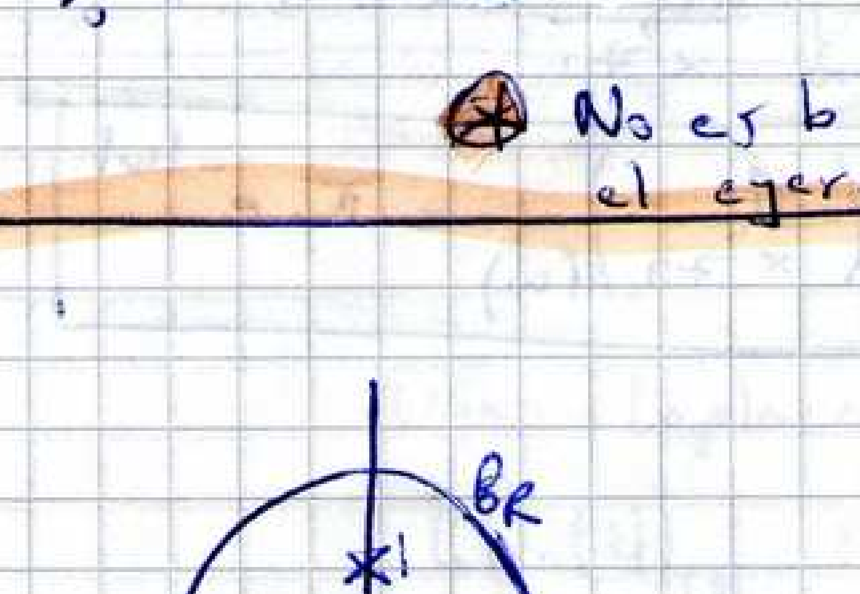
$\operatorname{Im}(z)>0$, $|z|\to\infty$

$$\boxed{\int_{-\infty}^{\infty} \frac{1}{x^2+1}\, dx = \pi}$$

(*) No es lo que pide el ejercicio

4) $\mathcal{F}\left(\frac{1}{x^2+1}\right)_{(w)} = \int_{-\infty}^{\infty} \frac{1}{x^2+1} e^{-iwx}\, dx$

La llevo al plano complejo



$$\int_{\ell_\infty} \frac{1}{z^2+1} \cdot e^{-iwz}\, dz = 2\pi i \operatorname{Res}\left(\frac{e^{-iwz}}{z^2+1}, i\right)$$

$$2\pi i \operatorname{Res}\left(\frac{e^{-iwz}}{z^2+1}, i\right) = 2\pi i \cdot \lim_{z\to i} \cancel{(z-i)} \frac{e^{-iwz}}{\cancel{(z-i)}(z+i)} = \cancel{2}\pi i \cdot \frac{e^{w}}{\cancel{2}i} = \boxed{\pi \cdot e^{w}}$$

NOTA

$$\int_{C}\frac{e^{-iwz}}{z^2+1}\,dz = \int_{C_L}\frac{e^{-iwz}}{z^2+1}\,dz + \int_{C_R}\frac{e^{-iwz}}{z^2+1}\,dz = \pi\cdot e^{w}$$

Si $w<0$

$\int_{C_R}\frac{e^{-iwz}}{z^2+1}\,dz$ $\qquad |z|=R \qquad \left|\frac{1}{z^2+1}\right| \le \frac{1}{R^2+1} = M_R \qquad \lim_{R\to\infty} M_R = 0$

Por lema de Jordan 2

$$\Rightarrow \int_{C_R}\frac{e^{\overset{>0}{(-w)}zi}}{z^2+1}\,dz \xrightarrow{R\to\infty} 0$$

$$\int_{-\infty}^{\infty}\frac{e^{-iwx}}{x^2+1}\,dx = \pi$$

con w

Si $w=0$

$\int_{C_R}\frac{1}{z^2+1}\,dz$ $\qquad$ Por lema de Jordan 1.

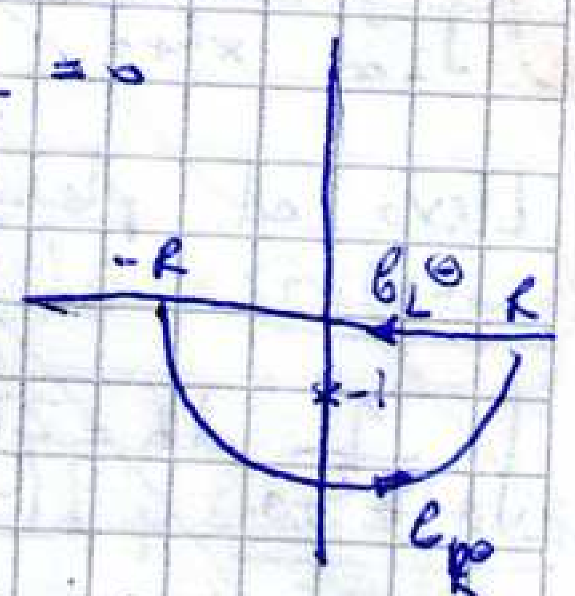
$$\lim_{|z|\to\infty}\left|z\,\frac{1}{z^2+1}\right| = 0 \Longrightarrow \int_{C_R}\frac{1}{z^2+1}\,dz \xrightarrow{R\to\infty} 0$$

Si $w>0$

$\int_{C_R}\frac{e^{-iwz}}{z^2+1}\,dz$ $\qquad$ usamos lema de Jordan 1.

$$e^{-iw(x+iy)} = e^{-iwx}\cdot e^{wy}$$

$$\lim_{\substack{|z|\to\infty\\ Im(z)<0}}\left|\frac{z}{z^2}\,\frac{e^{-iwz}}{\left(1+\frac{1}{z^2}\right)}\right| = \lim_{\substack{|z|\to\infty\\ Im(z)>0}}\frac{\left|e^{\overset{<0}{wy}}\right|}{|z|\cdot\left|1+\frac{1}{z^2}\right|} = 0$$



$$\Rightarrow \int_{C_{\ominus}}\frac{e^{-iwz}}{z^2+1}\,dz = \int_{C_L}\frac{e^{-iwz}}{z^2+1}\,dz + \underbrace{\int_{C_R}\frac{e^{-iwz}}{z^2+1}\,dz}_{\to 0} = 2\pi i\cdot Res\left(\frac{e^{-iwz}}{z^2+1}, -i\right)$$

$$2\pi i\, Res\left(\frac{e^{-iwz}}{z^2+1}, -i\right) = 2\pi i \lim_{z\to -i}\frac{\cancel{(z+i)}\,e^{-iwz}}{\cancel{(z+i)}(z-i)} = \cancel{2}\pi \cancel{i}\cdot\frac{e^{-w}}{-\cancel{2}\cancel{i}} = -\pi\cdot e^{-w}$$

$$\boxed{\int_{-\infty}^{\infty}\frac{e^{-iwx}}{x^2+1}\,dx = \pi - \left(-\pi\cdot e^{-w}\right) = \pi\cdot e^{-w}}$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{e^{-iwx}}{x^2+1}\,dx = \pi\cdot e^{-|w|}$$

$$\Rightarrow \boxed{\mathcal{F}\left(\frac{1}{x^2+1}\right)_{(w)} = \pi\cdot e^{-|w|}}$$

3)

3) Usando la Transformada de Laplace calculamos $y(t)$

$$y(t)'' + 3y'(t) + 2y(t) = \text{sen}\,t \cdot u(t)$$

siendo $y'(0^+) = 0$
$y''(0^+) = 0$

$$Y(s)\cdot s^2 + 3\,Y(s)\cdot s + 2Y(s) = \frac{1}{s^2+1}$$

$$\mathcal{L}(y(t))_{(s)} = Y_{(s)}$$

$$\mathcal{L}(y'(t))_{(s)} = Y(s)\cdot s - \underbrace{y(0^+)}_{0}$$

$$\mathcal{L}(y''(t))_{(s)} = Y(s)\cdot s^2 - \underbrace{y(0^+)}_{0}\cdot s - \underbrace{y'(0^+)}_{0}$$

$$Y(s)\cdot(s^2+3s+2) = \frac{1}{s^2+1}$$

$$Y(s) = \frac{1}{s^2+1}\cdot\frac{1}{s+1}\cdot\frac{1}{s+2} = \frac{1}{s^2+1}\cdot\left(\frac{1}{s+1} - \frac{1}{s+2}\right)$$

$$\boxed{\frac{-3\pm\sqrt{3^2-4\cdot1\cdot2}}{2} = \frac{-3\pm1}{2} = \begin{matrix}-1\\-2\end{matrix}}$$

$$y(s) = \int_0^t \text{sen}(z)\cdot\left(e^{-(t-z)} - e^{-2(t-z)}\right)dz$$

$$\boxed{y(s) = \text{sen}\,t * \left(e^{-t} - e^{-2t}\right)}$$

$$\frac{A}{s+1} + \frac{B}{s+2} = \frac{A(s+2)+B(s+1)}{(s+1)(s+2)} = \frac{1}{(s+1)(s+2)}$$

$A+B=0 \to A=-B$
$2A+B=1$

$$\boxed{\begin{matrix}A=1\\B=-1\end{matrix}}$$

$a>0$

$$e^{-t}\int_0^t \text{sen}\,z\cdot e^{az}\,dz = -\cos z\cdot e^{az}\Big|_0^t + \underbrace{\int_0^t a\cos z\cdot e^{az}\,dz}_{a\,\text{sen}\,z\cdot e^{az}\Big|_0^t + \int_0^t a^2\,\text{sen}\,z\cdot e^{az}\,dz}$$

$$\frac{1}{s^2+1}\cdot\frac{1}{s+1} = \frac{A(s+1)+B(s^2+1)}{(s^2+1)(s+1)} = 1$$

$$(1+a^2)\int_0^t \text{sen}\,z\,e^{az} = -a\cos z\cdot e^{az} + a\,\text{sen}\,z\,e^{az}\Big|_0^t$$

$$\boxed{\int_0^t \text{sen}\,t\,e^{az} = \frac{-\cos t\cdot e^{at} + a\,\text{sen}\,t\cdot e^{at} + 1}{1+a^2}}$$

(*) No es Necesario calcularlo

$$\Rightarrow \boxed{y(s) = e^{-t}\left(\frac{-\cos t\cdot e^{t} + \text{sen}\,t\cdot e^{t} + 1}{2} - \frac{-\cos t\cdot e^{2t} + 2\cdot\text{sen}\,t\cdot e^{2t} + 1}{5}\right)}$$

~~$y(s) = \cos t + \text{sen}\,t$~~

$$y(t) + \int_0^t \tau\cdot y(t-\tau)\,d\tau = m(t)$$

Uso Laplace

$$\mathcal{L}(m(t))_{(s)} = M(t)$$

$$y(t) + t * y(t) = m(t)$$

$$Y(s) + \mathcal{L}(t)\cdot\mathcal{L}(y(t)) = M(s)$$

$$Y(s) + \frac{1}{s^2}\cdot Y(s) = M(s)$$

$$M(s) = \frac{1}{s^2+1}\cdot\left(\frac{1}{s+1} - \frac{1}{s+2}\right) + \frac{1}{s^2}\cdot\frac{1}{s^2+1}\cdot\left(\frac{1}{s+1} - \frac{1}{s+2}\right) = \underbrace{\frac{1}{s^2+1}\left(\frac{1}{s+1} - \frac{1}{s+2}\right)}_{Y}\cdot\underbrace{\left(1+\frac{1}{s^2}\right)}_{\frac{s^2+1}{s^2}}$$

$$= \frac{1}{s^2}\left(\frac{1}{\ldots} - \frac{1}{\ldots}\right)$$

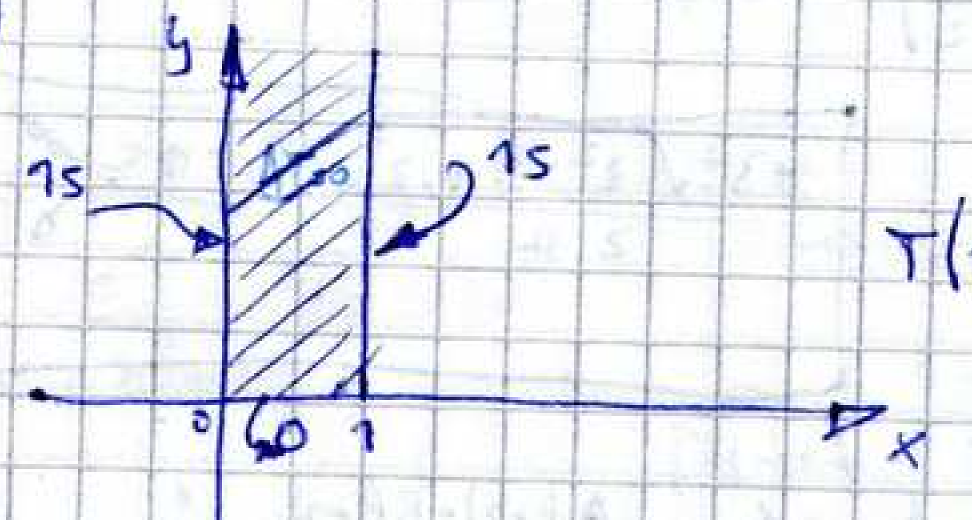
$m(t) = t * (e^{-t} - e^{-2t})$

$$m(t) = \int_0^t z\cdot\underbrace{\left(e^{-(t-z)} - e^{-2(t-z)}\right)}_{e^{-t}(e^{z} - e^{2z}\cdot e^{-2t})}dz = z\cdot\left(e^{-t}\cdot e^{z} - e^{-2t}\cdot\frac{e^{2z}}{2}\right)\Big|_0^t - \underbrace{\int_0^t \left(e^{-t}\cdot e^{z} - e^{-2t}\frac{e^{2z}}{2}\right)dz}_{-e^{-t}\cdot e^{z} + e^{-2t}\cdot\frac{e^{2z}}{4}\Big|_0^t}$$

$$m(t) = t\cdot\left(e^{-t}\cdot e^{t} - \frac{e^{-2t}\cdot e^{2t}}{2}\right) - e^{-t}\cdot e^{t} + \frac{e^{-2t}\cdot e^{2t}}{4} + e^{-t} - \frac{e^{-2t}}{4}$$

$$\boxed{m(t) = \frac{1}{2}t - 1 + \frac{1}{4} + e^{-t} - \frac{e^{-2t}}{4} = \frac{1}{2}t - \frac{3}{4} + e^{-t} - \frac{e^{-2t}}{4}}$$

4)



$T(x,y)$

$T(1,y) = T(0,y) = 15$

$T(x,0) = 0$

$\Delta T = 0$

$T_{xx} + T_{yy} = 0$

~~Propongo~~ $\cancel{T(x,y) = X(x)\cdot Y(y)}$

$\cancel{T_{xx}(x,y) = X''(x)\cdot Y(y)}$

$\cancel{T_{yy}(x,y) = X(x)\cdot Y''(y)}$

$\cancel{X''(x)Y(y) + Y''(y)X(x) = 0}$

$\cancel{\frac{X''(x)}{X(x)} = -\frac{Y''(y)}{Y(y)} = \alpha}$

~~Como el miembro izquierdo depende solo de x y el derecho solo de y y son iguales ⇒ deben ser igual a una constante~~

$\cancel{X''(x) - \alpha X(x) = 0}$

$\cancel{X(1) = 0}$ $\cancel{X(1) = 15}$

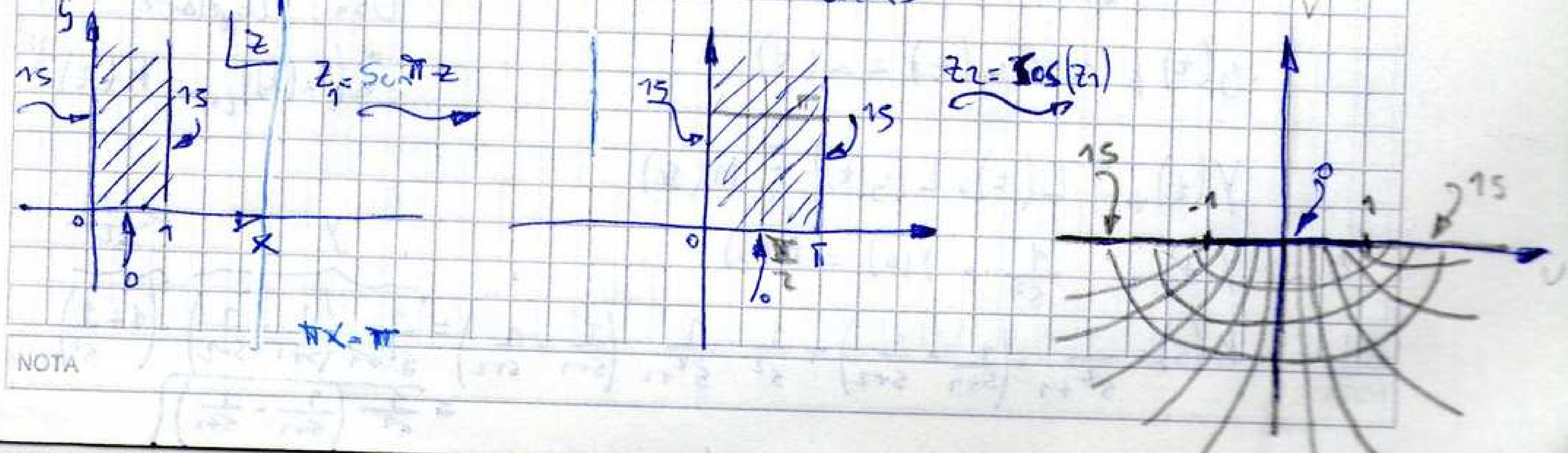
$\cancel{X(0) = 15}$ $\cancel{X(0) = 0}$

$\cancel{Y''(y) + \alpha Y(y) = 0}$

$\cancel{Y(0) = 0}$

~~Tengo dos condiciones iniciales no homogé... divido el problema 2~~

Resolvemos por Transformaciones Conformes



$\cos(z_1) = \cos(x_1)\cdot\cosh(y_1) - i\,\text{sen}(x_1)\cdot\text{senh}(y_1)$

$x_1 = \frac{\pi}{2}$ $\quad -i\,\text{senh}(y_1) \le 0 \quad y_1 \geq 0$

$x_1 = \pi$ $\quad -\cosh(y_1) \le -1$

$x_1 = 0$ $\quad \cosh(y_1) \geq 1$

$y_1 = 0$

$0 \le x_1 \le \pi$

$\cos(x_1)$ entre $[-1, 1]$

$0 < a < \frac{\pi}{2}$

$x_1 = a \quad \overbrace{\cos(a)\cdot\cosh(y_1)}^{(0,1)} - i\,\overbrace{\text{sen}(a)\cdot\text{senh}(y_1)}^{(0,1)}$

$\frac{U^2}{\cos^2(a)} = \cosh^2(y_1) \qquad \frac{V^2}{\text{sen}^2(a)} = \text{senh}^2(y_1)$

Resto

$\frac{U^2}{\cos^2(a)} - \frac{V^2}{\text{sen}^2(a)} = 1$

$\frac{\pi}{2} < b < \pi$

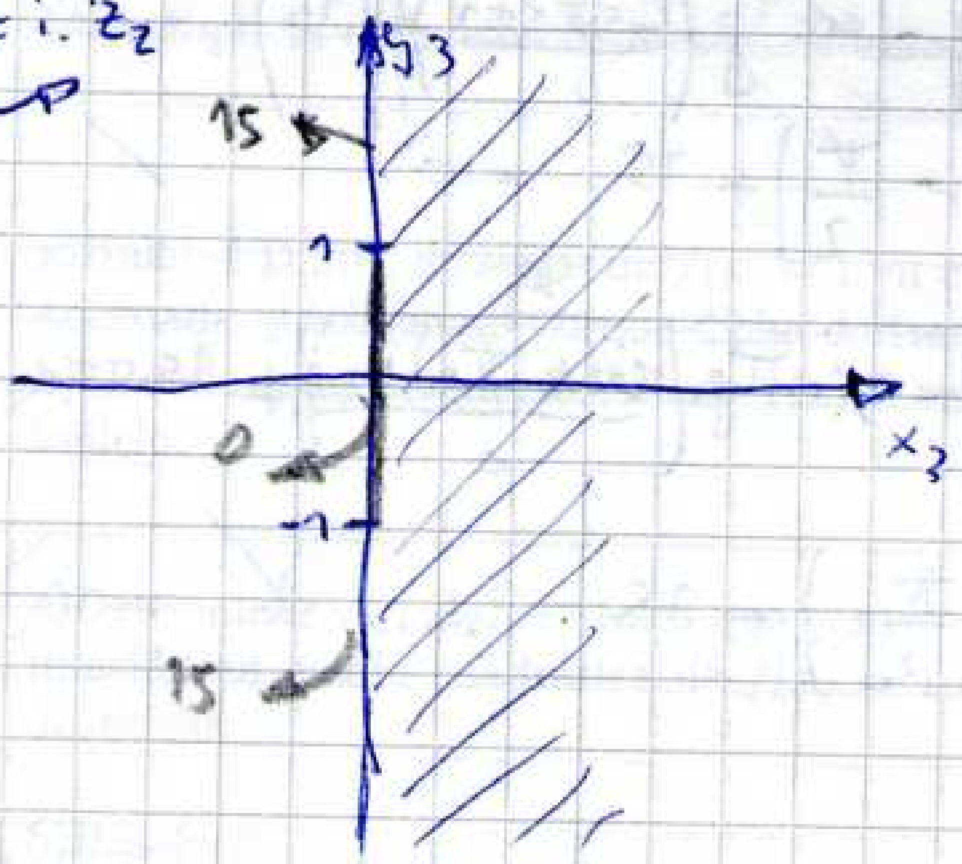
$x_1 = b \quad \overbrace{\cos(b)}^{(-1,0)}\cdot\cosh(y_1) - i\,\overbrace{\text{sen}(b)\cdot\text{senh}(y_1)}^{(0,1)}$

$U < 0$

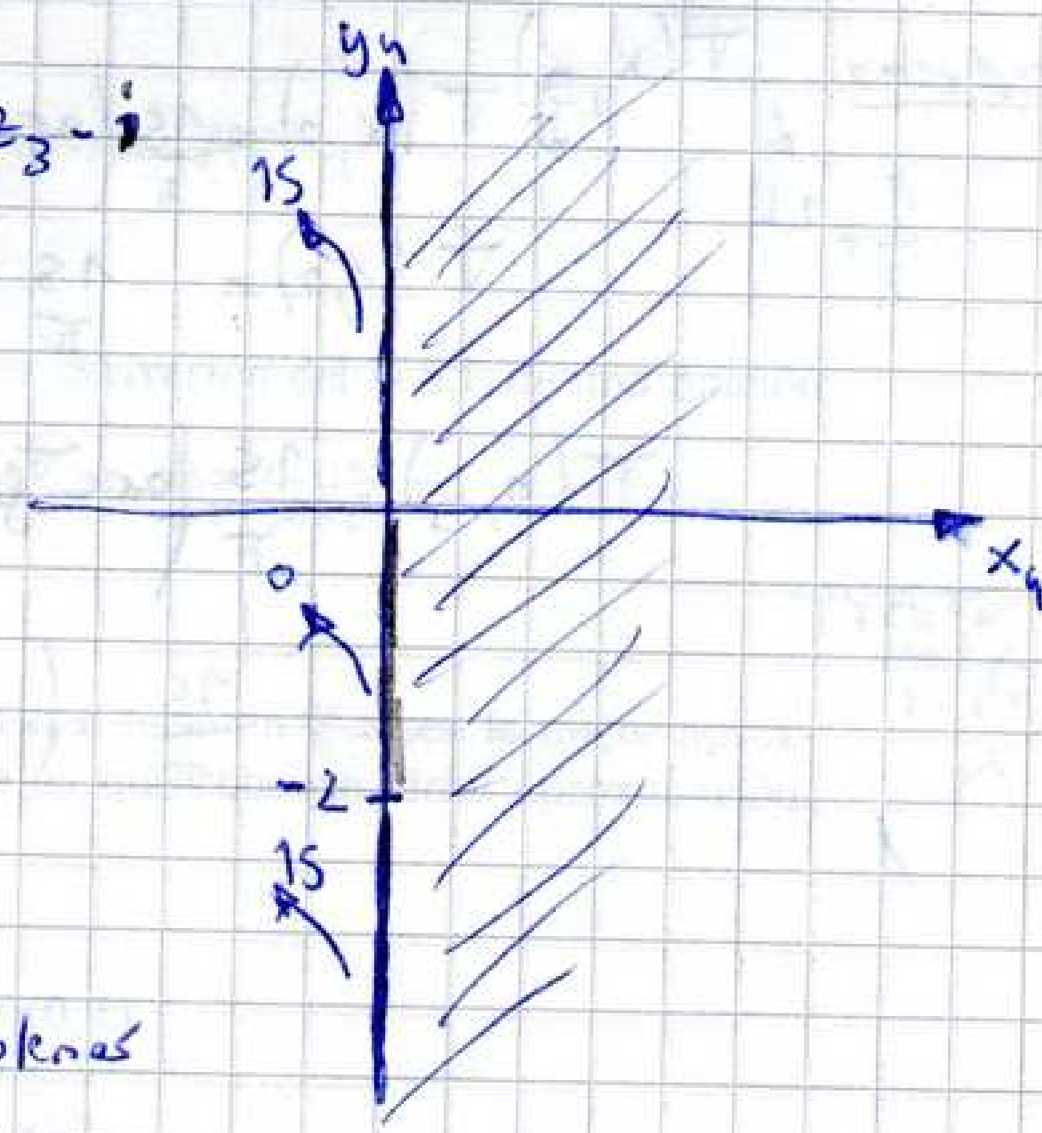
$V < 0$

$\frac{U^2}{\cos^2(b)} - \frac{V^2}{\text{sen}^2(b)} = 1$

$z_3 = i\cdot z_2$

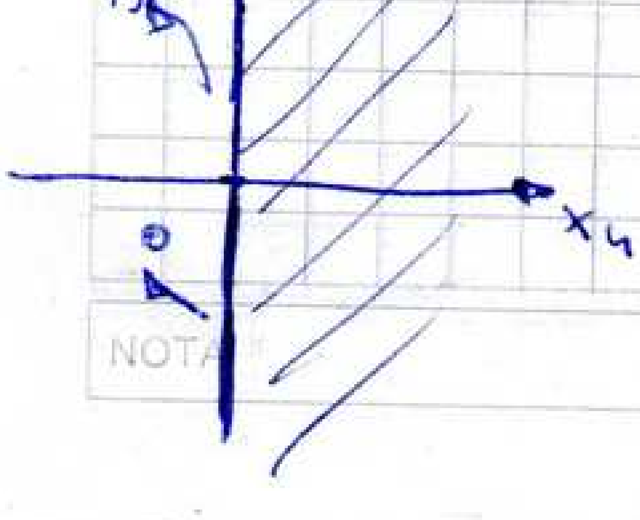


$z_4 = z_3 - i$



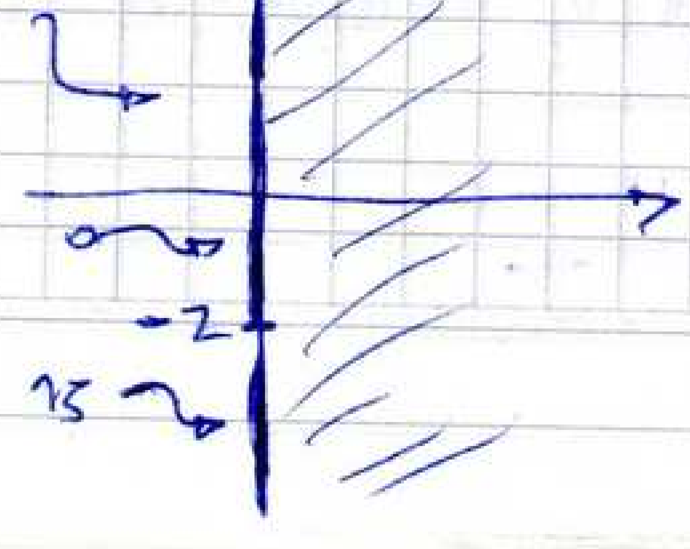
Resuelvo como la superposición de 2 problemas

①



$t = t_1 + t_2$

②



t

$t_1''(\theta)=0$

① $t_1(\theta)=A\theta+B$

$t_1\left(-\frac{\pi}{2}\right)=-\frac{\pi}{2}A+B=0$

$B=\frac{\pi}{2}A$

$t_1\left(\frac{\pi}{2}\right)=\frac{\pi}{2}A+B=15$

$2\pi A=15$

$\boxed{A=\frac{15}{2\pi}}$

$\boxed{B=\frac{15}{2}}$

$t_1(\theta)=\frac{15}{2\pi}\theta+\frac{15}{2}$

$t_1(z_4)=\frac{15}{2\pi}\cdot arcTg\left(\frac{Im(z_4)}{Re(z_4)}\right)+\frac{15}{2}$

$$\boxed{t_1(x,y)=\frac{15}{2\pi}\cdot arcTg\left(\frac{\cos(\pi x).\cosh(\pi y)-1}{Sen(\pi x).Senh(\pi y)}\right)+\frac{15}{2}}$$

$z_4=z_3-i=iz_2-i=i\cos(z_1)-i=i\cos(\pi z)$

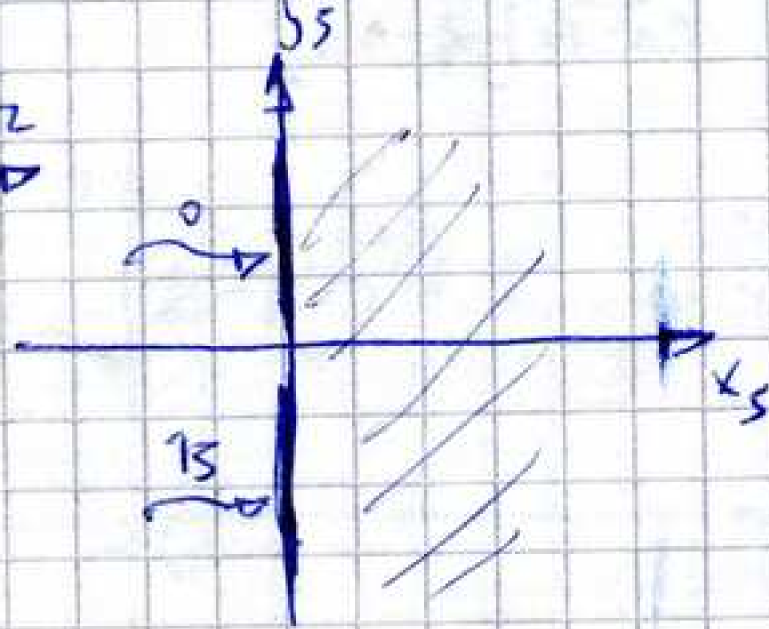
$z_4=i\cos(\pi x+i\pi y)-i=i(\cos\pi x.\cosh(\pi y)-i$

$Im(z_4)=\cos(\pi x).\cosh(\pi y)$

$Re(z_4)=Sen(\pi x).Sen(\pi y)$

②

$z_5=z_4+2$



$t_2(\theta)=A\theta+B$

$t_2\left(\frac{\pi}{2}\right)=A\frac{\pi}{2}+B=0$ $\quad \boxed{B=-A\frac{\pi}{2}=\frac{15}{2}}$

$t_2\left(-\frac{\pi}{2}\right)=-A\frac{\pi}{2}+\left(-A\frac{\pi}{2}\right)=-2A\frac{\pi}{2}=15$

$\boxed{A=-\frac{15}{2\pi}}$

$t_2(\theta)=-\frac{15}{2\pi}\theta+\frac{15}{2}$

$t_2(z_5)=-\frac{15}{2\pi}\, arcTg\left(\frac{Im(z_5)}{Re(z_5)}\right)+\frac{15}{2}$

$$\boxed{t_2(x,y)=-\frac{15}{2\pi}\cdot arcTg\left(\frac{\cos(\pi x).\cosh(\pi y)+1}{Sen(\pi x).Senh(\pi y)}\right)+\frac{15}{2}}$$

$z_5=z_4+2i=z_3+i=i$

$=i\cos z_1+i=i\cos(\pi$

$\Rightarrow Im(z_5)=\cos(\pi x).\cosh$

$Re(z_5)=Sen(\pi x).Senh(\pi y)$

$$\Rightarrow \boxed{T(x,y)=\frac{15}{2\pi}\left(arcTg\left(\frac{\cos(\pi x)\cosh(\pi y)-1}{Sen(\pi x).Senh(\pi y)}\right)-arcTg\left(\frac{\cos(\pi x)\cosh(\pi y)+1}{Sen(\pi x).Senh(\pi y)}\right)\right)+15}$$

Compruebo

$y=0 \quad x=\frac{1}{2}$

$\frac{1}{2}+2k$

$T(x,0)=\frac{15}{\pi}\left(arcTg\left(\frac{\overset{(0,1)}{\cos\pi x-1}}{0}\right)-arcTg\left(\frac{\overset{(0,1)}{\cos\pi x+1}}{0}\right)\right)+15$

$T(x,0)=\frac{15}{\pi}\left(-\frac{\pi}{2}-\frac{\pi}{2}\right)+15=0$ ✓

$T(0,y)=\frac{15}{\pi}\left(arcTg\left(\frac{\overset{>1}{\cosh(\pi y)-1}}{0}\right)-arcTg\left(\frac{\overset{>1}{\cosh(\pi y)+1}}{0}\right)\right)+15=$

$\frac{15}{\pi}\left(\frac{\pi}{2}-\frac{\pi}{2}\right)+15=0$ ✓

NOTA